Copula Study for Common Cause Failures under Asymmetric Conditions

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1. Introduction

Common Cause Failure (CCF) is a failure that significantly affects system reliability with redundant components. In general, Probabilistic Safety Assessment (PSA) considers the CCF using an alpha factor model. Several advantages of these parametric models are that it is easy to use and makes the estimation of the parameter easier [1][2].

These parametric models require the parameters to estimate the CCF probabilities based on the symmetry assumptions. Symmetry in estimating the CCF parameters means that the basic event probabilities of n failures are identical in a Common Cause Component Group (CCCG) consisting of n components. This assumption has an advantage in reducing the number of required parameters to estimate CCF probabilities. However, it is difficult to be used for the asymmetric conditions such as the case when components in a CCCG have different operation mode or different total failure probability due to degradation or partial dependency between specific components. The researches [2][3] have mentioned asymmetrical CCF such as the case that different operation mode or similar components in a CCCG and proposed the way to address this by adding the asymmetrical basic events into the existing fault tree or formulating the related equations approximately. In case of asymmetry in total failure probability due to degradation, the simple assumption that degradations do not affect CCFs have been suggested to consider an asymmetry in defendant failures [1].

In this paper, we attempted to address asymmetric CCF in terms of the joint probability distributions, not parametric approach. For this purpose the copula method that combines marginal distributions using their parameter to estimate joint failure probability was used. Copula is widely used to model the dependency between random variables in financial and reliability engineering [4][5]. This approach has advantages in estimating joint failure probability with different or asymmetric marginal distributions. In other words referring the technical words used in PSA, it is suitable for describing the components in a CCCG that have different total failure probabilities or partial dependency.

In order to build a copula structure, the type of copula and its parameter should be determined. However, original failure data to fit copula distribution is not available and alpha factors are the only available data for dependent failure practically. Therefore this paper focused on the development of the copula structure using alpha factors based on typical copulas to address asymmetric CCF.

2. Methods

2.1 Common Cause Failure

Figure 1 shows the fault tree of common cause basic events that have one-out-of-two success logic based on a CCF model.

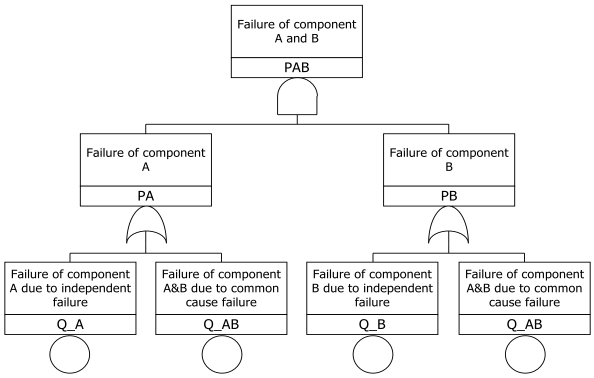

Fig. 1. Example of common cause basic events for CCCG 2

Using the BPM and alpha factor model based on the symmetry assumption, each basic event probability (for a non-staggered testing strategy) in figure 1 can be obtained from Eq. (1), (2), (3), respectively.

$$Q_1^A = Q_1^B = Q_1 = \frac{\alpha_1}{\alpha_T} Q_T \quad (1)$$

$$Q_2^{AB} = Q_2 = 2\frac{\alpha_2}{\alpha_T} Q_T \quad (2)$$

$$Q_T = Q_T^A = Q_T^B = Q_1 + Q_2 \quad (3)$$

Where, Q_1^A is independent failure of component A and Q_2^{AB} is common cause failure of component A and B. Q_T is total failure probability of an component and α_k is alpha factor for k failures and α_T is summation of α_k. As shown in Eq. (1), existing parametric CCF have been assumed to have same basic event probability for k failures. This assumption results in reducing parameters to be quantified.

However, this symmetry assumption makes the estimation of dependent failure probability difficult when a specific component may be degraded due to harsh environment (i.e. different total failure probability) or the component, which have partial dependency with specific one in high-redundant system, is under asymmetric conditions.

2.2 Copula

Copula developed by Sklar is a dependency structure between random variables, and it can model a multivariate joint distribution using marginal distributions and copula parameters. For a random variable x, if their joint and marginal cumulative distribution is $\mathrm{F}(x_1, x_2, \ldots, x_k)$, $\mathrm{F}(x_1) = u_1$, respectively, then copula (*C*) can be written in Eq. (4).

$$\mathrm{F}(x_1, x_2, \ldots, x_k) = \mathrm{C}(\mathrm{F}_1(x_1), \mathrm{F}_2(x_2), \ldots, F_k(x_k)) \quad (4)$$

For example, the normal copula is a multivariate distribution of uniform random variable *u* as follows:

$$\mathrm{C}(u_1, u_2, \ldots, u_k) = \mathrm{F}(F_1^{-1}(u_1), F_2^{-1}(u_2), \ldots F_k^{-1}(u_k)) \quad (5)$$

In case of a multivariate normal distribution, it consists of normal marginal distribution and normal copula distribution with a copula parameter.

Typical types of copula are normal and t-copula, which belong to the elliptical copula family. In Archimedean copula family, Frank copula, Gumbel copula and Clayton copula is widely used to model dependency because they need only a single parameter to build the copulas. Each copula needs a copula parameter. For example, Pearson's correlation coefficient is a kind of copula parameter in the normal copula. In Archimedean copula, they need other types of correlation coefficients such as Kendall's tau to evaluate the copula parameter (θ).

Figure 2 shows the example of copula distribution at a specific copula parameter. As shown in figure 2, normal and Frank copula have symmetric distribution, while Gumbel and Clayton have strong dependency at the one side of distribution.

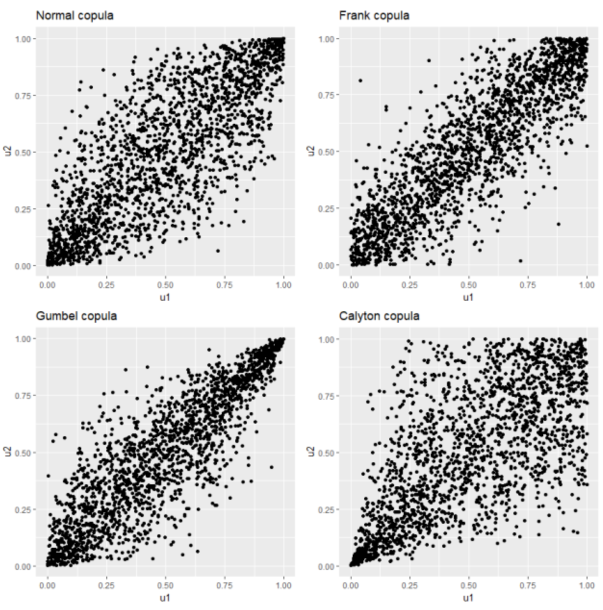

Fig. 2 Examples of Copula distributions for random number u_1, u_2

2.3 CCF modeling based on copula from alpha factor

In order to construct a joint probability distribution using copula, we need information for marginal distribution, copula distribution and copula parameter. Marginal distribution can be assumed as the exponential distribution for running failure or the binomial distribution for demand failure. In this paper, we have assumed that the marginal is an exponential distribution with failure rate given in NUREG/CR-6928 [6].

Copula and copula parameter should be determined by fitting to failure data. In other words, dependent failure data are required, however, it is not available in practice. In addition, the plant or component specific CCF data are not easy to be obtained because it is a rare event. Therefore, the existing alpha factor given in NUREG/CR-5497 was the only available data for dependent failure in this study.

This paper has dealt with typical 4 copulas introduced in section 1 and has developed the copula parameter obtained indirectly from alpha factors. The research on the relationship between CCF and correlation coefficients has been carried out in terms of seismic correlation studies [7]. In this study, system unreliability from Eq. (6) becomes joint failure probability in Eq. (7).

$$P_{sys}^{\alpha} = Q_1^2 + Q_2 = (\frac{\alpha_1}{\alpha_T} Q_T)^2 + 2\frac{\alpha_2}{\alpha_T} Q_T \quad (6)$$

In figure 1, the system unreliability (P_{sys}^{α}) in one-out-of-two success logic can be calculated in Eq. (6) using the given total failure probability and alpha factor under symmetric condition.

$$P_{sys}^{\theta} = \mathrm{F}(x_1, x_2) = P(x_1 < T, x_2 < T) \quad (7)$$

In terms of joint probability distribution, joint failure probability based on copula parameter (θ), P_{sys}^{θ} can be evaluated using Eq. (7).

Thus, if the joint failure probability, P_{sys}^{θ} at certain copula and parameter equals to P_{sys}^{α} from Eq. (6), then this copula parameter can be used to construct joint the probability distribution under asymmetric conditions.

2.4 Asymmetric CCF

In this section, asymmetric problems where the total probability of failure is different ($Q_T^A \neq Q_T^B$) have been addressed. It should be noted that the copula parameter calculated from Eq. (6) and (7) does not change even if total failure rates become changed. This assumption was driven or can be shared from the fact that the alpha factor does not change even if the total probability of failure changes.

Copula parameter from Eq. (6) and (7) are used to establish joint distribution under asymmetric conditions. Eq. (8), (9), (10) represents the failure probability of component A, B and system unreliability without a symmetry assumption.

$$P(A < T) = Q_T^A = Q_1^A + Q_2^{AB} \quad (8)$$

$$P(B < T) = Q_T^B = Q_1^B + Q_2^{AB} \quad (9)$$

$$P_{sys}^{asy} = P(A < T, B < T) = Q_1^A Q_1^B + Q_2^{AB} \quad (10)$$

The left side of Eq. (8), (9), (10) can be obtained from the joint distribution using the copula parameter. When CCCG = n, $2^n - 1$ system of equations are generated. Figure 3 shows the procedure for constructing the joint distribution using copula parameter from alpha factors and evaluating asymmetric CCFs.

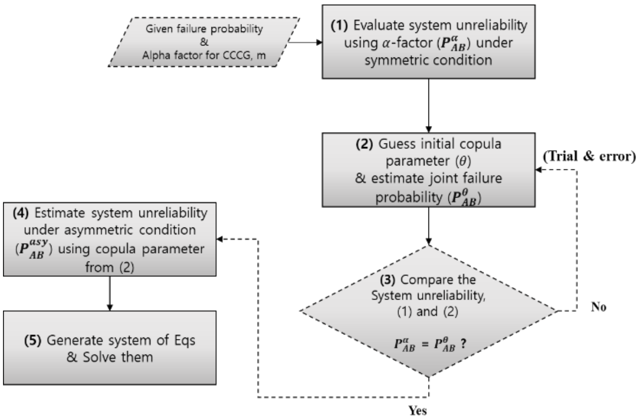

Fig. 3. Procedures for estimating joint failure probability of asymmetric CCF

3. Results

The asymmetric condition that failure rate of the one component increase due to degradation was assumed to study how copula works effectively. Table I includes failure rate of Emergency Diesel Generator (EDG) and alpha factor for CCCG=2 [6][8]. In addition, P_{sys}^{α} in Eq. (6) was calculated.

Table I: EDG failure rate and alpha factor

		Reliability data
Total failure rate (λ)		8.35E-04/hr
Mission Time (T)		24hrs
Alpha Factor	α_1	0.984
	α_2	0.0157
P_{sys}^{α} (from Eq.6)		9.83E-04

As mentioned in section 2.3 and 2.4, copula parameter (θ), which meets $P_{sys}^{\alpha} = P_{sys}^{\theta}$, can be found by optimization method as shown in figure 3. Table II shows each copula parameter depending on the type of copula.

Table II: Copula parameters (θ) that meet $P_{sys}^{\alpha} = P_{sys}^{\theta}$

Copula type	Copula parameter
Normal copula	0.1778
Frank copula	2.3691
Gumbel copula	1.2181
Clayton copula	0.0772

Figure 4 shows how P_{sys}^{asy} changes as the total failure probability of the component B (Q_T^B) increase using the copula parameter derived from Table 2. System unreliability (alpha factor in figure 4) that counts the increase of failure rate of component both A and B under symmetric condition was calculated to compare results from asymmetric condition using copula.

On the other hand, the joint failure probability (P_{sys}^{asy}) was calculated depending on copula under asymmetric condition that the failure rate of component A was fixed (8.35E-04/hr) and failure rate of component B increased by 1.0E-04/hr.

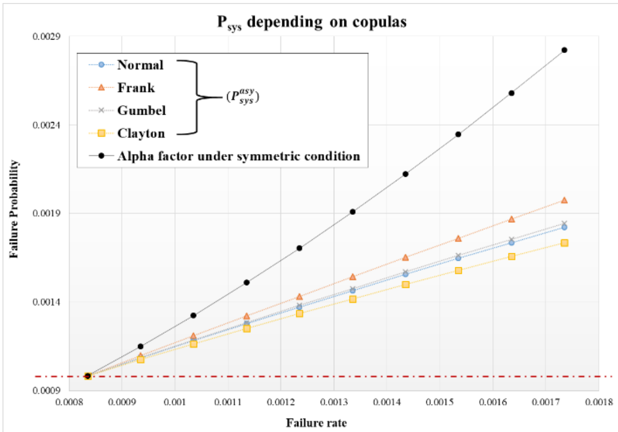

Fig. 4. P_{sys}^{asy} depending on copulas and failure rate

As shown in figure 4, the existing alpha factor model does not consider the difference in the failure rate of the components. Thus, all the components have an increased failure rate. This results in significantly conservative system unreliability. On the other hand, copula considers the difference in the failure rate of the components under asymmetric condition. P_{sys}^{asy} is located between the black (considering the increased failure of both components) and the red line (without considering the increased failures) in figure 4.

4. Conclusions

In this paper, the asymmetric CCF where the total failure probability is different has been described in terms of joint probability distributions. It turned out that copula is suitable for estimating joint failure probability under asymmetric conditions.

However, the copula approach mentioned in this paper has only one parameter to construct joint probability distribution. In other words, it is not proper in highly redundant systems to express all combinations of dependency between components by only a single parameter. Therefore, vine copula [9] can be used to consider partial dependency as future works.

In practice, this approach may not be valid in a typical PSA, but it can be applicable to specific reliability analysis problems, which is on-going through the study.

Acknowledgments

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