

## Estimation of PCT95/95 for LBLOCA using Monte-Carlo and Wilks Method

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### 1. Introduction

In the best estimate plus uncertainty (BEPU) methodology, the direct Monte-Carlo (MC) approach has emerged as an alternative uncertainty propagation and quantification method to remedy the shortcomings of the Wilks' method. Although a utilization of the direct MC method has been increasing, most of the previous studies using the MC method have not made statistical estimations, and it is still being debated that how many samples are required to obtain the result with low uncertainty and high convergence.

Therefore, in this study, assuming the 10% power uprate of APR-1400 nuclear power plant, the uncertainty quantification analysis of large break loss of coolant accident (LBLOCA) was conducted by using the direct MC method. Based on the PCT data obtained from different sample sizes and different sampling methods, their normality and trend of statistics were evaluated. The PCT95 (i.e., 95-percentile PCT) and their 95% confidence intervals (CIs) were estimated. Then, the results of MC method were compared with those of Wilks' method for PCT95/95.

### 2. Model description and MC calculations

Assuming the 10% power uprate of APR-1400 nuclear power plant, the LBLOCA by 100 % double-ended guillotine break at the reactor coolant pump discharge leg was considered to be analyzed, and the transient was analyzed by using MARS-KS code. In the scenario, two safety injection pumps (SIPs) and two safety injection tanks (SITs) were assumed to be available reflecting previous probabilistic risk assessment results [1]. The APR-1400 system modeling is shown in the reference [2], and 18 uncertainty parameters were considered for uncertainty propagation and quantification [3].

Based on the direct MC method, 100, 200, 500, 1000, 2000 samples were made by simple random sampling (SRS) and latin hypercube sampling (LHS), and corresponding calculations were performed. In addition, the calculations using 5000 samples with SRS were performed as the reference of MC calculations. Fig. 1 shows the probability density and cumulative probability of PCT for the reference calculation. The most of PCTs appeared in the reflood phase, and some cases beyond PCT limit of 1477 K were found.

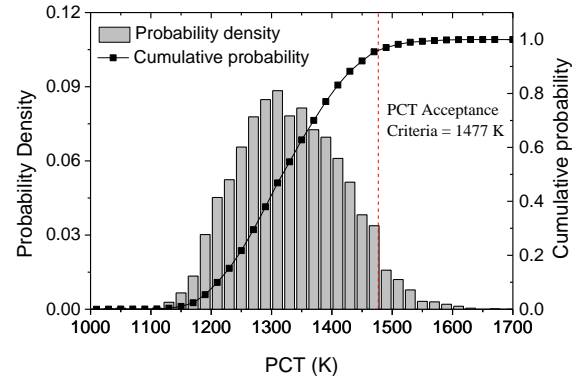


Fig. 1. Probability density and cumulative probability of PCT for reference calculation.

### 3. Results and Discussions

#### 3.1 Direct Monte-Carlo method

The normality of PCT data obtained by the direct MC calculations, was evaluated by Shapiro-Wilk normality test. Table 1 shows the p-values of MC calculations. For both SRS and LHS, if the sample size is more than 1000, the normality was not satisfied. Especially for LHS, the p-value decreased as the sample size increases.

Table I: Summary of normality test (p-value)

| Sample size | SRS       | LHS      |
|-------------|-----------|----------|
| 100         | 0.209     | 0.102    |
| 200         | 0.015     | 0.066    |
| 500         | 0.078     | 5.44E-4  |
| 1000        | 2.444E-5  | 1.824E-4 |
| 2000        | 1.833E-6  | 3.063E-9 |
| 5000        | 2.271E-13 |          |

The trends of several descriptive statistics for different MC sample sizes were additionally analyzed. Fig. 2 shows the trends of mean and some quantiles of PCT for the SRS and the LHS methods. The statistics except for the minimum and the maximum, almost converged after calculations with 500 and 200 samples for the SRS and the LHS, respectively. Fig. 3 shows the trends of standard deviation (SD) and standard error of mean (SEM) of PCT. The SD tended to converge from 1000 sample size, and the SEM decreased as the sample size increases. The SEM with 1000 samples was small enough (i.e., ~ 2.7 K) compared to PCT range (1100 ~ 1600 K), and for more sample sizes, the decreasing amounts of SEM due to the increase in sample size were too small (i.e., less than 1 K).

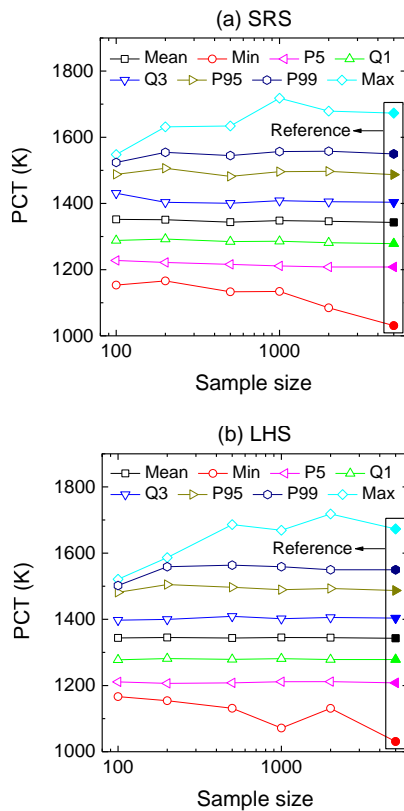


Fig. 2. Trend of mean and some quantiles of PCT; (a) SRS (b) LHS.

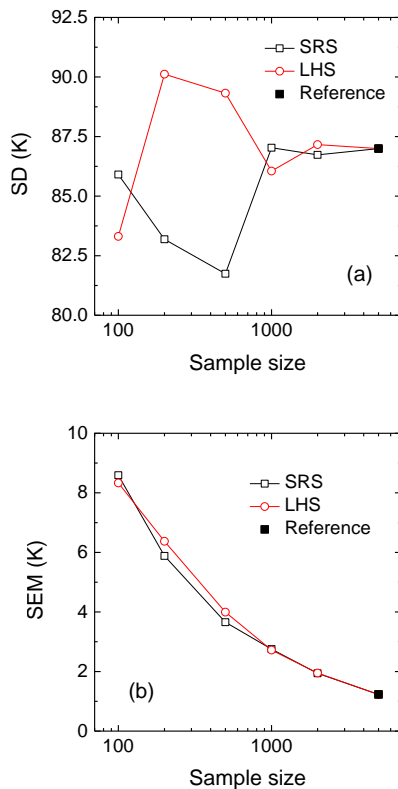


Fig. 3. Trend of SD and SEM of PCT; (a) SD (b) SEM.

From the PCT data, the PCT95 and its 95% CIs with respect to sample size and sampling method were estimated. According to the reference [4], the 95% CI of PCT95 ( $CI_{PCT95,0.95}$ ) can be expressed as following;

$$CI_{PCT95,0.95} = PCT95 \pm 2 \cdot SE_{Q_{0.95}} \approx PCT95 \pm 2 \cdot 2.11 \cdot SEM \quad (1)$$

where  $SE_{Q_{0.95}}$  is the standard error of 95 percentile and  $SEM$  is the standard error of mean. The  $SEM$  is defined as  $SD/\sqrt{n}$  where  $n$  is the sample size.

### 3.2 Comparison with Wilks' method for PCT95/95

Thirty uncertainty parameter data sets were newly sampled using the SRS method for the 2nd, 3rd, and 4th order Wilks' method. Then, 30 PCT95/95 tolerance limits (TLs) were estimated for each Wilks' method. When the 2nd, 3rd and 4th order statistics are employed, the minimum numbers of required samples are 93, 124 and 153, respectively. Fig. 4 shows the Box-Whisker plot of PCT95/95 TLs with respect to the order of Wilks' method. In particular, in Fig. 4, the reference result (i.e.,  $CI_{PCT95,0.95}$  obtained by MC method using 5000 samples) is also shown. The means of PCT95/95 TLs by Wilks' method were not within the reference result, showing more conservative results. The PCT95/95 TLs for the 4th order were distributed closest to the reference, and the standard deviation and range were also the smallest, showing that the variability is reduced as the order of statistics increases. Therefore, in order to obtain the result with better accuracy and less variability, the higher order of statistics should be used.

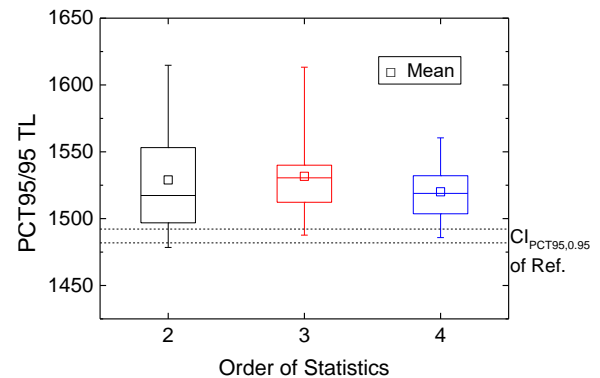


Fig. 4. Box-Whisker plot of PCT95/95 TLs using Wilks' method.

Fig. 5 shows the comparison of PCT95/95 obtained by the MC method and by the Wilks' method. For the MC method, the PCT95/95 to compare with acceptance criteria, was established as the upper confidence limit (CL) of PCT95. As aforementioned, the means of PCT95/95 TLs by Wilks' method were more conservatively estimated than the upper CL of reference result. When the sample size is 100, the width of 95% CI of PCT95 was  $\sim 70$  K, which was unacceptably large. As the sample size increases, the width of the CI became

smaller, and the more realistic PCT95/95 with narrower CI width can be obtained, as shown in the figure. Only when using 500 or more samples in the MC calculations, the upper CL could be lower than the mean of PCT95/95 TL using the 4th order of Wilks' method. However, in these cases, the decreasing amount of standard error due to the increase in sample size became too small (i.e., when the sample size increases from 1000 to 2000 and from 2000 to 5000, the standard error of PCT95 decreased only by less than  $\sim 2$  K).

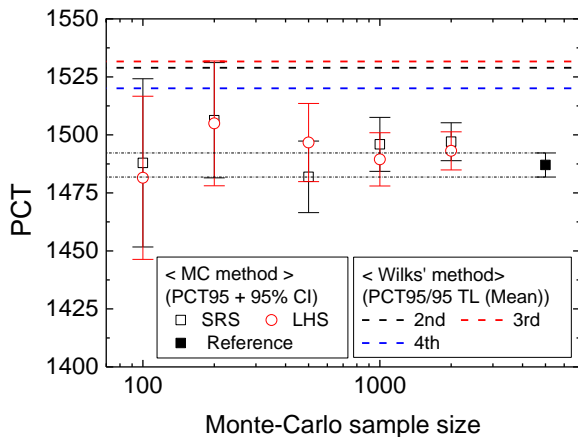


Fig. 5. Comparison of PCT95/95 by Monte-Carlo and Wilks' method.

Looking at the changes in PCT95 due to the change in the sample size, when the sample size is 500 or less, the variation was relatively large. However, when the sample size is more than 1000, the PCT95 did not fluctuate significantly. Therefore, considering both computational cost and benefit of increase in sample size, it was found that the MC method using 1,000 samples could remedy the shortcomings of Wilks' method (i.e., considerable conservatism and substantial variability) and provide reasonable PCT95/95 result. In addition, except for the case with sample size of 500, the difference in results due to the sampling method did not appear significantly.

#### 4. Conclusions

In this study, assuming the 10% power uprate of APR-1400, the uncertainty quantification analysis of LBLOCA with 18 uncertainty parameters was performed by using the direct MC method. Based on the PCT data obtained from different sample sizes and different sampling methods, their normality and the trend of statistics were evaluated. Then, the PCT95 and its 95% confidence intervals were estimated, and the results of MC method were compared with those of the Wilks' method for PCT95/95. The concluding remarks were summarized as below.

(1) When using direct MC method, a statistical estimation and sensitivity studies need to be made to obtain reliable result with low CI and high convergence.

It was confirmed that MC calculation results using a small number of samples have an unacceptably wide confidence interval.

(2) The limitations of the Wilks' method (i.e., considerable conservative bias and substantial variability) was identified, and it was confirmed that the MC method could replace them.

(3) Considering all of computational cost, benefit of increase in sample size and statistics convergence, it was found that the MC method using 1000 samples could remedy the shortcomings of Wilks' method, and it could yield reasonable PCT95/95 results. In addition, when the sample size was 1000 or more, the effect of sampling methods was not significant.

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