Single Bubble Simulation in Linear Shear Flow Using Smoothed Particle Hydrodynamics: Preliminary Results

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1. Introduction

Bubbly flows play an important role in light water reactor because it has large interfacial areas for heat and mass transfer. In bubbly flows, lift force greatly affects bubbles distribution, it is one of important forces. Therefore, lift force on bubbles in shear fields is studied intensively.

Various techniques have been developed in Eulerian method to capture multiphase interface. However, Level Set (LS) method and Volume of Fluid (VOF) method have problems in mass conservation or numerical merging of bubble [1]. In Lagrangian method, due to its meshless nature, multiphase interface can be captured uncomplicated and accurately. Smoothed Particle Hydrodynamics (SPH) is a one of the Lagrangian-based computational method used for simulation of fluid flows.

In this study, the motion of the bubble in linear shear flow was simulated and compared with the results of previous experimental studies. The SOPHIA code based on SPH method was used for analysis which was developed by Seoul National University. The SOPHIA code has been developed and parallelized using CUDA C language

A general Weakly Compressible Smoothed Particle Hydrodynamics (WCSPH) technique is utilized but density and continuity equations were re-formulated in terms of normalized density in order to handle multiphase flow. Interface sharpness force is considered to stabilize the interface with pressure, gravity, viscous and surface tension forces.

2. SPH Methodology

The SPH method is a Lagrangian method used for fluid flows. The entire system of the SPH method is represented by set of particles and the particles move by the governing equations.

2.1. SPH basics

The basic idea of the SPH method is to express the field function of a particles as a summation interpolant [2].

$$f_i(r) = \sum_j f_j W(r_i - r_j, h)\frac{m_j}{\rho_j} \quad (1)$$

Where j is the nearby particles of the particle i, m, ρ, r are mass, density and coordinate of particle, W is a kernel function, h is the smoothing length. The kernel function is a function of the distance of particles, kernel function value decreases as the distance increases. Using the divergence theorem and the Gauss integral formula, gradient of field function can be derived as follow equation [3].

$$\nabla f_i(r) = \rho_i \sum_j m_j\left(\frac{f_i}{{\rho_i}^2} + \frac{f_j}{{\rho_j}^2}\right)\nabla W\left(r_i - r_j, h\right) \quad (2)$$

2.2. Governing equations

In the SPH method, there are two approaches in density calculation, in this study, mass summation is used.

$$\rho_i(r) = \sum_j m_j W(r_i - r_j, h) \quad (3)$$

The conservation law of momentum can be rewritten in Lagrangian form as equation (4).

$$\frac{D\boldsymbol{u}}{Dt} = -\frac{\nabla p}{\rho} + \boldsymbol{g} + \nu\nabla^2\boldsymbol{u} \quad (4)$$

Where ρ is density, $\boldsymbol{u}$ is velocity, p is pressure, $\boldsymbol{g}$ is gravitational acceleration and ν is kinematic viscosity. The pressure force and laminar viscous force term can be discretized as equation (5) and (6).

$$\left(\frac{D\boldsymbol{u}}{Dt}\right)_i = -\sum_j m_j\left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2}\right)\nabla_i W_{ij} \quad (5)$$

$$\left(\frac{D\boldsymbol{u}}{Dt}\right)_i = \sum_j \frac{4m_j\mu_j\,\overrightarrow{r_{ij}}\cdot\nabla_i W_{ij}}{(\rho_i+\rho_j)\left(\left|\overrightarrow{r_{ij}}\right|^2+\eta^2\right)}\left(\overrightarrow{u_i} - \overrightarrow{u_j}\right) \quad (6)$$

In the SOPHIA code, macroscopic continuum surface force is adopted. In this model, color field is defined as equation (7), and normal vector can be obtained as equation (8) [4]

$$c_i{}^j = \frac{2\rho_i}{\rho_i+\rho_j} \text{ if } f_i \neq f_j \quad (7)$$

$$\overrightarrow{n_i} = \langle\nabla c_s\rangle_i = \frac{1}{V_i}\sum_j\left({V_i}^2 + {V_j}^2\right)\frac{{c_i}^i + {c_i}^j}{2}\nabla W_{ij} \quad (8)$$

The curvature and surface tension force is as follows [4].

$$\kappa_i = -\nabla\cdot\left(\frac{\overrightarrow{n_i}}{|\overrightarrow{n_i}|}\right) = -n\frac{\sum_j V_j\left(\frac{\overrightarrow{n_i}}{|\overrightarrow{n_i}|} - \varphi_{ij}\frac{\overrightarrow{n_j}}{|\overrightarrow{n_j}|}\right)\cdot\nabla W_{ij}}{\sum_j V_j|r_i - r_j||\nabla W_{ij}|} \quad (9)$$

$$\left(\frac{D\boldsymbol{u}}{Dt}\right)_i = -\frac{\sigma}{\rho}\kappa_i(\nabla c)_i \quad (10)$$

Where φ_j is a parameter that is 1 if particle i and particle j belong to same phase and 0 if they belong to different phases.

In the WCSPH method, fluids are assumed to be slightly compressible, and the equation of state is given by equation (11).

$$p = \frac{c_0{}^2\rho_0}{\gamma}\left[\left(\frac{\rho}{\rho_0}\right)^{\gamma} - 1\right] \quad (11)$$

Where ρ_0 is the standard reference density of fluid, $c_0 = c(\rho_0)$ is the reference speed of sound, γ is the polytrophic constant that determines the sensitivity of the pressure calculations [5].

2.3. SPH formulations

In the SPH multi-phase analysis, density distribution discontinuity occurs in the multi-phase interface particles. In smoothing region of particles at interface, there are particles with high density ratio, the interface becomes unstable. To handle multi-phase flow, the SOPHIA code introduced normalized-density formulations. The new method is to estimate the normalized density (ρ/ρ_0), instead of density (ρ), and the density of particle i can be calculated as follows.

$$\rho_i = \rho_{0,i} \cdot \left(\frac{\rho}{\rho_0}\right)_i \quad (12)$$

In the SPH multi-phase model, numerical instability may occur near multi-phase interface. Therefore, interface sharpness force was introduced to SOPHIA code to stabilize multi-phase interface. Interface sharpness forces are added to multi-phase interface to provide repulsive force between particles belonging to different phases. The interface force in the SOPHIA code is given by following equation (13).

$$\left(\frac{d\vec{u}}{dt}\right)_i = -\frac{\varepsilon}{m_i}\sum_{j_{(f_i \neq f_j)}}\left(|p_i|V_i{}^2 + |p_j|V_j{}^2\right)\nabla W_{ij} \quad (13)$$

Where ε ranges between 0.01 and 0.1, and summation is calculated in case of particle i and j belong to different fluid phases [6].

3. Simulation Set-up

In this study, transverse motion of single bubble in simple shear flow was simulated by SPH methodology. The experiment by Tomiyama et al. (2002) is selected as a reference and the simulation is conducted for four combinations of selected velocity gradients and bubble diameters [7].

3.1. Reference experiment

As shown in Figure 1, in the acrylic tank, glycerol-water solution is filled at atmospheric pressure and room temperature. A linear shear flow with a constant velocity gradient ω is formed between belt and fixed acrylic wall as the seamless belt was rotated at a constant speed. A single air bubble is emitted from the nozzle tip at the height where the shear flow is formed.

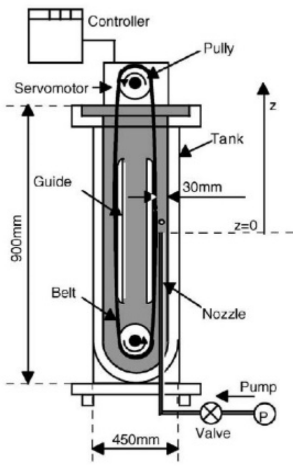


Figure 1. Schematic of experimental apparatus [7]

In the experiment, Morton number M, Eőtvős number Eo and the velocity gradient ω were selected as the parameters of experiments. The experimental conditions used in this study are shown in Table I, and the physical properties are shown in Table II (d is the diameter of bubbles).

Table III. Experimental conditions

	$\log_{10} M$	$\omega(s^{-1})$	Re_{D_h}	d(mm)
Exp. 1	-5.3	5.7	63	3.52
Exp. 2				5.54
Exp. 3		6.2	78	3.52
Exp. 4				5.54

Table IV. Fluid properties

	$\rho(\mathrm{kg}/m^3)$	$\mu(\mathrm{kg}/m \cdot s)$	$\sigma(N/m)$
Glycerol-water solution	1154	0.019	0.061
Air	1.22	0.001	-

3.2. Simulation set-up

To form a simple shear flow, a fixed wall and a wall moving at a constant speed are needed. As shown in Figure 2, the wall which is composed of fixed particles is located in the middle, and the walls composed of moving particles are located on both sides. Particles on both walls have velocity, but the position is not updated.

When the calculation has been performed for a sufficient time, simple shear flow is formed, and the y-direction velocity profile is shown in Figure 3. When the velocity gradient has been reached steady state, an air bubble is injected at a height of 0.15m which was located at the altitude where the flow established a simple shear flow.

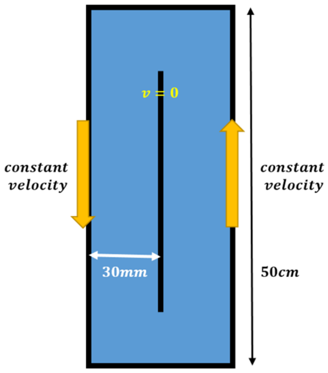


Figure 2. Simulation geometry

3.2. Initial conditions

For this study, a total of 4 simulations were performed. In all cases of simulation, initial particle distance was 0.25mm and total number of particles was 468260 including fluid, wall and air bubble. The properties of fluid particles were same as the glycerol water mixture used in the experiment, density was 1154kg/m^3 and viscosity was 0.091 kg/m·s. Also, simulation employs sound speed 30m/s, time step 10^{-6} s, Predictor-Corrector time stepping and Wendland2 kernel.

4. Result and Discussions

4.1. Simulation results

As shown in Figure 4, bubble in all simulations rose stably, and the direction of transverse movement of bubbles were different depending on the size of the diameter.

4.2. Comparisons with experiment

Comparisons of bubble trajectories in simulation results and experimental results are shown in Figure 5, and Figure 6. The errors between simulations and experiments are shown in Table III. Considering that the measurement error of the experiment was $\pm$0.3mm, the simulation results show good agreement with the experiment results.

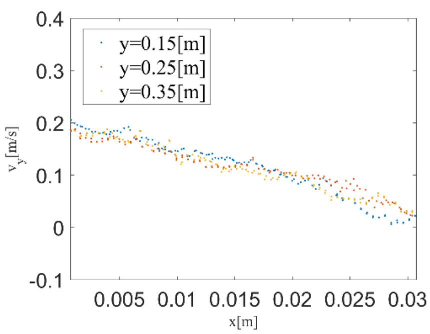


Figure 3. Y-direction velocity profile

Table III. Errors between simulations and experiments

Error		Bubble diameter (mm)	
		5.54	3.52
Velocity gradient (s^{-1})	5.7	0.29mm	0.76mm
	6.2	0.46mm	0.28mm

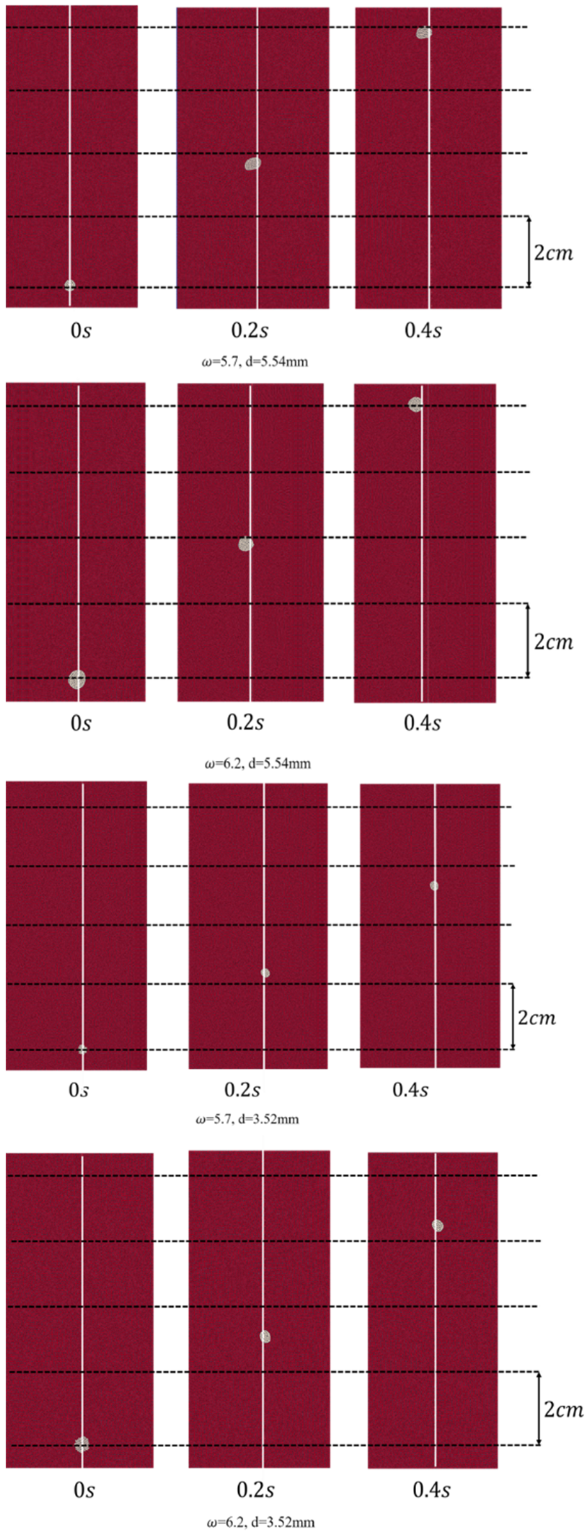
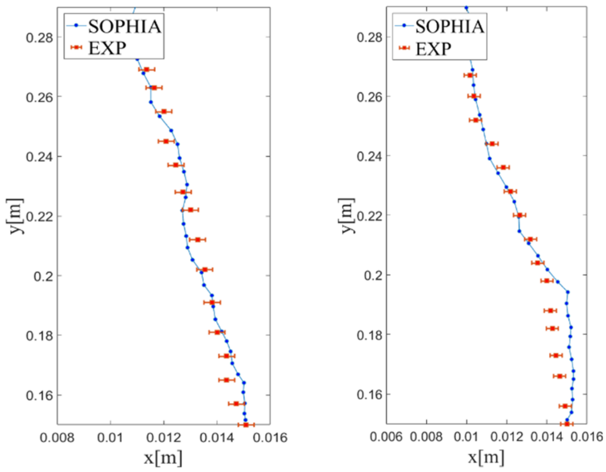
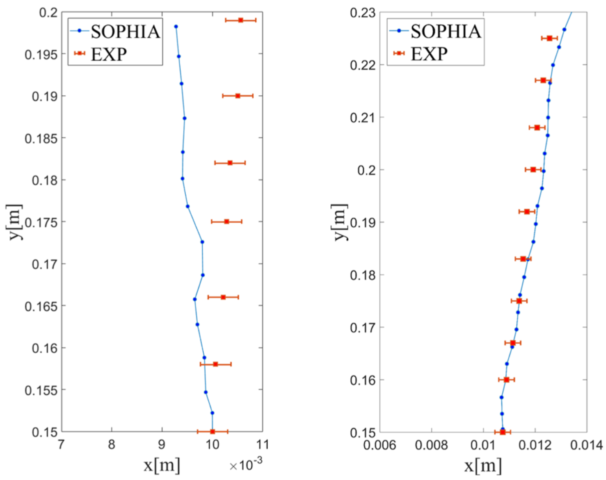


Figure 4. Bubble rising trajectories in simulations



$\omega = 5.7s^{-1}, d = 5.54mm$ $\omega = 6.2s^{-1}, d = 5.54mm$

Figure 5. Comparisons of simulation and experimental results of large diameter



$\omega = 5.7s^{-1}, d = 3.52mm$ **(to be revised)** $\omega = 6.2s^{-1}, d = 3.52mm$

Figure 6. Comparisons of simulation and experimental results of small diameter

5. Summary

In this study, single bubble in simple shear flow was simulated by the SPH method, and the experiment by Tomiyama et al. (2002) is selected as a reference. The SOPHIA code based on SPH method was used for analysis which was developed by Seoul National University. Interface sharpness force is adopted to stabilize multi-phase interface, density and continuity equations were re-formulated in terms of normalized density to handle multi-phase flow. The trajectories of the calculated bubbles show good agreement with the experimental results for various velocity gradients and bubble sizes in overall within the measurement uncertainties.

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