Investigation of optimized sample size of nuclear fuel pellets for national inspection

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1. Introduction

The International Atomic Energy Agency (IAEA) defines the safeguards as "the timely detection of diversion of nuclear material from peaceful nuclear activities to the manufacture of nuclear weapons or of other nuclear explosive devices…" [1]. Special nuclear material (SNM) is defined as material subjected to IAEA safeguards. The ROK, as a member state of IAEA, is obligated to control domestic SNMs based on state system of accounting and control (SSAC) [2]. Korea Institute of Nuclear non-proliferation and Control (KINAC) is committed to the control of SNM in the ROK by Nuclear Safety and Security Council (NSSC)

License holders are obliged to declare information of SNM in facility. KINAC verifies the declared information of SNM in bulk handling facilities (BHF) based on random sampling. KEPCO Nuclear Fuel Cooperation (KNFC), which manufactures commercial nuclear fuel pellets, is a target facility of random sampling. IAEA calculates the estimated amount of SNM in a facility using equation (1). Gross weight and enrichment of target material are conventional target of IAEA verification. However, IAEA uses material composition (U, Pu factor) declared by operator.

$$M_{SNM} = \sum_i M_{net}(i) \times f_{U,Pu}(i) \times w_{SNM}(i) \quad (1)$$

where,
M_{SNM}: Total amount of special nuclear material in facility,
$M_{net}(i)$: Net weight of items in stratum i,
$f_{U,Pu}(i)$: Composition of items in stratum i,
$w_{SNM}(i)$: Enrichment of items in stratum i.

KINAC is planning to enhance an independent national inspection due to the revised domestic notification on SNM (Article 4 of NSSC notification No. 2017-83) which requires verification of domestic SNM quantity and composition (U factor) [3]. The composition of SNM can only be verified using destructive analysis (DA) based methods: thermogravimetric analysis (TGA) and titration.

However, at the moment, IAEA (physical inventory verification, PIV) and KINAC (national inspection) have joint verification on the SNMs in KNFC except for DA sampling. KINAC has to determine the amount of random DA samples due to the reason above.

The purpose of this research is DA sample size calculation for national inspection to minimize administrative burden of KNFC as well as satisfy the level of verification required to KINAC. Since this research is at initial stage, the research focuses to calculate the optimized DA sample size of fuel pellet, which can be expanded to facility and state level.

2. Methods

Under the three conditions, IAEA calculates sample size in a stratum using equation (2) [4].

1) All items in the stratum have to be measured by operator and it has to be declared to inspector (KINAC or IAEA)
2) Inspectors have to sample items randomly
3) It has to be assured that operator do not falsify the sampled items between inspector's sampling and measurements

$$n = N\left(1 - \beta^{\left(\frac{1}{D}\right)}\right) \quad (2)$$

where,
n: Sample size in a stratum,
N: Total number of items in a stratum,
β : Non-detection probability,
D: Number of items required to divert 1 SQ (D=M⁄x)
M: 1 SQ of SNM,
x: Average amount of SNM per item in a stratum.

Entire random samples are classified into the samples for gross, partial and bias defect verification. The definition of each defect and verification is described in Table 1. Since U factor is a factor to calculate total ^{235}U mass in an item, U factor verification in national inspection can be performed by partial and bias defect verification. Equation (3) describes conventional IAEA method to classify the random sample into three different samples with different verification methods [5].

Table 1. Characteristics of different defect types.

Type of defects	Target	Accuracy	Methods
Gross defect	Enrichment(^{235}U)	Low	Gamma spectrometry (Method H)
Partial defect	Amount (^{235}U)	Intermediate	Weighing methods, Gamma spectrometry (Method F)
Bias defect	Amount (^{235}U)	High	DA based methods (Method D)

$$H = n - \eta_2,\ F = \eta_2 - \eta_3,\ \mathrm{D} = \eta_3 \qquad (3)$$

where,
H: Sample size for gross defect verification,
F: Sample size for partial defect vericiation,
D: Sample size for bias defect verification,
$\eta_i = \begin{cases} roundup\left(\frac{\ln(\beta_i)}{\ln(1-M/\gamma_i Nx)}\right), & \left(if\ \frac{M}{\gamma_i Nx} < 1\right) \\ 1, & (otherwise) \end{cases}$, (i = F or D),
β_i: Non-detection probability of method i,
$\gamma_i = 4.737\Delta_i - 5.490\Delta_i^2$,
$\Delta_i = \begin{cases} \max(\delta_2, 0.0075 - 0.0531\delta_1 + 2.369\delta_1^2), & (method\ F) \\ \max(\delta_3, 0.331\delta_2), & (method\ D) \end{cases}$,
δ_i: Relative standard deviation of method i (i=H, F or D).

Since U factor affects total ^{235}U mass in an item, U factor has to be verified using method F and D. However, only DA methods are available for U factor verification at the moment. Direct use of conventional DA sample size calculation (equation (3)) is inappropriate for national inspection due to the reason. This research investigated an empirical optimization index to determine optimized DA sample size for national inspection which is expected in between current DA and total sample size.

Compared to the conventional method which only considers detection probability, the optimization index includes three sub-factors (equation (4)): 1) Uncertainty of estimated ^{235}U in a stratum, 2) Cost required to analyze DA samples, 3) Detection probability. The optimization index can be described as equation (4), since lower uncertainty, lower cost and higher detection probability are desirable for national inspection.

$$\mathrm{S(S)} = \frac{1-\beta}{\sigma_{I(S)} \times C(S)} \qquad (4)$$

where,
S(S): Optimization index for a stratum,
$1-\beta$: Detection probability $\left(1-\beta = 1-\left(1-\frac{M}{\gamma_i Nx}\right)^{\mathrm{n}}\right)$,
C(S): Cost required to analyze DA samples
$\left(C(S) = \begin{cases} n, & (if\ n \le N_{th}) \\ n\left(e^{(n-N_{th})}\right), & (if\ n > N_{th}) \end{cases}\right)$,
$N_{th} = 14$ (Sample size limit for a stratum),
$\sigma_{I(S)}$: Uncertainty of estimated ^{235}U in a stratum
$(\sigma_{I(S)}^2 = \left(\overline{O(S)}\bar{\omega}\bar{f}\right)^2\left(\frac{\delta_{r,w}^2+\delta_{r,M}^2+\delta_{r,f}^2}{n} + \delta_{s,w}^2 + \delta_{s,M}^2 + \delta_{s,f}^2\right)$ [6]
$(\overline{O(S)})$: Operator declared stratum mass, $\bar{\omega}$: Average enrichment in a stratum,
$\bar{f}$: Average U factor in a stratum,
δ_X^2: Random and systematic RSD of weighting, U factor and enrichment)

3. Preliminary Results

Since the target of this research is to demonstrate stratum level DA sample size calculation, it only focused LEU pellet stratum in KNFC. Thus, the target SNM in this research is limited to ^{235}U in LEU pellet stratum. This research demonstrated the feasibility of the optimization index using a hypothetical PIV results. The goal of KINAC's DA uncertainty is ITV level. KINAC's DA uncertainty for various DA methods is described in Table 2 [7].

Table 2. Relative standard deviation of DA methods in ITV 2010.

Method(Target)	δ_r(rel. %)	δ_s(rel. %)
Grav., TGA (pure UO_2)	0.05	0.05
Grav., TGA (UO_2 with Gd)	0.10	0.10
TIMS (DU, **0** < **w** ≤ **0.3**%)	0.50	0.50
TIMS (U, [**0.3** < **w** ≤ **1.0**%])	0.20	0.20
TIMS (LEU, [**1.0** < **w** ≤ **20**%])	0.10	0.10
TIMS (HEU, [**w** > **20**%])	0.05	0.05

Table 3 summarizes operator declared information and calculated sample size using conventional method for the hypothetical PIV. The table indicates LEU pellet in KNFC are classified into two sub-strata: PL1-L (pellet without Gd) and PL2-L (pellet with Gd). The detection probability of IAEA for KNFC is 0.8 due to the integrated safeguards agreement between ROK-IAEA [9]. However, this research adopted detection probability for national inspection as 0.5, since national inspection does not include short notice random inspection (SNRI).

This research calculated optimized DA sample size for PL1-L and PL2-L stratum of the hypothetical PIV by applying the results of table 3 to equation (4). Figure 1, 2 and Table 4 depict the effect of DA sample size on optimization index and the three factors in equation (4) for two strata. The detailed results are Results indicate the optimized DA sample sizes for PL1-L and PL2-L stratum are 2 and 1, which is identical to conventional IAEA DA sample size in Table 3.

Results of the research can be a sampling basis for national inspection. Since current results were calculated by assuming KINAC's DA uncertainty is ITV level, optimized DA sample size may change once KINAC's DA uncertainty is quantified. It also can expand target material from single stratum (LEU pellet) in KNFC to all SNM in ROK.

Table 3. Operator declared information and calculated IAEA sample size for a hypothetical PIV

Stratum	Total item	Net W (kg)	U mass (kg)	U-235 mass (kg)	x (kg)	SQ (kg)	D (#)
PL1-L(JQLB)	2,179	101,500	89,462	3,690	1.694	75	45
beta	**n**	delta1	delta2	delta3	**H**	**F**	**D**
0.5	**34**	0.15	0.0362	0.0051	**26**	**6**	**2**
Stratum	Total item	Net W (kg)	U mass (kg)	U-235 mass (kg)	x (kg)	SQ (kg)	D (#)
PL2-L(JVLB)	3.0200E+02	1.2500E+04	10122	263	0.871	75	87
beta	**n**	delta1	delta2	delta3	**H**	**F**	**D**
0.5	**3**	0.15	0.0362	0.0051	**2**	**0**	**1**

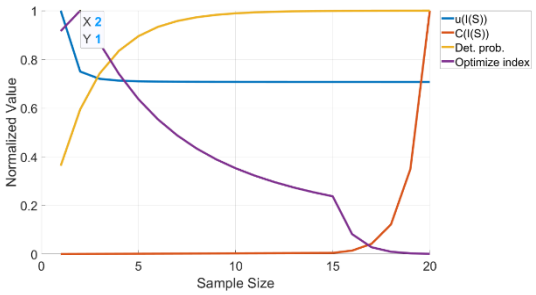

Fig. 1. Effect of DA sample size on three factors and optimization index (PL1-L stratum).

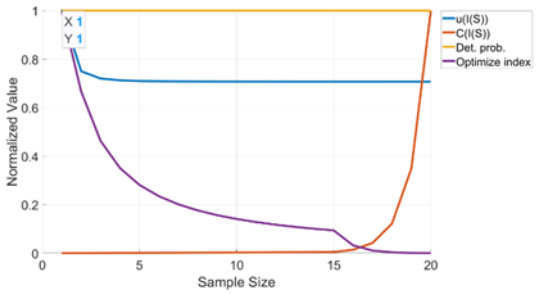

Fig. 2 Effect of DA sample size on three factors and optimization index (PL2-L stratum).

Table 4. Effect of DA sample size on optimization index for the two strata (normalized values).

Sample Size (n)	PL1-L				PL2-L			
	u(S)	C(S)	1−β	Opt. index	u(S)	C(S)	1−β	Opt. index
1	13.722	1	0.43027	0.031356	**1.5568**	**1**	**1.0000**	**0.64236**
2	**10.292**	**2**	**0.67541**	**0.032813**	1.1676	2	1.0000	0.42824
3	9.8812	3	0.81507	0.027496	1.1210	3	1.0000	0.29735
4	9.7786	4	0.89464	0.022872	1.1094	4	1.0000	0.22535
5	9.7418	5	0.93997	0.019298	1.1052	5	1.0000	0.18096
6	9.7255	6	0.96580	0.016551	1.1033	6	1.0000	0.15106
…	…	…	…	…	…	…	…	…

3. Conclusions

Direct application of conventional IAEA DA sample size calculation is impossible to ROK due to the revised domestic regulation on SNM. This research investigated an empirical optimization index to calculate optimized DA sample size for national inspection. Three factors were considered to calculate the optimization index: 1) uncertainty of inspector estimated ^{235}U mass, 2) DA cost required to analyze sample, 3) detection probability.

Results indicate optimized DA sample size is equivalent to conventional IAEA sample size if KINAC's DA uncertainty is equivalent to ITV. Since the target value of KINAC's DA uncertainty is ITV level or higher, estimated DA sample size of KINAC will be similar to current IAEA DA sample size.

The significance of this research is KINAC calculated DA sample size for nation inspection which considers ITV level DA uncertainty as well as minimizes facilities' administrative burden. Future research considers to apply relative importance between the three factors in the empirical equation to modify current empirical equation more realistic. It also will expand the target nuclear material from LEU pellet stratum in KNFC to entire SNM in the ROK.

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