Verification of Random Samples from Probability Distributions in Uncertainty Analysis of AIMS-PSA

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1. Introduction

Nuclear safety and security commission amended Enforcement Regulations of the NUCLEAR SAFETY ACT. By the regulation, all of Korean nuclear power plant have to perform Probabilistic Safety Assessment (PSA) about the ability to manage accidents. PSA is performed in various parts. Among the parts of PSA, Level 1 PSA is widely conduction using a software code AIMS-PSA to make the model of accident sequences to calculate the core damage frequency (CDF). Total CDF is calculated on the basis of failure rate of each component, human error probabilities (HEPs) from human reliability analysis (HRA), and so on. Because of the uncertainty associated with the failure rates of components, uncertainty analysis is considered to be an important part of Level 1 PSA in calculating total CDF result. As an attempt to make sure the validity of the uncertainty analysis performed using AIMS-PSA, the uncertainty analysis is verified with respect to theoretical analysis. For that purpose, we compared the result of uncertainty analysis using AIMS-PSA [1] to theoretical values and then judged whether the uncertainty analysis using AIMS-PSA is valid or not.

2. Methods and Results

AIMS-PSA provides three type of distributions, Lognormal, Beta and Gamma distributions. We made three types of one-top model which have an event that follows each of the three distributions. We performed uncertainty analysis 30 times with 100,000 samples and 5,000 intervals, and the results of AIMS-PSA in terms of mean of sample means and mean of sample standard deviations are compared with theoretical values. We define relative error as the difference between theoretical value and the value from random samples, divided by the theoretical value. We also calculated relative errors associated with samples to verify the random samples generated for uncertainty analysis in AIMS-PSA.

2.1 Lognormal distribution

Lognormal distribution is a distribution that logarithm of random variables takes the form of a normal distribution. Eq.(1) shows the basic form of a normal distribution. In terms of failure rates, lognormal distribution is a general distribution that is widely used to describe the uncertainty associated with the failure rates of component. In lognormal distributions, most failure occurs during relatively beginning time of component running [2].

$$f(x) = \frac{1}{x\sigma_t\sqrt{2\pi}} e^{\left(-\frac{(\ln x-\mu_t)^2}{2\sigma_t^{\,2}}\right)} \quad (x > 0) \qquad (1)$$

For lognormal distributions for uncertainty analysis, AIMS-PSA needs two parameters, mean value and error factor (EF). EF is a value that point of cumulative density function of lognormal distribution become 0.95 divided by point of 0.5. Mean value of lognormal distribution is, in general, is not same with the median value of logarithm distribution.

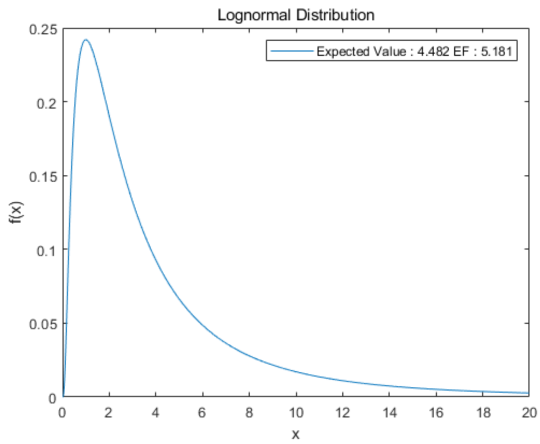

Fig. 1. Probability density function of a lognormal distribution with mean value of 4.482 and EF of 5.181

Random samples are generated from the lognormal distribution with mean value of 4.482 and EF of 5.181. Fig. 1 shows the probability density function for the lognormal distribution. From the random samples, AIMS-PSA provides sample mean and sample standard deviation for each set of random samples. The mean of sample means and mean of sample standard deviations for 30 sets of random samples are calculated to be 4.483 and 5.869, respectively. Relative errors for mean and standard deviation are calculated to be 0.0211% and 0.0949%, respectively.

2.2 Beta distribution

Beta distribution is widely used as a prior distribution in the Bayesian statistical inference for random variables with finite range between 0 and 1. Beta distribution has two parameters, α and β, which are related to the mean value and variance. Eq.(2) provides a general form of a beta distribution [2].

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} \quad (\alpha, \beta \geq 0, 0 < x < 1) \qquad (2)$$

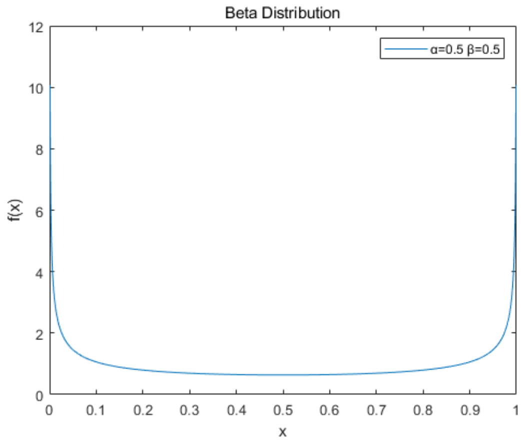

Fig. 2. Probability density function of Beta distribution with α = 0.5 and β = 0.5.

We performed an uncertainty analysis with a beta distribution with symmetric shape, by assigning parameters, α=0.5 and β=0.5. Fig. 2 shows the probability density function for the beta distribution. In this case, theoretical mean and standard deviation are 0.5 and 0.3536, respectively. From the random samples, AIMS-PSA provides sample mean and sample standard deviation for each set of random samples. The mean of sample means and mean of sample standard deviations for 30 sets of random samples are calculated to be 0.4998 and 0.3536, respectively. The relative errors for the mean of sample means and mean of sample standard deviations are calculated to be 0.04% and 0.003755%, respectively.

2.3 Gamma distribution

Gamma distribution is a distribution for the time to failure where a number of component failures are required to result in system failure, while the failure rates of components are same. It can be regarded as an extension of Poisson distribution in that the gamma distribution allows the number of failures in real numbers while Poisson distribution allows only the number of failures in integer. Gamma distribution has two parameters related with number of failures occurred and failure rate. When the number of failures is denoted as α and the single failure rate is denoted as β, Eq.(3) provides a general form of a gamma distribution.

$$f(x) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta t} \ (\alpha, \beta, x \geq 0) \qquad (3)$$

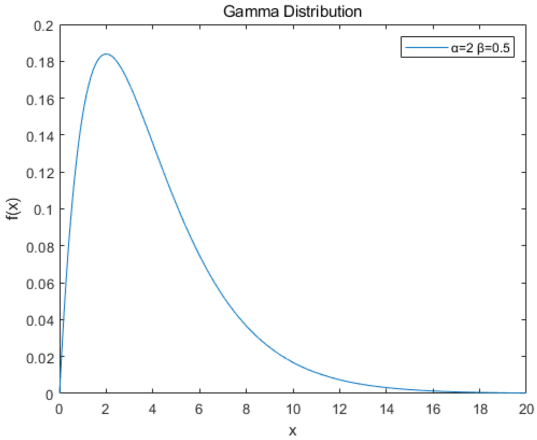

Fig. 3. Probability density function of Gamma distribution with α = 2 and β = 0.5

We performed uncertainty analysis with a gamma distribution whose parameters are α=2 and β=0.5. Fig. 3 shows the probability density function for the gamma distribution. The theoretical mean and standard deviation are calculated to be 4 and 2.828, respectively. From the random samples, the mean of sample means and mean of sample standard deviations for 30 sets of random samples are calculated to be 4.001 and 2.828, respectively. The relative errors for the mean of sample means and mean of sample standard deviations are calculated to be 0.02833% and 0.001398%, respectively.

3. Conclusions

By conducting uncertainty analysis with AIMS-PSA, mean of sample means and mean of sample standard deviations are calculated and compared with theoretical values of mean and standard deviation. By calculating relative errors, it is found that all of relative errors are lower than 0.1%. Therefore, it can be concluded that uncertainty analysis with AIMS-PSA have quite high accuracy and precision.

To calculate total CDF of a nuclear power plant, AIMS-PSA handles complex models which have components and systems with uncertainties. In such a case, the one-top model would have many basic events with associated distributions. Verification of uncertainty analysis can be extended to include logical relations such as OR gate and AND gate.

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