Transactions of the Korean Nuclear Society Autumn Meeting
Goyang, Korea, October 24-25, 2019

Optimization of fuel batch size and enrichment by means of the simplex method

Jinman Kim, Bumhee Jo, Alexandru Catalin Stafie, Chang Joo Hah*
KEPCO International Nuclear Graduate School, Nuclear Power Plant Engineering Dept.,1456-1, Shinam-ri, Seosaeng-myeon, Ulju-gun, Ulsan 689-882, Republic of Korea
kim.jinman@khnp.co.kr; bumhey@gmail.com; alexandru.stafie@gmail.com; changhah@hanmail.net
**Corresponding author: changhah@hanmail.net;*

1. Introduction

As nuclear energy developed throughout the decades it became the mature technology we know today. With significant advances it has become a more viable option to supply electricity to an ever-developing world. For it to be competitive as an energy source it must be more cost efficient compared to conventional energy sources. In this respect high fuel cycle efficiency is very desirable.

The nuclear fuel cycle accounts for approximatively 20% of the total cost of operation [1]. The two main factors that influence the fuel component are given by enrichment and discharge burnup, which are directly connected to fuel management optimization. Because there is significant time lag between purchasing and utilizing the fuel coupled with the time value of money associated with it, the situation creates reason for the search of new methods in order to minimize the fuel cost.

In a previously conducted study, the notion of correctly predicting the required core enrichment for a given energy requirement was developed in the tool named Fuel Management Net Graph (FMNG) [2]. This tool will be used to aid in predicting the enrichment required for the desired cycle burnup.

The focus of this paper will be on optimizing the required fuel batch size and enrichment for the initial core load for given energy requirements, through the simplex method [3], in order to reduce the total cost. While performing the analysis, the energy requirements for the second and third cycle are also considered.

2. Methods and Results

In the process of achieving the target goal of reducing fuel cost, two different optimization approaches have been created for this purpose. The APR1400 design specifications [4] will be used as a model, with the help of CASMO3 code, for burnup calculations [5].

2.1 *Enrichment Based Approach (EBA)*

EBA is intended to determine the optimal enrichment and the number of fuel assemblies for each batch in order to reduce fuel costs. To that end, the simplex problem here is modeled based on changes in reactivity dependent on enrichment by using the linear reactivity model [6]. When a required cycle length exists, the optimum combination is determined by finding enrichment and the batch size satisfying the average enrichment of the whole core obtained from the FMNG. The simplex algorithm is based on a linear model, while the fuel cost is a nonlinear function comprised of the product cost and quantity of fuel. Therefore, the problem has been split into two steps. EBA first optimizes the fuel quantity for each batch and then simulates the fuel cost minimization through the method of varying the enrichment for the specified number of fuel assemblies from the first simplex method result as shown in Fig. 1.

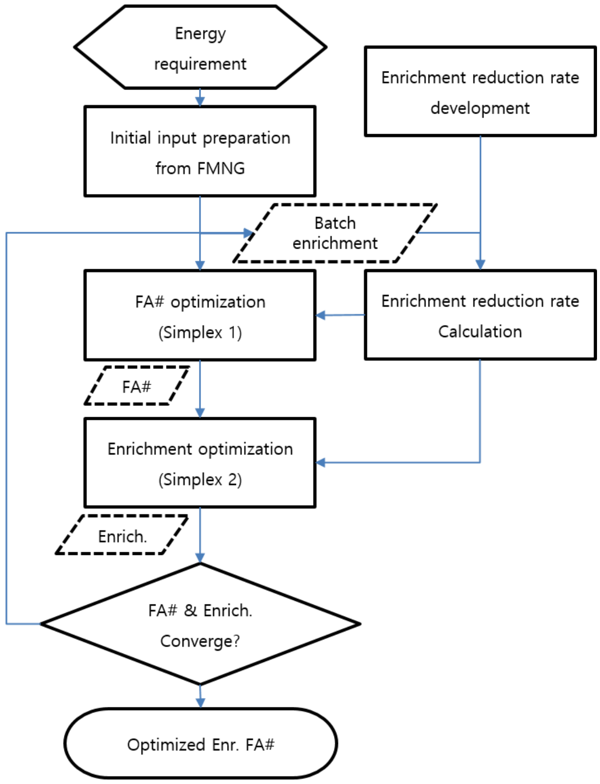

Fig. 1 Enrichment based, simplex problem procedure

Table 1: Enrichment change with cycles

Cy	Batches							
	a	b	c	d	e	f	g	h
1	e1	e2	e3					
2		e2 -A	e3 -B	e4				
3			e3-B -B1	e4 -C	e5			
4				e4-C -C1	e5 -D	e6		
5					e5-D -D1	e6 -F	e7	
6						e6-F -F1	e7 -G	e8

where, A, B, C, D, F, G, B1, C1, D1, F1 represent the enrichment reduction after depletion.

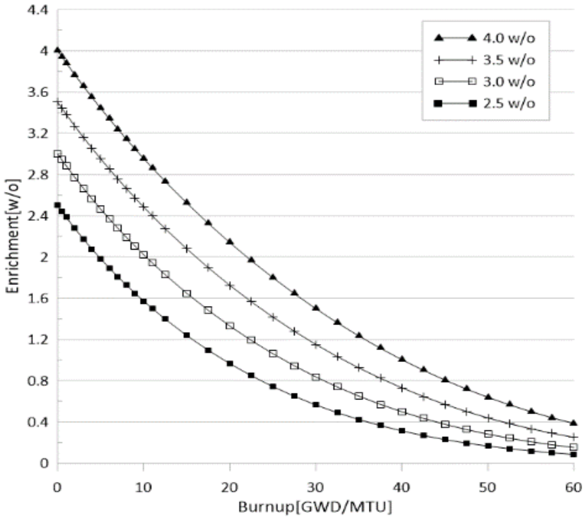

Fig. 2 Enrichment change with depletion

The entire core is assumed consisting of 3 batches in the EBA. Therefore, it is required to know how much enrichment will remain after the 1st and 2nd depletion in order to calculate the core average enrichment. Table 1 shows the enrichment change of each batch as cycles progress.

In Fig. 2, the variation of enrichment decreases exponentially depending on the burnup. However, it can be said that enrichments in certain intervals of interest decrease linearly. Such a linear change appears identically for all enrichment levels, therefore, Eq. (1) can be used to calculate the average enrichment after depletion.

$$Enrichment = K \times Burnup + T \tag{1}$$

where, K: burnup coefficient for enrichment
T: enrichment constant

After developing the enrichment formula, two simplex problems are modeled as followings.

The objective of the first simplex problem is to minimize the total number of assemblies in 6 cycles. And the objective function will be expressed in the form of Eq. (2).

$$Z_n = x_1 + \sum_{i=2}^{8} (x_{ir} + x_{id}) \tag{2}$$

where, x_i : number of fuel assemblies in batch i
r : reloaded
d : discharged

In this problem, decision variables are the value to be determined, that is the number of fuel assemblies of each batch: x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8. Constraints of the first simplex problem are as follows:

- Constraint 1: Core average enrichment of each cycle should be more than the required core average enrichment;
- Constraint 2: The number of FAs of each batch must be within 76 and 100;
- Constraint 3: The total number of FAs in the core is 241 for each cycle;
- Constraint 4: Decision variables are integers.

The objective of the second simplex problem is to minimize the total enrichment for 6 cycles, with the objective function is expressed in Eq. (3).

$$Z_e = \sum_{i=1}^{8} e_i \tag{3}$$

where, e_i is the enrichment of *i* batch

In the simplex 2 problem, decision variables are the enrichment of each batch: e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8. Constraints of the second simplex problem are as follows:

- Constraint 1: Calculated core average enrichment of each cycle should be more than the expected core average enrichment read from the FMNG;
- Constraint 2: Enrichment of each batch should be within boundaries such as $2.0 \le e_1 \le 3.0$.

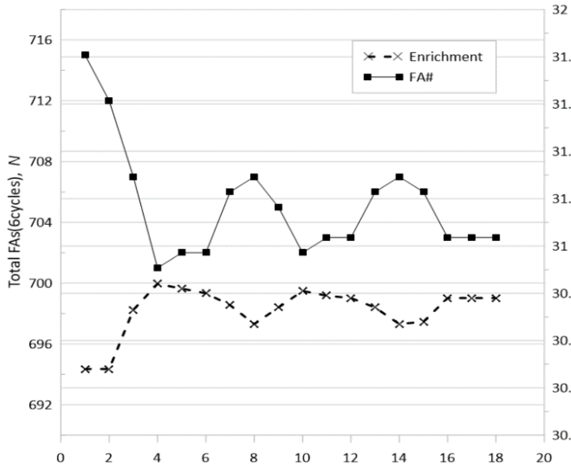

Fig. 3 Simplex method results after several iterations

Table 2: Final two simplex results

Batch / Item	a	b	c	d	e	f	g	h
Number of Ass.	77	76	88	93	99	86	90	94
Enrichment (%)	2.0	2.5	3.5	4.5	4.38	4.7	4.7	4.5

Microsoft Excel is used for solving the two simplex problems. After several iterations, two problem results converge successfully at a certain point with stable values. The final iteration represents the optimized values for enrichment and the total number of FAs, as illustrated in Fig. 3.

2.2 Burnup Based Approach (BBA)

The purpose of BBA is to minimize the batch-wise assembly numbers (N_i) and enrichment (E_i) using batch-wise burnup (B_i) information. In this approach, the simplex method is used, and the B_i values of each batch are used as constraints of the simplex problem. Therefore, calculating the correct B_i values are key point of this technique. Fig. 4 shows the logic flow of the proposed method.

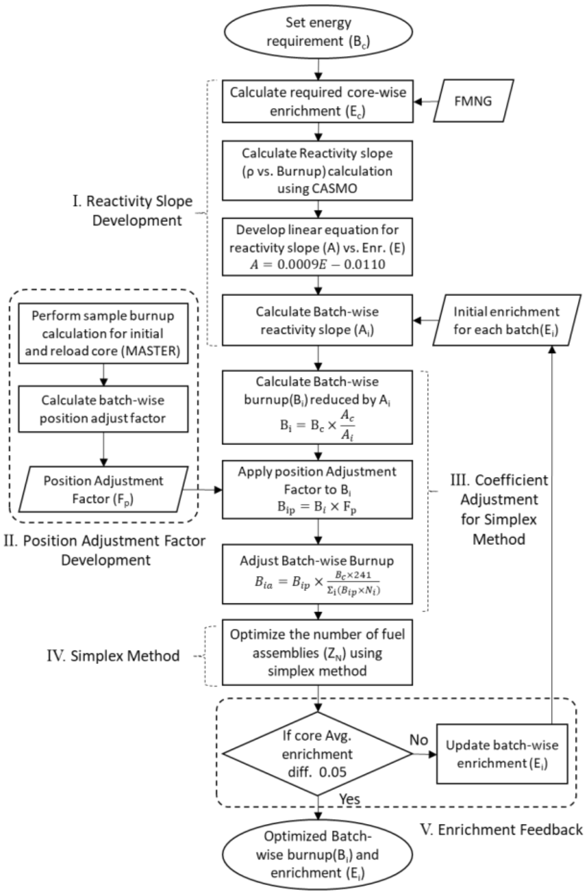

Fig. 4 Logic flow of BBA method

In a simple one-batch depletion, the reactivity is considered as a linear function of core average burnup [6]:

$$\rho = \rho_0 - A_c B_{c,d} \tag{4}$$

where $B_{c,d}$ is the core average discharge burnup and A_c is a slope constant dependent on core average enrichment. However, the slope constant for each batch is different because of various enrichment levels. The batch-wise burnup (B_i) can be expressed with Eq. (5).

$$B_i = B_c \times \frac{A_c}{A_i} \tag{5}$$

where B_c is the core average burnup for each cycle to satisfy the imposed energy requirement. The core average slope constant (A_c) is calculated with CASMO3 code by inputting the core average enrichment (E_c) obtained from the FMNG [2]. The batch-wise reactivity slope constant (A_i) in part I of Fig. 4 is calculated also with CASMO3 as a function of batch-wise enrichment (E_i):

$$A_i = 0.0009E_i - 0.0110 \tag{6}$$

For the general loading pattern, batch-wise discharge burnups are dependent on their leakage. In the area with high leakage, the loss of neutrons is high, and the discharge burnup is low compared to the inside zone. Therefore, the position adjustment concept was introduced as shown in part II of Fig. 4. In order to develop position adjustment factor, the whole core was divided into four (4) different regions that have different leakage levels, shown in Fig. 5. An example of the region-wise burnup calculation result for the APR1400 initial cycle simulation is illustrated in Fig. 6. The discharge burnups of assemblies in the different region are plotted as a relative value in this figure. The average values of each region called as position adjustment factor (F_p) will be used to adjust the batch-wise burnup (B_i) in Step III in Fig. 4.

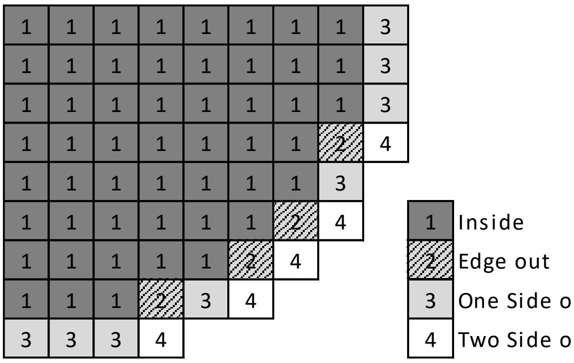

Fig. 5 Example of regions considered for leakage adjustment.

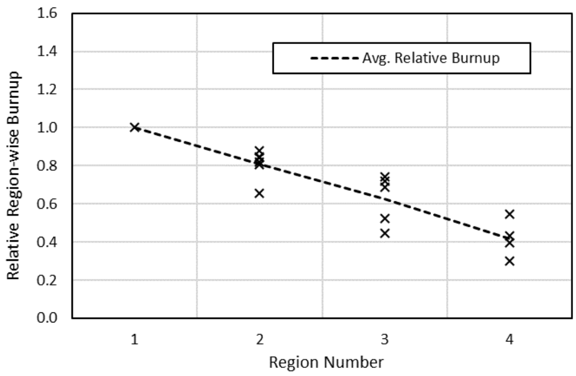

Fig. 6 Example of relative region-wise burnup.

After the batch-wise burnup (B_i) for all batches is adjusted, the simplex problem is solved in part IV of Fig. 4 with these values as coefficients (Eq. 8). The objective of the simplex problem is to minimize the total number of assemblies (Eq. (7).

$$Z_n = \Sigma_i N_i \tag{7}$$

where, N_i = the number of assemblies in batch i. Constraints of the simplex problem are as follows:

- Constraint 1: Core average burnup of each cycle must achieve the required burnup ($BU_{Required}$);

$$\frac{\Sigma_i B_i N_i}{\Sigma_i N_i} \geq BU_{Required} \tag{8}$$

- Constraint 2: The number of FAs of each batch must be within 76 and 100;
- Constraint 3: The total number of FAs in the core is 241 for each cycle;
- Constraint 4: Decision variables are integers.

Table 3 shows the results of the 1st simplex problem. Here, the calculated burnup by the simplex method satisfies the required burnup. However, the difference between the two values from the 2nd cycle is higher than 500 MWd/T, which indicates that the fuel assemblies have too much reactivity compared to the required energy. Also, the core average enrichment calculated with the set of assemblies is 2.79% which is 0.18% higher than the value read from the FMNG (2.61%). Therefore, the core reactivity should be reduced by changing enrichments.

In part V of Fig. 4, if the difference between the required core average enrichment and the optimized core average enrichment is higher than 0.05%, then batch-wise enrichments are updated. Table 4 shows that the number of batch-wise assemblies for the initial cycle (A, B, and C) is not changed while the required number of fresh fuels for reload cycles were reduced.

Table 3: Simplex Result (1st iteration)

Item \ Batch	A	B	C	D	E	F
Number of Ass.	60	81	100	100	100	100
Enrichment (%)	1.71*	2.90*	3.34*	4.25	4.25	4.25
Req. BU (GWd/T)	17.50			18.00	19.00	19.50
Calc. BU (GWd/T)	17.51			18.57	19.55	19.88
Diff. BU (GWd/T)	0.01			0.57	0.55	0.88

*Reference values from SKN-3 NDR [4]

Table 4: Simplex Result (2nd iteration)

Item \ Batch	A	B	C	D	E	F
Number of Ass.	60	81	100	98	97	93
Enrichment (%)	1.71	2.60	3.10	4.25	4.25	4.25
Req. BU (GWd/T)	17.50			18.00	19.00	19.50
Calc. BU (GWd/T)	17.52			18.29	19.14	19.50
Diff. BU (GWd/T)	0.02			0.29	0.14	0.00

3. Conclusions

The determination of the fuel enrichment and number of fuel assemblies for each batch for fuel cost optimization has been traditionally performed by heuristic rules through engineering experience.

Two simplex approaches for optimizing the fuel cost were proposed in this study and produced meaningful results that satisfied all constraints. Although it is still necessary to confirm the applicability and difference between the two methods in actual design, by using the obtained results for an actual loading pattern (LP) search.

After verification, by applying the FMNG tool to calculate the design parameters of the whole core, and using the two simplex methods to determine the optimized fuel batch size and the enrichments, an engineer can obtain useful input values as to narrow down their search of a LP.

Acknowledgment

This research was supported by the 2019 Research Fund of the KEPCO International Nuclear Graduate School (KINGS), the Republic of Korea.

REFERENCES

[1] Nuclear Energy Institute, "White Paper: The Economic Benefits and Challenges with Utilizing Increased Enrichment and Fuel Burnup for Light-Water Reactors," Nuclear Energy Institute, Washington DC, February 2019.
[2] A. C. Stafie, J. Kim and Bumhee Jo, "Approximation method of determining initial core design parameters," in Transactions of the Korean Nuclear Society Spring Meeting, Jeju, 2019.
[3] R. Fourer, "A simplex algorithm for piecewise-linear programming I: Derivation and proof," Mathematical Programming, vol. 33, no. 2, p. 204–233, November 1985.
[4] Korea Nuclear Fuel, "The Nuclear Design Report for Shin-Kori Nuclear Power Unit 3 Cycle 1," KEPCO Nuclear Fuel Companly, LTD., 2016.
[5] M. Edenius and B. H. Forssen, CASMO-3 A Fuel Assembly Burnup Program User's Manual, Studsvik Nuclear, 1991.
[6] M. J. Driscoll, T. J. Downar and E. E. Pilat, The Linear Reactivity Model for Nuclear Fuel Management, Illinois: American Nuclear Society, 1991.