Monte Carlo integration approach for uncertainty analysis in PSA

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1. Introduction

Uncertainty analysis is one of major elements of probabilistic safety assessment (PSA) for nuclear power plants. The analytic solution is represented by multiple integral with Jacobian transformation [1]. However, the numerical calculation of the analytic solution has limitations when the number of basic events becomes large. That is why Monte Carlo simulation is widely applied in practical situations. However, Monte Carlo simulation have problems in estimating the tails of a distribution and in providing the resultant probability densities in a continuous manner.

On the other hand, the top event of a fault tree is generally represented by logical OR of minimal cutsets. When uncertainties of all basic events are modeled as lognormal random variables, the uncertainties of minimal cutsets are also represented with lognormal random variables. And then the uncertainty of the top event is given by the sum of lognormal random variables when the rare event approximation is applied. Song and Kim [2] propose method to calculate the probability density function (PDF) for sum of lognormal random variables with change of variable and Monte Carlo integration. In this paper, the Monte Carlo integration method is applied to uncertainty analysis for simple example and the result is compared with those of Monte Carlo simulation.

2. Analytic solution for uncertainty in top event

The exact expression of PDF for sum of lognormal random variables is derived from Jacobian transformation and change of variables.

$$f_S(s) = \int_D f_X(\boldsymbol{x}, s - x_1 \cdots - x_{n-1})\, d\boldsymbol{x} \qquad (1)$$

where f_X is joint PDF of multivariate lognormal random variables, D is region of integral, $x_1 + \cdots + x_{n-1} \leq t$, and $\boldsymbol{x}$ is $n-1$ dimensional vector, $\boldsymbol{x} = [x_1, \cdots, x_{n-1}]^T$.

It is hard to generate samples for Monte Carlo integration from the region of integral. Song and Kim [2] provides how the region of integral is transformed to the region to unit hypercube through several sequences. The probability density function in Eq. (1) is transformed to integral on $(n-1)$ dimensional unit hypercube.

$$f_S(s) = \int_{[0\ 1]^{n-1}} f_W(\boldsymbol{w}, s)\, d\boldsymbol{w} \qquad (2)$$

where

$$f_W(\boldsymbol{w}, s) = \frac{\left(\prod_{i=1}^{n-1} b_i\right) \cdot e^{-\frac{1}{2}\left[\Phi^{-1}(b_n)\right]^2}}{\sqrt{2\pi}\, C_{n,n} \left(s - \sum_{j=1}^{n-1} e^{\mu_j + \sum_{k=1}^{j} c_{j,k} \Phi^{-1}(w_k b_k)}\right)} \qquad (3)$$

Monte Carlo integration approximates the integral in Eq. (2) with the sample mean of the integrand function as

$$f_S(s) = E_W[f_W(\boldsymbol{w}, s)] \approx \frac{1}{N} \sum_{i=1}^{N} f_W(\boldsymbol{w}_i, s) \qquad (4)$$

The error of Monte Carlo integration is related to standard deviation of integrand function. The error of Monte Carlo integration is given as

$$\epsilon = 1.96 \cdot \frac{\sigma_N}{\sqrt{N}} \qquad (5)$$

where σ_N is the unbiased estimate of the standard deviation and N is the number of samples.

3. Application to an example

To demonstrating the accuracy of the proposed method, the proposed method is applied to a simple example. The example is uncertainty analysis for the simplified auxiliary feedwater system (AFWS) for a steam generator. One of main purposes of AFWS is to provide feedwater to a steam generator when main feedwater system is not available. AFWS include two types of pumps, motor-driven pumps and turbine-driven pumps, to protect AFWS from common cause failures. Each pump has two failure modes, fail-to-start and fail-to-run. It is assumed that the auxiliary feedwater pumps received feedwater from the condensate storage tank (CST).

Fig. 1 shows the fault tree for the example AFWS with two pumps and CST. The top event of the fault tree can be transformed to logical OR of minimal cutsets. Table I shows minimal cutsets for the top event of the fault tree with an initiating event. When the all the events are modeled as lognormal random variables, the minimal cutsets are also lognormal random variables. Table II shows the reliability data of basic events used in the fault tree and the initiating event data [3, 4]. The initiating event is assumed to be a lognormal random variable which have same mean. The error factor of the initiating event is assumed to be 3. The minimal cutsets are not independent because the minimal cutsets may have same events. The covariance matrix of logarithm of random variables which is a parameter of joint PDF of multivariate lognormal random variables is as follows:

$$\Sigma = \begin{bmatrix} 1.34 & 0.89 & 0.89 & 0.45 & 0.45 \\ 0.89 & 2.85 & 0.45 & 2.41 & 0.45 \\ 0.89 & 0.45 & 2.85 & 2.41 & 0.45 \\ 0.45 & 2.41 & 2.41 & 4.36 & 0.45 \\ 0.45 & 0.45 & 0.45 & 0.45 & 2.41 \end{bmatrix} \quad (6)$$

Fig. 2 shows the probability density function for the top event calculated with Monte Carlo simulation and Monte Carlo integration. Monte Carlo simulation is performed with 100,000 samples for each event. Monte Carlo integration is performed with 1% tolerance error and maximum iteration is set to 100,000. The result of Monte Carlo integration has good agreement with that of Monte Carlo simulation and provides continuous values for the whole interval of abscissas.

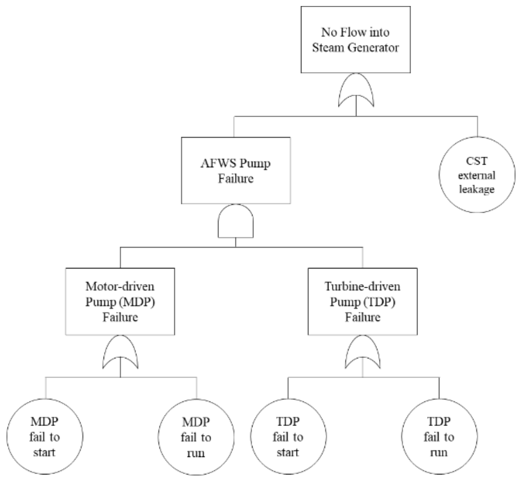

Fig. 1. Fault tree for the example AFWS

Table I: Minimal cutsets for the fault tree with an initiating event

Minimal cutset	Basic events		
1	IE-LOMFW	MDP fail to start	TDP fail to start
2	IE-LOMFW	MDP fail to start	TDP fail to run
3	IE-LOMFW	MDP fail to run	TDP fail to start
4	IE-LOMFW	MDP fail to run	TDP fail to run
5	IE-LOMFW	CST External Leakage	

Table II: Reliability data from NUREG/CR-6928 and EGG-SSRE-8875 [3, 4]

Failure Mode	Type	Parameters	
		μ_x	EF
Loss of Main Feedwater	Lognormal	1.0E-1	3
Motor-Driven Pump Fail to Start	Lognormal	1.0E-3	3
Motor-Driven Pump Fail to Run	Lognormal	6.5E-4	10
Turbine-Driven Pump Fail to Start	Lognormal	1.2E-3	3
Turbine Driven Pump Fail to Run	Lognormal	6.5E-5	10
CST external leakage	Lognormal	1.4E-3	10

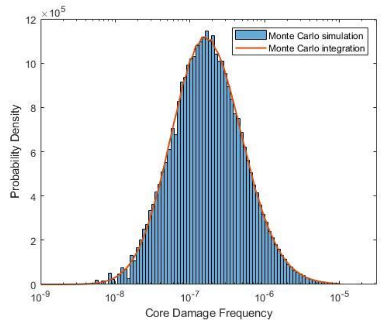

Fig. 2. Probability density function for the example with Monte Carlo simulation and Monte Carlo integration

3. Conclusions

In this paper, the Monte Carlo integration with the change of variables method is presented for the uncertainty analysis in PSA. The accuracy of the proposed method is demonstrated with a simplified uncertainty analysis example. The result is compared with that of Monte Carlo simulation. The Monte Carlo integration provides more reasonable result compared to Monte Carlo simulation. Typical numerical integration methods have problems when the number of random variables is large or when the random variables have correlation. On the other hand, the Monte Carlo integration with the change of variables can be applied to relatively large number of variables even when the random variables have correlation. In practical situations, the top event is determined by dominant minimal cutsets. Hence, the top event can be represented by the sum of lognormal random variables when the basic events are modeled as lognormal random variable. The Monte Carlo integration method with the change of variables can be used as a complementary method with Monte Carlo simulation in uncertainty analysis of PSA for nuclear power plants.

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