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How to Convert Correlated Seismic Failures into Seismic CCFs

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1. Introduction

One of important basic assumptions in the fault tree analysis (FTA) is that all the component failures are independent. The failure probability of a fault tree or minimal cut sets (MCSs) is calculated based on this assumption.

For a seismic PSA, the dependency among seismic failures is not explicitly modeled. This dependency is separately assigned with numbers ranging from zero to unity for reflecting mutual correlation level among seismic failures. The determination and calculation of the mutual correlation level among seismic failures is an important issue in a seismic PSA.

Because of complexity and difficulty in calculating combination probabilities of correlated seismic failures, there has been a great need of development to explicitly model seismic correlation in terms of seismic common cause failures (CCFs). If seismic correlation can be converted into seismic CCFs, it is possible to obtain an accurate value of a top event probability or frequency of a complex seismic fault tree by using the same procedure as for internal PSA.

The paper[1] proposes a methodology to model explicitly dependency among seismic failure by converting correlated seismic failures into seismic CCFs. This method was implemented into a new tool COREX (CORelation EXplicit). A detailed discussion of this methodology is provided in Section 3.

Notations and definitions of failures and probabilities used in this study are listed below.

$X_i = \text{Failure of component i}$
$X_{ij} = X_i X_j = \text{Joint failures of components i and j}$
$C_i = \text{Failure of componet i excluding CCFs}$
$C_{ij} = \text{CCF of components i and j}$
$X_i = C_i + C_{ij} + C_{ik} + C_{ijk} + \cdots$
$P_i = P(X_i)$
$P_{ij} = P(X_{ij})$
$Q_i = P(C_i)$
$Q_{ij} = P(C_{ij})$

2. Seismic PSA

The seismic fragility of a component is defined as the conditional probability of its failure at a given value of peak ground acceleration [2]. The entire fragility family for a component corresponding to a particular failure mode can be expressed in terms of the best estimate of the median ground acceleration capacity and two random variables. Therefore, the ground acceleration capacity of a component is given by

$$A = A_m \varepsilon_R \varepsilon_U \quad (1)$$

where A_m is a median ground acceleration capacity, ε_R and ε_U are random variables with unit medians, representing, respectively, aleatory (randomness) uncertainty of A_m, and epistemic (modelling) uncertainty of A_m. In this model, these two random variables ε_R and ε_U are assumed to be lognormally distributed with logarithmic standard deviations, β_R and β_U, respectively.

A conditional failure probability of a component at any ground acceleration a is represented by the following equations.

$$P(a) = P(A < a) = P(\ln(A/Am) < \ln(a/Am)) \quad (2)$$

$$P(a) = \Phi\left(\frac{\ln(a/Am) + \beta_U \Phi^{-1}(Q)}{\beta_R}\right) \quad (3)$$

Here, $\Phi(\ )$ is a standard normal cumulative distribution function, and $\Phi^{-1}(\ )$ is a standard normal cumulative distribution inverse function. Q is subjective probability or confidence with a value ranging from 0 to 1.

If some component failures are simultaneously affected by a seismic event, it is defined that seismic failures of these components are correlated.

For a seismic PSA, dependency among seismic failures of components is not explicitly modeled. Instead of explicit modeling, the dependency is separately identified and assigned an approximate number ranging from zero to unity by taking into account the mutual correlation among seismic failures.

Some integration methods to implicitly calculate simple AND/OR combination probabilities of correlated seismic failures were already developed[2-5]. Simple combination probability of correlated seismic failures such as $P(X_i X_j X_k)$ or $P(X_i + X_j + X_k)$ is calculated by Multi-Variate Normal (MVN) or Reed-McCann integration method as discussed in NUREG/CR-7237[3].

3. Conversion Method of Correlated Seismic Failures into Seismic CCFs

3.1 Conversion Procedure

As shown in Fig. 1, the conversion of correlated seismic failures into seismic CCFs is performed. Since correlations among X_1, X_2, and X_3 are explicitly converted and modeled with seismic CCFs, the seismic

PSA model can be solved in the same manner as solved for internal PSA model.

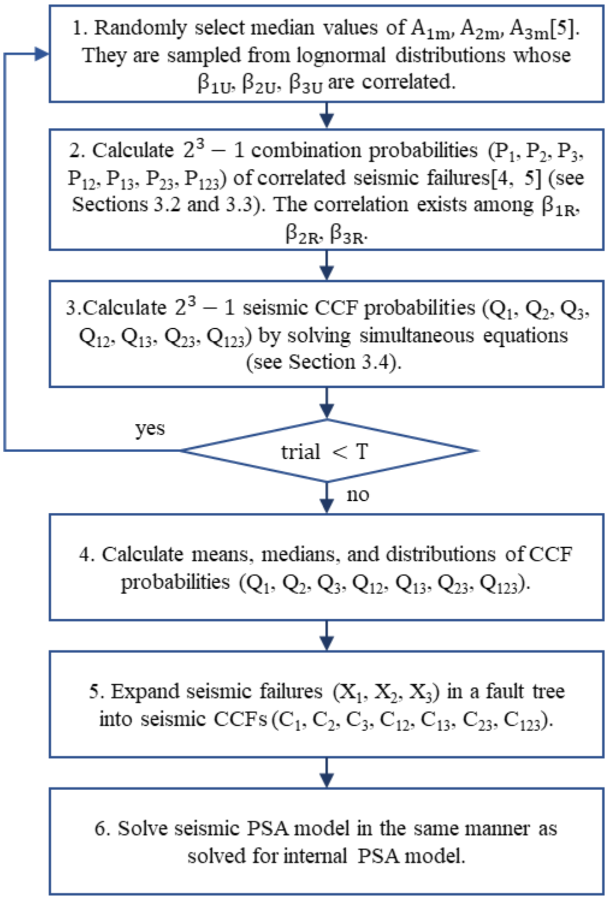

Fig. 1. Conversion procedure

3.2 MVN Integration

Combination probability of seismic failures of $P_{12\ldots n}(a) = P(\cap_{i=1}^{n} A_i < a)$ is calculated by using Monte-Carlo integration of multivariate normal (MVN) distribution[4].

$$P_{12\ldots n}(a) = \int_{-\infty}^{\ln\left(\frac{a}{A_{1m}}\right)} \int_{-\infty}^{\ln\left(\frac{a}{A_{2m}}\right)} \ldots \int_{-\infty}^{\ln\left(\frac{a}{A_{nm}}\right)} \frac{1}{\sqrt{|\Sigma|(2\pi)^n}} \exp\left(-\frac{1}{2}x^t\Sigma^{-1}x\right) dx_1 dx_2 \ldots dx_n \quad (4)$$

Where, $x^t = [x_1 \quad x_2 \quad \ldots \quad x_n]$. Σ is a symmetric positive definite covariance matrix as shown below.

$$\Sigma = \begin{bmatrix} \beta_1^2 & \beta_{12}^2 & \ldots & \beta_{1n}^2 \\ \beta_{21}^2 & \beta_2^2 & \ldots & \beta_{2n}^2 \\ \ldots & \ldots & \ldots & \ldots \\ \beta_{n1}^2 & \beta_{n2}^2 & \ldots & \beta_n^2 \end{bmatrix}, \beta_{ij}^2 = cov(X_I, X_j) \quad (5)$$

(6)

Where, $|\Sigma|$ is a determinant of Σ, and Σ^{-1} is an inverse matrix of Σ.

If x_i is replaced with $\beta_i z_i$ as $x_i = \beta_i z_i$, Eq. (4) can be converted into

$$P_{12\ldots n}(a) = \int_{-\infty}^{\frac{\ln(a/A_{1m})}{\beta_1}} \int_{-\infty}^{\frac{\ln(a/A_{2m})}{\beta_2}} \ldots \int_{-\infty}^{\frac{\ln(a/A_{nm})}{\beta_n}} \frac{1}{\sqrt{|\Sigma_\rho|(2\pi)^n}} \exp\left(-\frac{1}{2}z^t\Sigma_\rho^{-1}z\right) dz_1 dz_2 \ldots dz_n \quad (7)$$

Here, $z^T = [z_1 \quad z_2 \quad \ldots \quad z_n]$. Σ_ρ is a symmetric positive definite correlation matrix as shown below.

$$\Sigma_\rho = \begin{bmatrix} 1 & \rho_{12} & \ldots & \rho_{1n} \\ \rho_{21} & 1 & \ldots & \rho_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ \rho_{n1} & \rho_{n2} & \ldots & 1 \end{bmatrix}, \rho_{ij} = \frac{\beta_{ij}^2}{\beta_i\beta_j} \quad (8)$$

3.3 Reed-McCann Integration

As an alternate method of MVN integration in Section 3.2, $P_{12\ldots n}(a) = P(\cap_{i=1}^{n} A_i < a)$ can be calculated with Reed-McCann integration[5]. Reed-McCann integration is essentially identical to MVN integration.

$$P_{12\ldots n}(a) = \int_0^\infty \ldots \int_0^\infty f_{12\ldots n}(x) dx_{12} \ldots dx_{(n-1)n} \quad (9)$$

$$f_{12\ldots n}(x) = g_1(x) g_2(x) \ldots g_n(x) \quad (10)$$

$$g_i(x) = \Phi\left(\frac{\ln\left(\frac{a}{A_{im}\prod_{j\neq i} x_{ij}}\right)}{\beta_i^-}\right) \quad (11)$$

Here, $x_{ij} = x_{ji}$, $\beta_i^- = \sqrt{\beta_i^2 - \sum_{j=1, j\neq i}^{n} \beta_{ij}^2}$, and $\beta_{ij} = \beta_{ji}$. $\Phi(\ )$ is a standard normal cumulative distribution function.

If two failures are correlated, $P_{12}(a)$ is calculated as

$$P_{12}(a) = \int_0^\infty f(x) g(x) dx_{12} \quad (12)$$

$$f(x) = \Phi\left(\frac{\ln\left(\frac{a}{A_{1m}x_{12}}\right)}{\beta_1^-}\right) \Phi\left(\frac{\ln\left(\frac{a}{A_{2m}x_{12}}\right)}{\beta_2^-}\right) \quad (13)$$

$$g(x) = \varphi\left(\frac{\ln x_{12}}{\beta_{12}}\right) \frac{1}{\beta_{12}x_{12}} \quad (14)$$

Here, $\Phi(\ )$ is a standard normal cumulative distribution function, and $\varphi(\ )$ is a standard normal probability density function.

If three failures are correlated, and then $P_{123}(a)$ is calculated with equations below.

$$P_{123}(a) = \int_0^\infty \int_0^\infty \int_0^\infty f(x) g(x) dx_{12} dx_{13} dx_{23} \quad (15)$$

$$f(x) = \Phi\left(\frac{\ln\left(\frac{a}{A_{1m}x_{12}x_{13}}\right)}{\beta_1^-}\right) \Phi\left(\frac{\ln\left(\frac{a}{A_{2m}x_{12}x_{23}}\right)}{\beta_2^-}\right) \Phi\left(\frac{\ln\left(\frac{a}{A_{3m}x_{13}x_{23}}\right)}{\beta_3^-}\right) \quad (16)$$

$$g(x) = \varphi\left(\frac{\ln x_{12}}{\beta_{12}}\right) \frac{1}{\beta_{12}x_{12}} \varphi\left(\frac{\ln x_{13}}{\beta_{13}}\right) \frac{1}{\beta_{13}x_{13}} \varphi\left(\frac{\ln x_{23}}{\beta_{23}}\right) \frac{1}{\beta_{23}x_{23}} \quad (17)$$

$$\beta_1^- = \sqrt{\beta_1^2 - (\beta_{12}^2 + \beta_{13}^2)} \quad (18)$$

$$\beta_2^- = \sqrt{\beta_2^2 - (\beta_{12}^2 + \beta_{23}^2)} \quad (19)$$

$$\beta_3^- = \sqrt{\beta_3^2 - (\beta_{13}^2 + \beta_{23}^2)} \quad (20)$$

3.4 Seismic Failures into Seismic CCFs

A detailed discussion on the conversion process of correlated seismic failures into seismic CCFs is presented below with a case having three seismic failures correlated:

1. Calculate combination probabilities of correlated seismic failures; by using either MVN or Reed-McCann integration, $2^3 - 1$ combination probabilities (P_1, P_2, P_3, P_{12}, P_{13}, P_{23}, P_{123}) is calculated.

2. Construct MCUB or REA probability equations that consist of $2^3 - 1$ seismic CCF probabilities (Q_1, Q_2, Q_3, Q_{12}, Q_{13}, Q_{23}, Q_{123}) for $2^3 - 1$ combination probabilities (P_1, P_2, P_3, P_{12}, P_{13}, P_{23}, P_{123}).

$$\begin{aligned}
P_1 &= Q_1 + Q_{12} + Q_{13} + Q_{123} \\
P_2 &= Q_2 + Q_{12} + Q_{23} + Q_{123} \\
P_3 &= Q_3 + Q_{13} + Q_{23} + Q_{123} \\
P_{12} &= Q_1Q_2 + Q_{12} + Q_{123} + Q_1Q_{23} + Q_2Q_{13} + Q_{13}Q_{23} \\
P_{13} &= Q_1Q_3 + Q_{13} + Q_{123} + Q_1Q_{23} + Q_3Q_{12} + Q_{12}Q_{23} \\
P_{23} &= Q_2Q_3 + Q_{23} + Q_{123} + Q_2Q_{13} + Q_3Q_{12} + Q_{12}Q_{13} \\
P_{123} &= Q_1Q_2Q_3 + Q_{123} + Q_1Q_{23} + Q_2Q_{13} + Q_3Q_{12} + \\
&\quad Q_{12}Q_{13} + Q_{13}Q_{23} + Q_{12}Q_{23}
\end{aligned} \tag{21}$$

$$\begin{aligned}
P_1 &= 1 - (1 - Q_1)(1 - Q_{12})(1 - Q_{13})(1 - Q_{123}) \\
P_2 &= 1 - (1 - Q_2)(1 - Q_{12})(1 - Q_{23})(1 - Q_{123}) \\
P_3 &= 1 - (1 - Q_3)(1 - Q_{13})(1 - Q_{23})(1 - Q_{123}) \\
P_{12} &= 1 - (1 - Q_1Q_2)(1 - Q_{12})(1 - Q_{123}) \\
&\quad (1 - Q_1Q_{23})(1 - Q_2Q_{13})(1 - Q_{13}Q_{23}) \\
P_{13} &= 1 - (1 - Q_1Q_3)(1 - Q_{13})(1 - Q_{123}) \\
&\quad (1 - Q_1Q_{23})(1 - Q_3Q_{12})(1 - Q_{12}Q_{23}) \\
P_{23} &= 1 - (1 - Q_2Q_3)(1 - Q_{23})(1 - Q_{123}) \\
&\quad (1 - Q_2Q_{13})(1 - Q_3Q_{12})(1 - Q_{12}Q_{13}) \\
P_{123} &= 1 - (1 - Q_1Q_2Q_3)(1 - Q_{123})(1 - Q_1Q_{23}) \\
&\quad (1 - Q_2Q_{13})(1 - Q_3Q_{12})(1 - Q_{12}Q_{13}) \\
&\quad (1 - Q_{13}Q_{23})(1 - Q_{12}Q_{23})
\end{aligned} \tag{22}$$

3. Calculate $2^3 - 1$ CCF probabilities (Q_1, Q_2, Q_3, Q_{12}, Q_{13}, Q_{23}, Q_{123}) by solving nonlinear simultaneous equations in Eq. (21) or (22).

4. Expand seismic failures (X_1, X_2, X_3) in a fault tree into seismic CCFs.

$$\begin{aligned}
X_1 &= C_1 + C_{12} + C_{13} + C_{123} \\
X_2 &= C_2 + C_{12} + C_{23} + C_{123} \\
X_3 &= C_3 + C_{13} + C_{23} + C_{123}
\end{aligned} \tag{23}$$

4. Conclusions

There has long been a great need of development to explicitly model seismic correlation with seismic CCFs. Existing methodologies such as MVN and Reed-McCann methods have some limitation in use for seismic PRA application. This study proposes a streamlined methodology to explicitly model dependency among seismic failures by converting correlated seismic failures into seismic CCFs. Furthermore, this method was implemented into a new tool COREX(CORelation EXplicit).

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