System Reliability Evaluation by using Factor Graph Analysis

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1. Introduction

The nuclear power plant is safety critical system. The event in the nuclear power plant may lead serious consequences. Therefore, to seek the weak point of the system and analyze the risk of the system, fault tree method is proposed. The method is not only powerful to figure out the risk probability of the system, but also the method can find the minimal cut sets of the system. And also, several importance measures [1] were invented (Fussel-Vesely (FV), Risk increase of Risk Achievement Worth (RAW)) based on fault tree analysis.

However, as the system goes more and more complex, the fault tree becomes even more complex. Therefore, now a day, it is hard to recognize the system with fault tree.

Therefore, in this paper, to analyze the risk of system intuitively, belief propagation based factor graph analysis is proposed.

2. Factor Graph and Belief Propagation Algorithm

Factor graph is the graph that has both factor node and variable node. The factor graph is a bipartite graph. Therefore, the edge can construct when two nodes are different type. For instance, edge from variable node to factor node doesn't have any problem. But for the case of variable node to variable node, the edge can't be constructed. The Fig. 1 shows the example of Factor graph.

Fig. 1 The structure of Factor graph

In the Fig. 1 the rectangular node represents factor node and the circular node represents variable node. As shown in the Fig. 1 the edge is only connected in between different node. Compared to other probability graph model, the major difference of factor graph is that the factor graph has undirected edges.

The information in the Bayesian network is transferred to other node based on conditional probability theory. As same as the conditional probability in Bayesian network, Factor graph transports information based on belief propagation algorithm, which is called as sum-product algorithm. The sequence of sum-product belief propagation is listed as follows.

- i: set of factor node, α: set of variable node, E: set of edge

- 1. Initialize the message to the uniform distribution

$$\mu_{i\to\alpha}(x_i) = 1, \mu_{\alpha\to i}(x_i) = 1, \forall(i,\alpha) \in E$$

- 2. Choose a root node
- 3. Send message from the leave to the root and send message from the root to the leaves

$$\mu_{i\to\alpha}(x_i) = \prod_{\alpha\in N(i)\setminus\alpha} \mu_{\alpha\to i}(x_i)$$

: factor node to variable node

$$\mu_{\alpha\to i}(x_i) = \sum_{x_\alpha : x_\alpha[i]=x_i} \psi_\alpha(x_\alpha) \prod_{j\in N(\alpha)\setminus i} \mu_{j\to\alpha}(x_\alpha[i])$$

: Variable node to factor node

- 4. Compute the beliefs $(b_i(x_i))$

$$b_i(x_i) = \prod_{\alpha\in N(i)\setminus\alpha} \mu_{\alpha\to i}(x_i)$$: Belief of factor node

$$b_\alpha(x_\alpha) = \psi_\alpha(x_\alpha) \prod_{i\in N(\alpha)} \mu_{i\to\alpha}(x_\alpha[i])$$

: Belief of factor node

- 5. Normalize belief (Depends on value)

3. Reliability Evaluation by using Factor Graph

The two major structure of the system is parallel system and serial system. Therefore, in this section, two reliability evaluation example will be shown and solved by using Factor graph and belief propagation algorithm. The example contains two components (x_1 *and* x_2). In the first example, two components are serially connected. In case of second example, two components are connected in parallel. The Fig. 2 shows the example structure.

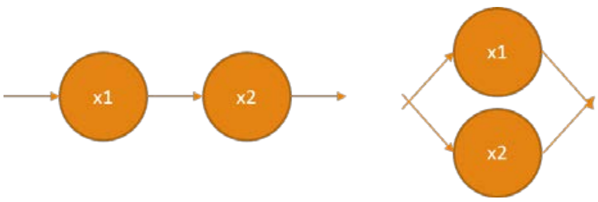

Fig. 2 Example structure

3.1. Serial connection

The factor graph structure of serially connected system is described in Fig. 3.

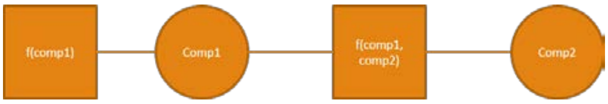

Fig. 3 Factor graph (Serial)

At first, to initialize messages to uniform distribution, all nodes sends 1 to adjacent node. As described in section 2, exchange message between factor node to variable node and variable node and factor node. Factor node point of view, each factor node sends message to adjacent variable node as follows.

- $\mu_{f(comp1) \to v(comp1)} = f(comp1)$
- $\mu_{f(comp1,comp2) \to v(comp1)} = \sum_{comp2} f(comp1, comp2)$
- $\mu_{f(comp1,comp2) \to v(comp2)} = \sum_{comp1} f(comp1, comp2)$

Variable node point of view, each variable node sends message to adjacent factor node as follows.

- $\mu_{v(comp1) \to f(comp1)} = \sum_{comp2} f(comp1, comp2)$
- $\mu_{v(comp1) \to f(comp1,comp2)} = f(comp1)$
- $\mu_{v(comp2) \to f(comp1,comp2)} = 1$

After transfer message from variable node to factor node, a single iteration is finished. By using the result of first iteration, the result of second iteration can be calculated as follows.

- $\mu_{f(comp1) \to v(comp1)} = f(comp1)$
- $\mu_{f(comp1,comp2) \to v(comp1)} = \sum_{comp2} f(comp1, comp2)$
- $\mu_{f(comp1,comp2) \to v(comp2)} = \sum_{comp1} f(comp1)\, f(comp1, comp2)$
- $\mu_{v(comp1) \to f(comp1)} = \sum_{comp2} f(comp1, comp2)$
- $\mu_{v(comp1) \to f(comp1,comp2)} = f(comp1)$
- $\mu_{v(comp2) \to f(comp1,comp2)} = 1$

After the second iteration, values are saturated because the factor node f(comp1) and variable node v(comp2) are sink node. Therefore, the final value of the system reliability is:

- $b_{comp(2)} = \sum_{comp1} f(comp1)\, f(comp1, comp2)$

In this example, factor values are described as follows (s:success, f:fail, c1:comp1, c2:comp2):

- $f(comp1) = \begin{matrix} p(c1 = s) \\ p(c1 = f) \end{matrix}$
- $f(comp1, comp2) = \begin{matrix} p(c2 = s|c1 = s) & p(c2 = s|c1 = f) \\ p(c2 = f|c1 = s) & p(c2 = f|c1 = f) \end{matrix}$

As a result, $b_{comp(2)}$ is calculated as follow.

$$b_{comp(2)} = \begin{matrix} p(c2 = s|c1 = s) * p(c1 = s) + p(c2 = s|c1 = f) * p(c1 = f) \\ p(c2 = f|c1 = s) * p(c1 = s) + p(c2 = f|c1 = f) * p(c1 = f) \end{matrix}$$

The first row, represents system success probability and the second row represents system failure probability.

3.2. Parallel connection

The factor graph structure of parallel connected system is described in Fig. 4.

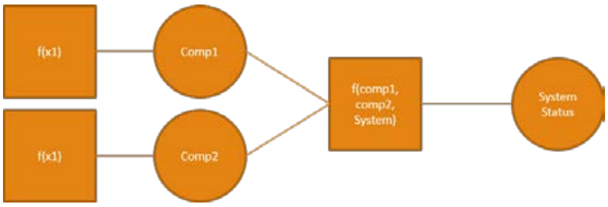

Fig. 4 Factor graph (Parallel)

By following the similar sequence, the value after the first iteration can be calculated as follows.

- $\mu_{f(comp1) \to v(comp1)} = f(comp1)$
- $\mu_{f(comp2) \to v(comp2)} = f(comp2)$
- $\mu_{f(comp1,comp2,System) \to v(comp1)} = \sum_{comp2,System} f(comp1, comp2, System)$
- $\mu_{f(comp1,comp2,System) \to v(comp2)} = \sum_{comp1,System} f(comp1, comp2, System)$
- $\mu_{f(comp1,comp2,System) \to v(System)} = \sum_{comp1,comp2} f(comp1, comp2, System)$

- $\mu_{v(comp1)\rightarrow f(comp1)} = \sum_{comp2,System} f(comp1, comp2, System)$
- $\mu_{v(comp1)\rightarrow f(comp1,comp2,System)} = f(comp1)$
- $\mu_{v(comp2)\rightarrow f(comp2)} = \sum_{comp1,System} f(comp1, comp2, System)$
- $\mu_{v(comp2)\rightarrow f(comp1,comp2,System)} = f(comp2)$

Also, the message is saturated after the second iteration. Therefore, the result can be calculated as follow.

- $b_{System} = \sum_{comp1,comp2}[f(comp1) * f(comp2) * f(comp1, comp2, System)]$

4. Concluding Remarks

To improve the safety of the nuclear power plant, accurate evaluation of system reliability is essential. To estimate the risk of the system, fault tree method is developed and widely used. However, as the system goes more and more complex, the structure of the fault tree becomes even more complex. Therefore, figuring out the system structure from fault tree result is almost impossible.

To overcome this limitation, we proposed factor graph based reliability estimation. And to validate the proposed methodology, we solved two different type of system structure (series, parallel) and the method showed correct result.

The factor graph and belief propagation algorithm is a promising algorithm cause it can modelling the system intuitively. Reliability is calculated in a graph like the P&ID diagram of the system therefore, more intuitive risk analysis can be achieved. And also, factor graph can apply to the graph with loop structure. To estimate the reliability with conventional probability graph model, pre-processing to cut the logical loop is required. However, by using factor graph method with specific stopping criteria, logical loop can be also treated.

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