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Application of Adjoint Based Node Optimization Method to MARS

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1. Introduction

In nuclear engineering, thermal-hydraulic (TH) system analysis code plays a crucial role in safety analysis of nuclear power plant (NPP). For the analysis, nodalization should be determined by users, and it is well-known that this node configuration affects the simulation results, i.e. user effect. In general, such uncertainty could be quantified and analyzed with multi simulations in the code. Instead, the authors use a sensitivity-based optimization method to quantify the node uncertainty with the adjoint method, it is possible to perform the sensitivity analysis efficiently for many parameters such as the geometric position of the nodes. In this paper, the node uncertainty of Loss Of Fluid Test (LOFT) is discussed and MARS-KS 1.4 is used as the reference code.

2. Method

In this section, the procedure how to calculate the node sensitivity by applying the adjoint method to discretized governing equations of MARS-KS 1.4 is presented. The implementation process for optimization is described as well.

2.1 Adjoint based sensitivity analysis procedure

The MARS (Multi-dimensional Analysis of Reactor Safety) code is developed by KAERI for a multi-dimensional and multi-purpose realistic thermal-hydraulic system analysis of light water reactor transients [2]. It is basically consisted of hydrodynamic equations containing non-condensable gas and conduction equation for heat structure. Above equations are discretized with semi-implicit scheme to solve numerically and these discretized equations are expressed in Eq.1 with independent variable vector set $\boldsymbol{X}$ and parameter $\boldsymbol{N}$.

$$\boldsymbol{F}(\boldsymbol{X},\boldsymbol{N}) = \boldsymbol{A}(\boldsymbol{X}^n,\boldsymbol{N})\boldsymbol{X}^{n+1} + \boldsymbol{B}(\boldsymbol{X}^n,\boldsymbol{N}) = \boldsymbol{0} \quad (1)$$
$$\text{where } \boldsymbol{X} = (\boldsymbol{\alpha}_g, \boldsymbol{P}, \boldsymbol{U}_f, \boldsymbol{v}_f, \boldsymbol{U}_g, \boldsymbol{v}_g)$$

Adjoint method is a widely used technique for sensitivity computation of parameters. It retrieves derivatives of a cost function respect to parameters efficiently. When an objective function is $\boldsymbol{G}(\boldsymbol{X},\boldsymbol{N})$, parameters i.e. node position are $\boldsymbol{N}$. The sensitivity is defined as in Eq.2

$$\frac{d\boldsymbol{G}(\boldsymbol{X},\boldsymbol{N})}{d\boldsymbol{N}} = \frac{\partial \boldsymbol{G}(\boldsymbol{X},\boldsymbol{N})}{\partial \boldsymbol{N}} + \frac{\partial \boldsymbol{G}(\boldsymbol{X},\boldsymbol{N})}{\partial \boldsymbol{X}}\frac{d\boldsymbol{X}}{d\boldsymbol{N}} \quad (2)$$

In Eq.3, differentiating the discretized governing equations with respect to the parameters, an approach called discrete adjoint approach, is used.

$$\frac{\partial \boldsymbol{F}(\boldsymbol{X}^{n+1},\boldsymbol{X}^n,\boldsymbol{N})}{\partial \boldsymbol{X}^{n+1}}\boldsymbol{X}_N^{n+1} + \frac{\partial \boldsymbol{F}(\boldsymbol{X}^{n+1},\boldsymbol{X}^n,\boldsymbol{N})}{\partial \boldsymbol{X}^n}\boldsymbol{X}_N^n = -\frac{\partial \boldsymbol{F}(\boldsymbol{X}^{n+1},\boldsymbol{X}^n,\boldsymbol{N})}{\partial \boldsymbol{N}} \dots (3)$$
$$\text{where } \frac{d\boldsymbol{X}^{n+1}}{dN} = \boldsymbol{X}_N^{n+1}, \frac{d\boldsymbol{X}^n}{dN} = \boldsymbol{X}_N^n$$

Eq.3 can be expressed in a matrix form, Eq.4.

$$\begin{bmatrix} \frac{dF(X^{n+1},X^n,N)}{\partial X^{n+1}} & \frac{dF(X^{n+1},X^n,N)}{\partial X^n} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \frac{dF(X^n,X^{n-1},N)}{\partial X^n} & \frac{dF(X^n,X^{n-1},N)}{\partial X^{n-1}} & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \frac{dF(X^1,X^0,N)}{\partial X^1} & \frac{dF(X^1,X^0,N)}{\partial X^0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{X}_N^{n+1} \\ \boldsymbol{X}_N^n \\ \vdots \\ \boldsymbol{X}_N^1 \\ \boldsymbol{X}_N^0 \end{bmatrix} = \begin{bmatrix} -\frac{dF(X^{n+1},X^n,N)}{\partial N} \\ -\frac{dF(X^n,X^{n-1},N)}{\partial N} \\ \vdots \\ -\frac{dF(X^1,X^0,N)}{\partial N} \\ \boldsymbol{X}_N^0 \end{bmatrix} \dots (4)$$

Eq.6 shows that the node sensitivity can be obtained with adjoint function $\boldsymbol{\lambda}$ that satisfies Eq.5

$$\begin{bmatrix} \frac{dF(X^{n+1},X^n,N)}{\partial X^{n+1}} & \frac{dF(X^{n+1},X^n,N)}{\partial X^n} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \frac{dF(X^n,X^{n-1},N)}{\partial X^n} & \frac{dF(X^n,X^{n-1},N)}{\partial X^{n-1}} & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \frac{dF(X^1,X^0,N)}{\partial X^1} & \frac{dF(X^1,X^0,N)}{\partial X^0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \boldsymbol{I} \end{bmatrix}^T \begin{bmatrix} \boldsymbol{\lambda}_{n+1} \\ \boldsymbol{\lambda}_n \\ \vdots \\ \boldsymbol{\lambda}_1 \\ \boldsymbol{\lambda}_0 \end{bmatrix} = \begin{bmatrix} \frac{\partial G(X,N)}{\partial X^{n+1}} \\ \frac{\partial G(X,N)}{\partial X^n} \\ \vdots \\ \vdots \\ \frac{\partial G(X,N)}{\partial X^0} \end{bmatrix} \dots (5)$$

$$\frac{dG(X,N)}{dN} = \frac{\partial G(X,N)}{\partial N} + [\lambda_{n+1} \quad \lambda_n \quad \dots \quad \dots \quad \lambda_0] \begin{bmatrix} -\frac{dF(X^{n+1},X^n,N)}{\partial N} \\ -\frac{dF(X^n,X^{n-1},N)}{\partial N} \\ \vdots \\ -\frac{dF(X^1,X^0,N)}{\partial N} \\ \boldsymbol{X}_N^0 \end{bmatrix}$$

(6)

2.2 Implementation process on the code

Figure 1 is an algorithm for node optimization using adjoint based sensitivity analysis. For node sensitivity analysis, necessary data are extracted and saved for adjoint sensitivity module from MARS code and sensitivity is calculated in adjoint sensitivity module code written in MATLAB. After then, rearranging the node configuration using a gradient descent method and modifying the input file of MARS is proceeded in optimization module written in MATLAB.

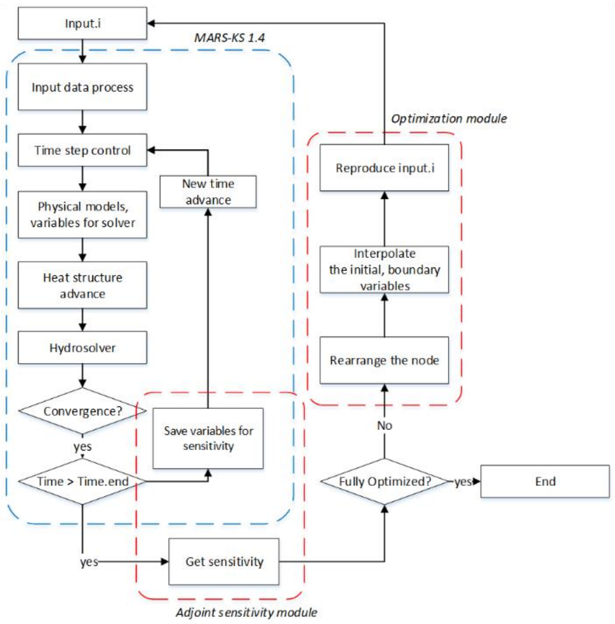

Fig. 1. Algorithm of adjoint based node optimization

3. Results

In this section, the node uncertainty at broken cold leg in LOFT L2-3 is calculated using the proposed methodology and it is compared to a result that is obtained using Wilks' formula.

3.1 Node uncertainty at broken cold leg in LOFT L2-3

LOFT L2-3 represents the postulated double ended LBLOCA in the typical Westinghouse four loop type PWR. A nodalization of LOFT L2-3 is shown in Figure 2 and blue box is a region of interest corresponding to the broken cold leg. The region is related to bleed of coolant and the authors simulated with various node configurations in the region to observe the node effect before the optimization. From the simulations, it was found that the influence of the node configuration of the region is significant from the time of blow down.

Thus, in this paper, the authors quantify the node uncertainty in the broken cold leg by finding the node structure having the maximum and the minimum of break mass flow. The region of interest consists of two nodes and a case that the region is divided into two nodes uniformly is set as a nominal. The objective function is defined as in Eq.7 and it is the sum of mass flow during 0~1.5 sec.

$$G = \sum_{n=1} (\rho_g^n v_g^n A + \rho_f^n v_f^n A)\Delta t \quad (7)$$

The stopping criteria for iteration is set when the sign of sensitivity is changed or the shortest node length is smaller than 1/10 of the total length. The step size was set to decrease by a half for every iteration. The normalized and optimized node configurations are presented in Figure 3 and each mass flow rate and the hottest cladding temperature profile is shown in Figures 4 and 5. These results show the situation that the small change of loss of coolant causes the significant change in the hottest cladding temperature. The reason of the significant change is that a variation of void fraction in the core causes a bifurcation of heat transfer modes in the code calculation.

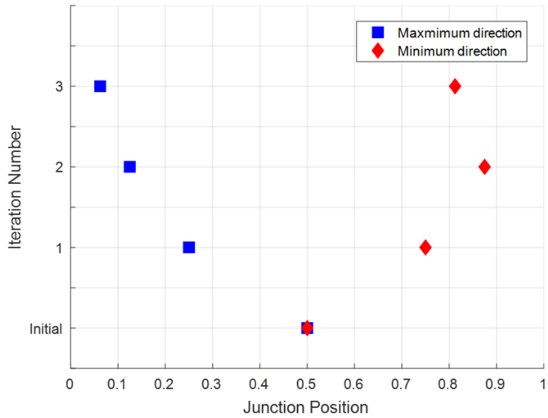

Fig. 3. Optimization process

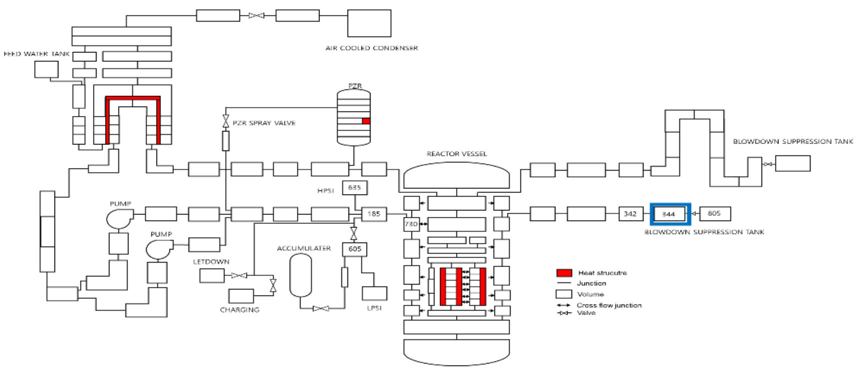

Fig. 1. Nodalization of L2-3

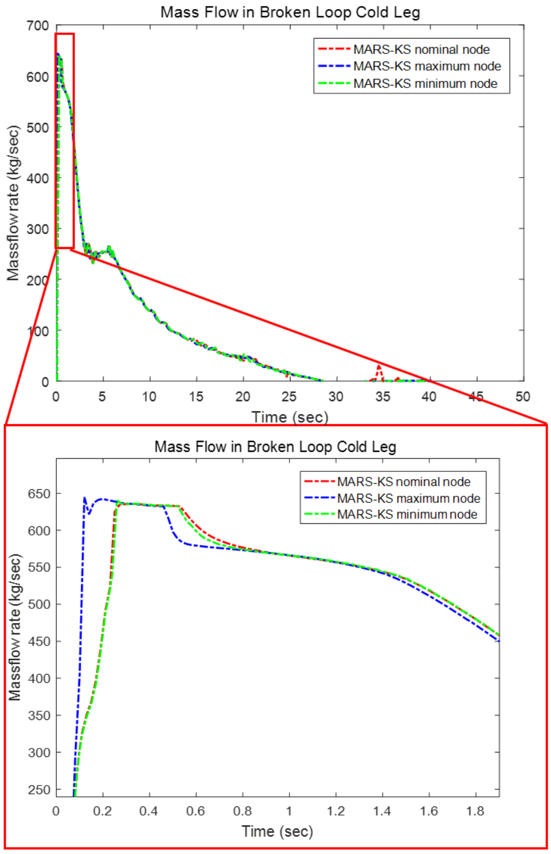

Fig. 4. Calculated node uncertainty in the mass flow rate

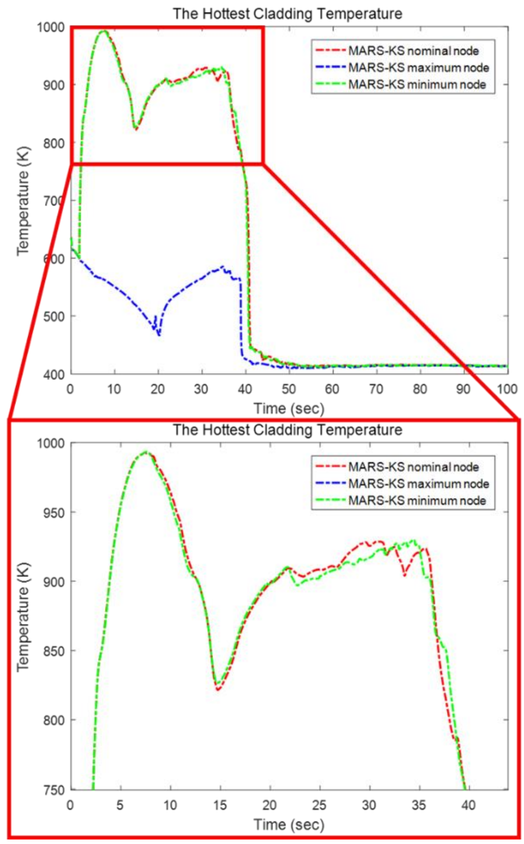

Fig. 5. Calculated node uncertainty in the hottest cladding temperature

3.2 Comparison with Wilks' formula

According to the first order of Wilks formula, 93 is the required number of simulations for two-sided statistical tolerance limit with a 95% confidence level and 95 % probability. The highest and the lowest values are corresponding to the two-sided statistical tolerance limit with 95/95. These 93 simulation results of the hottest cladding temperature are shown in Figure 6. It seems to fit well with the result in Figure 5.

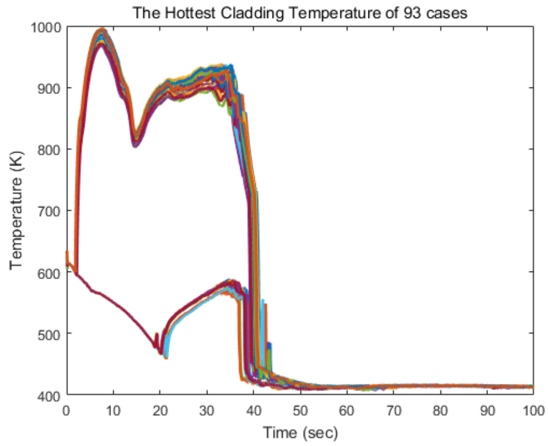

Fig. 6. Node uncertainty in the hottest cladding temperature using Wilks formula

4. Summary and Conclusions

In this study, the authors calculated the node uncertainty at broken cold leg of LOFT L2-3 using an adjoint based node optimization method. The comparison with Wilks' formula shows that adjoint based node optimization could quantify the node uncertainty reasonably well. It is also identified that the bifurcation could be occurred depending on the break flow node configuration and confirmed that the bifurcation is included in the quantified uncertainty.

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REFERENCES

[1] D. G. Cacuci, Sensitivity and Uncertainty Analysis: Volume I Theory, Chapman & Hall/CRC, 2003.
[2] KAERI, "MARS CODE MANUAL", KAERI/TR-2811/2004, 2009.
[3] J. J. Lee, M. G Kim, S. M. Son and J. I. Lee, "A study on the applicability of adjoint based optimization method for nuclear thermal hydraulic system analysis code nodalization", BEPU 2018, May 13-19, 2018, Real Collegio, Lucca, Italy.
[4] J. J. Lee, M. G Kim, S. M. Son and J. I. Lee, "Study on Adjoint Based Node Sensitivity Analysis of Two Phase

Thermal-Hydraulic System Analysis Code", Transactions of the Korean Nuclear Society fall Meeting, Oct. 25-26, 2018, Yeosu, Korea.
[5] Bang, Y.S., Kim, H.J., 1992. Assessment of RELAP5/MOD2 Cycle 36.04 with LOFT Large Break LOCE L2-3, NUREG/IA-007