A Refinement of the AFEN Response Matrix Method in the Two-dimensional Trigonal Geometry

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1. Introduction

We introduced the trigonal node based Analytic Function Expansion Nodal (AFEN) method which have been being developed to deal with the asymmetric inhomogeneity inside the hexagonal fuel assembly. [1] Considering that the number of nodes increases by six times in the trigonal AFEN method compared to the hexagonal method, a relatively simple form of the AFEN method was attempted, in which only the neutron flux was used as the nodal unknown per interface. The hexagonal refined AFEN method that the trigonal method is being benchmarked against uses both the neutron flux and the flux moment as the nodal unknown per interface. This method is the first AFEN method tried for the trigonal geometry, while various forms of the AFEN method have successfully been developed in the rectangular geometry [2,3] and in the hexagonal geometry[4-7].

In spite that this method increases not only the number of nodes by six times but also the number of interface unknowns by 1.5 times, it failed in showing better accuracy than the hexagonal refined AFEN method. The most probable reason analyzed as being responsible for this poor performance was the looser flux continuity constraint applied at each interface between two adjacent hexagonal blocks in the triangular AFEN method. While the refined hexagonal AFEN method with the step weighting function in the transverse direction in the definition of the flux moment divides a hexagonal block interface in half and applies the continuity condition to each half, the triangular method applied only one continuity condition for the entire block interface.

In order to improve the loose continuity constraint of the trigonal AFEN method applied at each interface of two adjacent hexagonal blocks, it is conceivable to introduce a triangular refined AFEN which employs the flux moments as an additional unknowns at all the triangular interfaces. However, this refined AFEN method may be too much refinement in terms of increasing the number of interface unknowns by 6 times compared to the hexagonal refined AFEN method. Instead, this paper introduces a simpler refined AFEN method that adopts the flux moment as an additional interface unknown only on one of three sides in a trigonal node that interfaces with an adjacent hexagonal block. This method increases the number of interface unknowns by 2.5 times compared to the hexagonal refined AFEN method. It should also always be superior to the hexagonal refined AFEN method at least in terms of unknown numbers and constraints, because it compasses all the nodal unknowns and interface constraints of the hexagonal refined AFEN method.

2. Methodology

The trigonal refined AFEN method developed in this paper asymmetrically introduces the flux moment as an additional unknown on only one of three sides of the trigonal node. For example, for Node n in Fig. 1, the flux moment is asymmetrically introduced only at the x interface shared by the two adjacent hexagonal blocks as shown in Fig. 2.

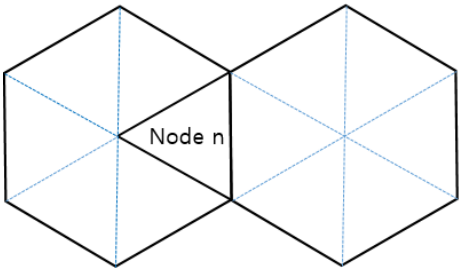

Fig. 1. Two adjacent hexagonal fuel blocks divided into trigonal nodes

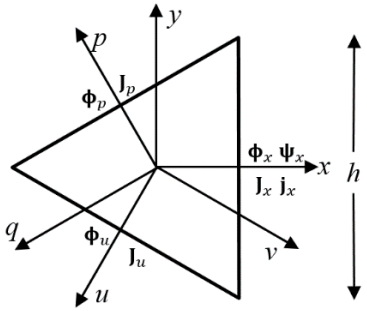

Fig. 2. Coordinate systems and nodal unknowns of Node n

In this figure, $\boldsymbol{\psi}_x$ and $\mathbf{j}_x$ are the flux moment and its corresponding current moment at the interface, respectively. The other quantities in Figure 2 are defined as in Reference 1, but $\boldsymbol{\psi}_x$ and $\mathbf{j}_x$ are defined by.

$$\boldsymbol{\psi}_x = \frac{1}{h}\int_{-\frac{h}{2}}^{\frac{h}{2}} w(y)\boldsymbol{\phi}\left(\frac{\sqrt{3}}{6}h, y\right) dy \tag{1}$$

and

$$\mathbf{j}_x = \frac{\mathbf{D}}{h}\int_{-\frac{h}{2}}^{\frac{h}{2}} w(y)\frac{\partial}{\partial x}\boldsymbol{\phi}(x,y)\, dy\Big|_{x=\frac{\sqrt{3}}{6}h} \tag{2}$$

respectively. Here, $w(y)$ is a weighting function in the y direction and we use the step function as the weighting function which is a unit function with the sign changing across $y = 0$.

The response matrix for the refined AFEN method is derived for Node n by a two-step procedure similar to that applied in Reference 1: the first step to solve a single node problem to obtain the relationship between the interface fluxes and the interface currents and the second step to derive the response matrix by replacing the interface fluxes and interface currents in this relationship with the incoming and outgoing interface partial currents.

2.1 Refined AFEN Solution of Single-Node Problem

2.1.1 Intranodal Flux Expansion

We have retained the three fluxes symmetrical to the three interfaces of a node, and additionally introduced the asymmetrical flux moment at only one interface. Therefore, the intranodal flux expansion function corresponding to the three symmetrical interface fluxes can be exactly same to that used in Reference 1. This flux expansion function consists of only valid three of six component functions: $\sinh(\sqrt{\mathbf{\Lambda}}x)$, $\cosh(\sqrt{\mathbf{\Lambda}}x)$ and their four symmetric partners to the 120-degree and 240-degree directions. Please see Reference 1 for the detailed procedure of obtaining this flux expansion function which is shown below:

$$\begin{aligned}\boldsymbol{\phi}_s(x,y) = &\left\{2\cosh\left(\tfrac{\sqrt{\Lambda}}{2}x\right)\cosh\left(\tfrac{\sqrt{3\Lambda}}{2}y\right)+\cosh(\sqrt{\mathbf{\Lambda}}x)\right\}\mathbf{A}_\theta\\ &+\left\{\sinh\left(\tfrac{\sqrt{\Lambda}}{2}x\right)\cosh\left(\tfrac{\sqrt{3\Lambda}}{2}y\right)+\sinh(\sqrt{\mathbf{\Lambda}}x)\right\}\mathbf{A}_\varepsilon\\ &-3\cosh\left(\tfrac{\sqrt{\Lambda}}{2}x\right)\sinh\left(\tfrac{\sqrt{3\Lambda}}{2}y\right)\mathbf{A}_\chi\end{aligned} \quad (3)$$

where A_θ, A_ε and A_χ are the three transformed coefficients resulted by applying the direction decoupling transformation described in Reference 1. Of course, this function satisfies the neutron diffusion equation everywhere within the node to conform to AFEN's philosophy.

Now, the expansion function component corresponding to the flux moment at the x interface must be added to this symmetric expansion function. Since we asymmetrically introduced the flux moment only at the x interface, the expansion function component to be added no longer needs to be symmetric in the three side-directions. Rather, $\sinh(\sqrt{\mathbf{\Lambda}}x)$ multiplied by a linear function in the y direction is the most natural one which complies the diffusion equation in the node because the symmetric expansion function Eq. (3) does not contain a component that is odd in both the x and y directions.

Then, the intranodal flux expansion becomes

$$\boldsymbol{\phi}(x,y) = \boldsymbol{\phi}_s(x,y) + \sinh(\sqrt{\mathbf{\Lambda}}x)\mathbf{B}_x \quad (4)$$

Note that this flux expansion includes all the combinations in which even and odd functions in the x and y directions can be combined. The even-odd test described in Reference 1 performed in each of x- and y-directions for the four expansion components in Eq. (4) revealed that all four of these are valid for the flux expansion of a trigonal node.

2.1.2 Refined AFEN Solution of Single-Node Problem

Solving the single node problem in Fig. 2 to obtain the intranodal flux distribution means expressing four coefficients of the flux expansion Eq. (4) in terms of three transformed interface fluxes and one interface flux moment. According to the direction decoupling transformation, the transformed fluxes and currents are given as a linear combination of the original interface fluxes and currents shown in Fig. 2 as follows:

$$\boldsymbol{\phi}_\theta = \tfrac{\boldsymbol{\phi}_x+\boldsymbol{\phi}_u+\boldsymbol{\phi}_p}{3} - \bar{\boldsymbol{\phi}}, \boldsymbol{\phi}_\varepsilon = \tfrac{2\boldsymbol{\phi}_x-\boldsymbol{\phi}_u-\boldsymbol{\phi}_p}{3}, \boldsymbol{\phi}_\chi = \tfrac{\boldsymbol{\phi}_u-\boldsymbol{\phi}_p}{3} \quad (5)$$

$$\mathbf{J}_\theta = \tfrac{\mathbf{J}_x+\mathbf{J}_u+\mathbf{J}_p}{3},\ \ \mathbf{J}_\varepsilon = \tfrac{2\mathbf{J}_x-\mathbf{J}_u-\mathbf{J}_p}{3},\ \ \mathbf{J}_\chi = \tfrac{\mathbf{J}_u-\mathbf{J}_p}{3} \quad (6)$$

where $\bar{\boldsymbol{\phi}}$ is the volume average flux of the node.

When we express the unknowns in Eq. (4) in terms of the transformed expansion coefficients, the simple AFEN method introduced in Reference 1 decouples completely the original 3x3 matrix system into three scalar systems due to the decoupling transformation. However, the decoupling transformation of the refined AFEN method in this paper decouples the 3x3 system into two scholar systems and one 2x2 matrix system. For θ and ε components, two scholar systems are obtained:

$$\boldsymbol{\phi}_\theta = \mathbf{P}_\theta\mathbf{A}_\theta, \quad \boldsymbol{\phi}_\varepsilon = \mathbf{P}_\varepsilon\mathbf{A}_\varepsilon \quad (7)$$

or

$$\mathbf{J}_\theta = \mathbf{D}\mathbf{Q}_\theta\mathbf{A}_\theta, \quad \mathbf{J}_\varepsilon = \mathbf{D}\mathbf{Q}_\varepsilon\mathbf{A}_\varepsilon \quad (8)$$

For χ component, one 2x2 system is obtained:

$$\begin{pmatrix}\boldsymbol{\phi}_\chi\\ \boldsymbol{\psi}_x\end{pmatrix} = \mathbf{P}_\chi\begin{pmatrix}\mathbf{A}_\chi\\ \mathbf{B}_x\end{pmatrix} \quad (9)$$

or

$$\begin{pmatrix}\mathbf{J}_\chi\\ \mathbf{j}_x\end{pmatrix} = \mathbf{D}\mathbf{Q}_\chi\begin{pmatrix}\mathbf{A}_\chi\\ \mathbf{B}_x\end{pmatrix} \quad (10)$$

By eliminating the coefficient vector, we can get the relationship between the transformed interface fluxes and currents:

$$\boldsymbol{\phi}_\alpha = \mathbf{T}_\alpha\mathbf{J}_\alpha, \ \begin{pmatrix}\boldsymbol{\phi}_\chi\\ \boldsymbol{\psi}_x\end{pmatrix} = \mathbf{T}_\chi\begin{pmatrix}\mathbf{J}_\chi\\ \mathbf{j}_x\end{pmatrix} \quad (11)$$

where α is θ or ε and

(12)

$$\mathbf{T}_\beta = \mathbf{P}_\beta\mathbf{Q}_\beta^{-1}\mathbf{D}^{-1},\ \beta = \theta, \varepsilon\ or\ \chi \quad (13)$$

Actually, all relationship matrixes **T**'s just before multiplying the inverse of **D** to the far right of Eq. (13) are a matrix function of $\mathbf{\Lambda}$. By expanding the relationship matrix, e.g., $\mathbf{T}_\theta$ in Taylor series of $\mathbf{\Lambda}$,

$$\mathbf{T}_\theta = \left\{2\sqrt{3} + \tfrac{\mathbf{\Lambda}}{8\sqrt{3}} - \tfrac{\mathbf{\Lambda}^2}{5760\sqrt{3}} + O(\mathbf{\Lambda}^3)\right\}\mathbf{D}^{-1} \quad (14)$$

we can show that the flux expansion function is physically valid because the relationship matrix is not singular across $\mathbf{\Lambda} = \mathbf{0}$.

2.2 Refined AFEN Response Matrix

The response matrix that computes the interface partial currents going out of a node from the interface partial currents coming into the node is derived by noting that the interface partial current at the interface *s* is expressed in terms of the interface flux and current.

$$\mathbf{P}_s^f = \tfrac{\mathbf{J}_s^f}{2} + \tfrac{\boldsymbol{\Phi}_s}{4} \quad (15)$$

where flow direction index *f* is *in* or *out*, interface index *s* is *x*, *u*, or *p*. This relation for the partial current moments is also similarly given by

$$\mathbf{p}_x^f = \frac{\mathbf{j}_x^f}{2} + \frac{\boldsymbol{\psi}_x}{4} \quad (16)$$

Then, the interface flux and current are equivalently given by

$$\mathbf{J}_s^{in} = \mathbf{P}_s^{in} - \mathbf{P}_s^{out}, \;\; \boldsymbol{\phi}_s = 2(\mathbf{P}_s^{in} + \mathbf{P}_s^{out}) \quad (17)$$

A similar expression is also given for the flux moment and the current moment.

$$\mathbf{j}_x^{in} = \mathbf{p}_x^{in} - \mathbf{p}_x^{out}, \;\; \boldsymbol{\psi}_x = 2(\mathbf{p}_x^{in} + \mathbf{p}_x^{out}) \quad (18)$$

Since the relationship (16) is linear and the direction transformation explained in Reference 1 is also linear, the partial current shall have its transformed form with respect to the transformation and this form shall be just the form in which interface index *s* is substituted the direction index θ, ε, or χ.

$$\mathbf{J}_\alpha^{in} = \mathbf{P}_\alpha^{in} - \mathbf{P}_\alpha^{out}, \;\; \boldsymbol{\phi}_\alpha = 2(\mathbf{P}_\alpha^{in} + \mathbf{P}_\alpha^{out}), \;\; \boldsymbol{\alpha = \theta, \varepsilon, or\ \chi} \quad (19)$$

Substituting the relationships Eq. (18) and Eq. (19) into Eq. (11) and solving for the transformed outgoing partial current and the outgoing partial current moment, we finally obtain the response matrix for the refined AFEN method as follows,

$$\mathbf{P}_\alpha^{out} = \mathbf{R}_\alpha \mathbf{P}_\alpha^{in}, \;\; \alpha = \theta \; or \; \varepsilon \quad \begin{pmatrix} \mathbf{P}_\chi^{out} \\ \mathbf{p}_x^{out} \end{pmatrix} = \mathbf{R}_\chi \begin{pmatrix} \mathbf{P}_\chi^{in} \\ \mathbf{p}_x^{in} \end{pmatrix} \quad (20)$$

where $\mathbf{R}_\alpha = -(2\mathbf{I} + \mathbf{T}_\alpha)^{-1}(2\mathbf{I} - \mathbf{T}_\alpha)$ for α = θ, ε, or χ.

Note that the interface partial currents can be easily transformed into their linearly transformed partners and vice versa. Once the interface incoming partial currents are given for a node, the interface outgoing partial currents can be calculated by the response matrix Eq. (20). Then, these outgoing partial currents become the partial currents incoming into its neighboring nodes. This provides an iterative process to solve the global core eigenvalue problem through the well-known inner-outer iteration. As in Reference 1, the number of inner iterations per outer iteration is set to be one and the RGB-BW sweeping scheme is adopted in this paper. This kind of iteration schemes is known to be good in convergence and stability due to geometrical balance. It further enhances the advantage in parallel-computing that the response matrix method has already.

3. Numerical Results and Discussion

Note that we developed this triangular refined AFEN method in order to improve the accuracy of the simple AFEN method introduced in Ref. 1, which is suspected of degrading performance due to the loose continuity constraints applied to each interface of two adjacent hexagonal blocks. This refined AFEN method encompasses the refined hexagonal AFEN method because it contains all the node unknowns and continuity constraints of the refined hexagonal AFEN method and further more et cetera.

The ability of this refined AFEN method to overcome the inferiority of the simple AFEN method was verified by solving the MHTGR-350 problem which was also used in Reference 1 as a benchmark problem.

In Fig. 3, the assembly-wise relative powers of two trigonal AFEN methods were compared with those of the hexagonal refined AFEN method. Since all the AFEN methods are based on the response matrix method, the zero incoming partial current boundary condition was applied. As mentioned in Reference 1, the accuracy of the hexagonal AFEN method was well shown in the reference 3. This accuracy is also demonstrated by showing that, when compared with the CAPP solution of the MHTGR-350 problem with the cubic finite element option, a 22 pcm error occurs in the effective multiplication factor and up to 0.2% error in the block-wise power distribution. Therefore, the hexagonal method can be estimated to show an error of this level or less for the extrapolated fine mesh solution of the diffusion equation for this problem.

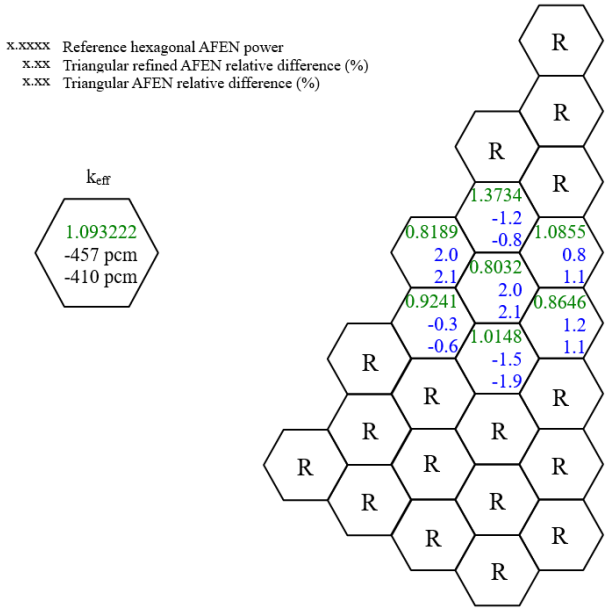

Fig. 3. Results of the MHTGR-350 benchmark problem (1/12 core).

Following the disappointment we received in Reference 1, this figure delivers another disappointment to us. Obviously, the refined trigonal AFEN method provides better results than the simple AFEN method. However, the amount of improved accuracy is very small. Moreover, it is natural that the refined trigonal AFEN method should show better results than even the hexagonal refined AFEN method, because the whole set of the nodal unknown and continuity constraints is a subset of the set of he trigonal refined AFEN method. Contrary to our expectation, the trigonal method has a power error of 2% magnitude against the hexagonal method which is estimated to show only a power error of within 0.2% to the exact reference solution. Now, there is a situation where we have to suspect a program bug. To reduce the risk of such a situation, we solved the MHTGR-350 benchmark problem by reducing the size of the hexagonal block. The core configuration was left as it was, and the block size was sequentially reduced to 1/2, 1/4, 1/8, 1/16, and 1/32 times. The reason for reducing the overall problem size is simply because the core made of hexagonal blocks cannot be divided into smaller hexagonal meshes.

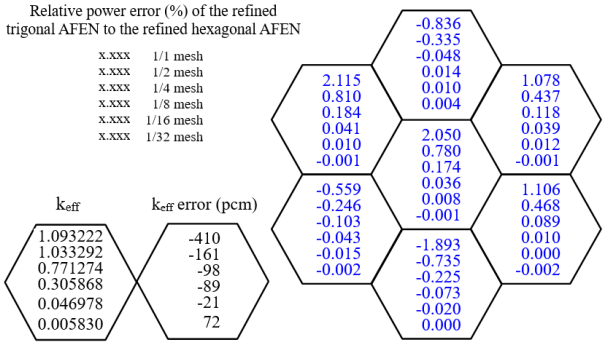

Fig. 4. Results of the MHTGR-350 benchmark problem with mesh refinement (1/12 core).

In Fig. 4, the results of the refined trigonal AFEN method were compared with those of the refined hexagonal AFEN method. Unlike Fig. 3, this figure has only the active core in order not to get too busy. Looking at this figure, it can be seen that as the node size decreases, the solutions of the two methods converge to each other. However, the relative error of the effective multiplication factor cannot be ignored even when the node size is very small, which may be because the absolute value of the effective multiplication factor is small. This result reduces the likelihood that the trigonal AFEN's low performance is due to a bug. The remaining cause that can now be suspected is that the AFEN expansion function is inefficient in representing the neutron flux distribution within a trigonal node. It will be a future study to compare the various expansion functions including polynomials and finally to find out the cause of the performance degradation of the trigonal AFEN method.

4. Conclusions

In Reference 1, we introduced the AFEN method in the trigonal geometry which uses only the neutron flux per interface as the nodal unknown. However, we reported the poor performance of this method despite increasing the number of interface unknowns by 1.5 times compared to the refined hexagonal AFEN method. The most probable cause of this poor performance analyzed in Reference 1 was the loose flux continuity constraint applied to each interface between two adjacent hexagonal blocks. To eliminate this cause, we developed the refined AFEN method in this paper that adopts the flux moment as an additional interface unknown only on one of three sides in a trigonal node that interfaces with an adjacent hexagonal block.

This method should have shown better results than the hexagonal refined AFEN method, because it compasses all the nodal unknowns and constraints of the hexagonal method. However, this method was not only inferior to the hexagonal method, but also showed similar accuracy to the trigonal method in Reference 1, which we intended to improve.

Now we are suspecting that the inferiority of the trigonal method is due to the AFEN expansion function being inefficient in representing the neutron flux distribution within the node. It will be a future study to find the final cause of the performance degradation of the trigonal AFEN method by trying various flux expansion functions including polynomials to represent the intranodal flux distribution within a trigonal node properly.

Alternatively, this triangular nodal method, along with the hexagonal nodal method, may constitute a global-local iteration method. In this method, the triangular nodal method solves a single asymmetrically heterogeneous hexagonal block with the boundary condition obtained by the global hexagonal nodal calculation and passes the homogenization constants of the block to the hexagonal nodal calculation. This global-local iteration method facilitates treatment of heterogeneous hexagonal blocks while maintaining the accuracy of the hexagonal nodal method.

Acknowledgements

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