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A Preliminary Study on Modeling for Inverse Tracking Through the Markov Chain Monte Carlo

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1. Introduction

Estimating the location and distribution of radioactive materials released to the ground in the event of a nuclear power plant accident is fundamental to establishing response strategies to protect residents and the environment. The process of determining and quantifying the surrounding area affected by the emitted radioactive material can be performed by measuring the intensity at any point in space. The data from monitoring the concentration by forming a static sensor on the ground or by placing a detector on a mobile platform are used to estimate the information of source term. In the field of meteorology for many years, it has been used a lot in estimating the location of the fundamental emission source that discharges harmful substances in the atmosphere, but studies on inverse estimation of the source emitter for accident response at nuclear power plants considering the topography of Korea are insufficient. In this study, we introduced the parameters and equations necessary to construct the algorithm and described its application.

2. Methods and Results

In this section some of the techniques used to model the Algorithm of source term estimation are described. This includes a Source-Receptor Relationship, Bayesian inference, Dynamic Bayesian Model and Markov Chain Monte Carlo (MCMC).

2.1 Source-Receptor Relationship

A concept defined using an adjacent framework assuming that the measured value is a linear ratio the emission.

$$Q \;=\; q_s\delta(x-x_s)\;\;[H(t-t_{on})-H(t-t_{off})] \quad (1)$$

The above equation is the forward advection diffusion equation used when there is a given measurement data. First, it is necessary to model the amount of source emission through the change in the concentration distribution over time. And then, consider the concentration measurement at the i_{th} dectector.

$$R\ {}_{i}^{(j)}(m) \;=\; q_s \int_{t_{on}}^{min(t_i^{(j)},t_{off})} C*\ {}_{i}^{(j)}(x_s,t_s)\ \ dt_s \quad (2)$$

Herein, term R is the expected average concentration, and C is the measured adjacent concentration over time during the T period, and The total number of fields that must be measured and generated during time T to ensure that source M is accurately characterized must be equal to the multiplied by the number of time intervals sampled at detector.

2.2 Bayesian inference

Bayesian theory could be used to predict the probability that a hypothesis of inference is true given new evidence..

$$Posterior \;\propto\; \frac{Prior\ \times\ Likelihood}{Evidence} \;\rightarrow\; P(\theta/\ D,M,I\ ) \quad (3)$$

In the above equation, θ is the parameter constituting the model are indicated, D is the concentration measured by the monitor, M is the atmospheric dispersion model, I is the hypothesis given prior information $P(\theta/DMI)$. Herein, to estimate the probability that θ is true, the prior distribution and the actual sampling distribution must be considered. At this time, the Monte Carlo method is used to accurately estimate the posterior distribution for the parameter.

2.3 Dynamic Bayesian Model

Defining θ_t as the set of model parameters for time and y_t as the potential data that can be obtained at time t, the gap between the variable and the data can be expressed as a likelihood function, which is time-involving data model.

$$p(x_y|\theta_{1:t}) : t = 1, 2, \cdots, \quad (4)$$

However, since the actually measured data y_t does not depend only on time and parameters, it can be expressed as follows because of the joint distribution of all data in which time is considered.

$$p(y_{1:t}|\theta_{1:t}) = \prod_{t'=1}^{t} p(y_t, \theta_{1:t})$$
$$where\, y_{1:t} = (y_1, \cdots, y_t) \quad (5)$$

The input and model used in the algorithm can be specified through the Probability Density Function (PDF), and an estimate can be generated along with the Confidence Level (C.L) in consideration of the uncertainty of the input data and the selected atmospheric model. The method used Kalman filtering (KF).

2.4 Markov Chain Monte Carlo (MCMC)

Markov Chain is the definition that could be predicted the motion of a certain system, only need to know the current state with having to know all the past history. Considering the distribution through Markov Chain, the mobile kernel should be configured so that each chain has an equilibrium distribution.

$$p(X_{n+1} = s_i | X_0 = s_j, \cdots, X_n = s_k) = p(X_{n+1} = s_i | X_n = s_k) \quad (6)$$

For this, the reversible chain that the kernel satisfies must be considered. Methods to implement MCMC include Gibbs Sampling and Metropolis-Hastings (MH) method. The MH technique is a rejection sampling algorithm used to generate a sequence of samples from a probability distribution that is difficult to obtain a direct sample. It is used to simulate MCMC approximating a given distribution or to obtain a predicted value. The following is the scheme of MH.

1. Adopt a proposal or candidate sample from the proposal distribution

2. Calculate the probability of deciding whether to accept or not (Acceptance Rate)

$$\alpha(\theta, \varphi) = \mathrm{Min}\ 1, \frac{\pi(\varphi)q(\varphi, \theta)}{\pi(\theta)q(\theta, \varphi)}$$

3. Accept or Reject the proposed sample based on probability

Fig. 1. As a sampling method for using the MCMC technique, the MH algorithm proceeds in the following flow.

In general, it is easy to execute a Markov Chain with the desired characteristics. A more difficult problem is determining how many steps are required to converge to a static distribution of acceptable margin of error.

3. Conclusions

The Parameter and Modeling were described to study STE algorithm through the Markov Chain technique. Using the MCMC-MH algorithm, The paper written by Keat et al. estimated the source strength and location of the pollutant column in the urban environment, and confirmed this through experimental data. As a way to improve the calculation speed and accuracy of the MCMC algorithm, Many researches are being conducted using various sampling methods and closer sensor measurement data. It is thought that a lot of basic research should be conducted to increase stability of the emergency response system of domestic nuclear power plants by referring to these overseas study trends.

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