# Random Vibration Theory Methodology for Probabilistic Site Response Analysis

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### 1. Introduction

Seismic design of new nuclear facilities and improvement of seismic performance of existing ones have been major concerns for engineers in the area of earthquake engineering. In order to guarantee their seismic safety, seismic actions on their structural behaviors must be estimated by considering the properties of seismic sources, propagation paths of seismic waves, and local soil sites where nuclear facilities are built. The seismic actions are usually represented by a design response spectrum. The design response spectrum can be obtained from deterministic seismic hazard analysis of actual records of earthquake ground motions in the region. The standard design response spectra, specified in the United States Nuclear Regulatory Commission (USNRC) Regulatory Guide (RG) 1.60 [1], is one example of the spectra obtained from the deterministic approach. After the concept of probabilistic seismic hazard analysis was introduced, the deterministic approach began to change to a probabilistic approach. Specifically, a uniform hazard response spectrum (UHRS) was employed for seismic design of nuclear facilities. The level of earthquake ground motion in a UHRS is determined for seismic hazard, which is obtained from a probabilistic seismic hazard analysis, to be uniform for all considered frequencies. With the introduction of performancebased designs, a uniform risk response spectrum (URRS), which is also referred to as ground motion response spectrum (GMRS), was proposed to have uniform seismic risk for all frequencies [2].

It should be noted that earthquake responses of structures at soil sites are greatly affected by the soilstructure interaction. Therefore, their seismic safety must be evaluated by considering the effects of flexible soil. Four approaches were proposed in order to obtain UHRS/GMRS at soil sites from those for bedrock outcrop motions [3, 4]. Because Approach 4 considers the attenuation of seismic waves from their source to specific soil sites directly, it is the most accurate approach. Approach 3, in which considers soil amplification of seismic hazard curves for control motions at bedrock, is the best alternative among currently available approaches for most soil sites since an attenuation relation for a specific soil site is available only for well-instrumented regions with high seismicity.

Frequency contents of seismic waves, which propagate in layered soil, can be very different from those of bedrock outcrop motions. The bedrock outcrop motions have random frequency contents which depend on properties of seismic sources and propagation paths from the sources to considered sites. Therefore, when free-field motions for soil-structure interaction analysis are evaluated, the randomness must be considered in the site response analysis.

The ASCE/SEI 4-16 standard describes how to consider the mentioned randomness in site response analysis to obtain seismic input for soil-structure interaction analysis [5]. The randomness in a local soil site can be considered by simulation techniques. The Monte Carlo simulation is one possible approach for the techniques. For the simulation, the probabilistic properties for the low-strain shear-wave velocity, the relationships of shear modulus and hysteretic damping to shear strain levels, and the layer thickness must be described.

The randomness in a bedrock outcrop motion can be considered by two approaches. In the first approach of the response-history methodology, an input ground motion history consistent with a UHRS is input into the soil column as a bedrock outcrop motion. A sufficient number of input ground motions are required to consider the randomness of bedrock outcrop motion in this approach because soil responses depend heavily on the characteristics of input ground motions. On the other hand, the random vibration theory (RVT) methodology can be employed for the probabilistic site response analysis. In this approach, an input UHRS is necessary instead of time histories of input ground motions for the response-history methodology.

In this study, a RVT methodology for probabilistic site response analysis will be employed to consider the randomness in bedrock outcrop motions for UHRS/GMRS at soil sites. Specifically, earthquake ground motions, which have dominant contents at high frequencies of 10 Hz or more, will be considered. The UHRS/GMRS at rock/soil sites in the regions, where high-frequency ground motions can be observed, were evaluated in Lee et al. [6]. It was observed that UHRS/GMRS at soil sites have peaks at soil natural frequencies and the amplification in soil sites depends on the frequency contents of bedrock outcrop motions. In the study, the randomness in bedrock outcrop motions only was considered by the response-history methodology with ground motions from real earthquakes. However, the randomness in bedrock outcrop motions will be considered by the RVT methodology in this study. The effects of randomness on UHRS/GMRS at soil sites will be studied.

## 2. Probabilistic Site Response Analysis by RVT Methodology

Dynamic responses of a layered soil site subjected to seismic waves can be calculated by solving onedimensional wave-propagation problems [7]. The equilivalent linear analysis method can be employed to consider nonlinear effects in the soil site. A ground response  $r(\omega)$  can be represented as follows in the solution:

$$r(\omega) = H_r(\omega)a_{rock}(\omega) \tag{1}$$

where  $H_r(\omega)$  is a transfer function for the response  $r(\omega)$ ,  $a_{rock}(\omega)$  is the Fourier transform of an incident bedrock outcrop motion, and  $\omega$  is the excting frequency. The power spectral density (PSD) function  $G_r(\omega)$  of the response  $r(\omega)$  can then be obtained from Eq. (1).

$$G_r(\omega) = \left| H_r(\omega) \right|^2 G_{a_{rack}}(\omega) \tag{2}$$

where  $G_{a_{rock}}(\omega)$  is the PSD function for the bedrock outcrop motion. Codes and standards specify design response spectra for earthquake ground motions for seismic design of facilities. However, there is no explicit one-to-one relation between the spectra and their corresponding PSD functions. Therefore, even though design response spectra are specified in seismic design codes and standards, earthquake responses of a soil site cannot be obtained from Eq. (2) until  $G_{a_{rock}}(\omega)$ 

is defined consistent with the design response spectra.

When the bedrock outcrop motion is applied to a single degree-of-freedom (SDF) system, the PSD function  $G_{SDF}(\omega)$  for acceleration of the SDF system can be obtained as follows:

$$G_{SDF}(\omega) = |H_{SDF}(\omega)|^2 G_{a_{rack}}(\omega)$$
(3a)  
$$H_{SDF}(\omega) = \frac{1}{1 - (\omega / \omega_n)^2 + 2i\xi(\omega / \omega_n)}$$
(3b)

where  $\omega_n$  and  $\xi$  are the natural frequency and damping ratio for the SDF system, respectively. Based on the random vibration theory [8], the mean value of peak acceleration or spectral acceleration of the SDF can be estimated from its PSD function  $G_{SDF}(\omega)$ .

$$S_{a_{rock}} = \rho \sqrt{\lambda_0} \tag{4}$$

where  $\rho$  is the peak factor and  $\lambda_n = \int_0^\infty \omega^n G_{SDF}(\omega) d\omega$ ,  $n = 0, 1, 2, \cdots$ , is the *n*th-order moment of the PSD function  $G_{SDF}(\omega)$ . The peak factor can be derived based on Vanmarckle's formula.

When the PSD function  $G_{a_{rock}}(\omega)$  for a bedrock outcrop motion is defined, the corresponding acceleration response spectrum can be obtained from Eq. (4). However, it should be noted that the spectrum

does not match a design response spectrum. A PSD function, which is consistent with a design response spectrum, can be obtained through an iterative process. The PSD function is modified by the squared ratio of the estimated response spectrum during the iteration [9]. After the iteration for the PSD function  $G_{a_{ref}}(\omega)$  for

a bedrock outcrop motion to be consistent with a design response spectrum, earthquake responses of a soil site can be obtained from Eq. (2). The responses include strains in soil layers. The mean peak values of the strains can be obtained from Eq. (4) with the corresponding PSD functions  $G_{\varepsilon}(\omega)$  from Eq. (2). The mean peak values are utilized to determine straincompatible material properties for the equivalent linear analysis for nonlinear behavior of soil. Using the equivalent linear analysis, the PSD function  $G_a(\omega)$  of acceleration in a soil site and the acceleration response spectrum  $S_a$  can be obtained from Eqs. (2) and (4), respectively.

Based on the RVT methodology in the above, earthquake responses and corresponding response spectra of a soil site can be obtained when subjected to a bedrock outcrop motion which is consistent with a design response spectrum. It should be noted that no time histories of bedrock outcrop motions are required for the RVT methodology for the probabilistic site response analysis.

The RVT methodology was applied to calculate transfer functions for six generic soil sites [6]. They are compared in Figure 1 with those from the conventional response-history (RH) methodology using time histories of bedrock outcrop motions [7]. It can be observed that the RVT methodology produces reliable results without time histories of earthquake ground motions.

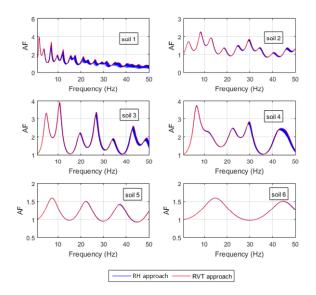


Fig. 1. Transfer functions for 6 generic soil sites.

#### 3. Conclusions

In this study, a RVT methodology for probabilistic site response analysis was employed to consider the randomness in bedrock outcrop motions for UHRS/GMRS at soil sites. Specifically, earthquake ground motions, which have dominant contents at high frequencies of 10 Hz or more, was considered. The RVT methodology was applied to calculate transfer functions for six generic soil sites. They are compared with those from the conventional RH methodology using time histories of bedrock outcrop motions. It was observed that the RVT methodology produced reliable results without time histories of earthquake ground motions.

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