Probabilistic Model of PWSCC in Alloy 690 Steam Generator Tubing

Dayu Fajrul Falaakh [a], Chi Bum Bahn [a*]
[a] *School of Mechanical Engineering, Pusan National University, Busan 46241, Republic of Korea*
[*] *Corresponding author: bahn@pusan.ac.kr*

1. Introduction

Alloy 690 has been used as replacement of Alloy 600 for components of nuclear power plants (NPPs), such as reactor pressure vessel head penetration nozzles in pressurized water reactors (PWRs) and steam generator (SG) tubing. Compared to its predecessor, Alloy 690 offers much better resistance to stress corrosion cracking (SCC) in the primary water environment of PWRs, which is also known as primary water stress corrosion cracking (PWSCC) [1]. There has not been PWSCC observed in Alloy 690-based components in PWRs to date. Nevertheless, developing an ability to predict PWSCC initiation time of Alloy 690 is indispensable.

Probabilistic modeling has been used for PWSCC initiation time prediction. However, due to difficulty in acquiring data concerning with PWSCC initiation time, methods that can deal with the absence of failure in the test should be used. Weibayes method has been developed and proposed in reference [1]. In the current work, a probabilistic model adopting Bayesian method is proposed.

2. Database

The dataset used here are obtained from experience of NPPs with Alloy 600 MA and Alloy 690 TT SG tubing up to the year of 2008 summarized in reference [2]. In the current analyses, the axially-oriented PWSCC in hot leg expansion is considered and the failure criterion is when 0.1% tubes have developed PWSCC. There are 12 NPPs with Alloy 600 MA tubing being considered, out of which 10 plants reached the failure criterion threshold. The time to reach the failure criterion varies from 2.29 to 11.48 effective full power years (EFPYs). Two of the 12 NPPs had operated 12.08 and 23.50 EFPYs without experiencing PWSCC. Out of 54 plants with Alloy 690 TT tubing being considered, none had developed PWSCC. The plant operating time varies from 2.41 to 18.91 EFPYs.

3. Methods

3.1. Probabilistic model

The time to PWSCC initiation has been modeled based on Weibull distribution, with the cumulative distribution function and probability density function given by (1) and (2), respectively.

$$F(t) = 1 - exp\left\{-\left(\frac{t}{\eta}\right)^{\beta}\right\} \quad (1)$$

$$f(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \exp\left\{-\left(\frac{t}{\eta}\right)^{\beta}\right\} \quad (2)$$

where, t is time, $\eta > 0$ is the scale parameter and $\beta > 0$ is the shape parameter of the Weibull distribution [2]. The Weibull scale parameter η can be re-parameterized as the time to reach the earliest (first) failure t_{1st}:

$$t_{1st} = \eta\left\{ln\left(\frac{n}{n-1}\right)\right\}^{1/\beta} \quad (3)$$

The probabilistic models (1) and (2) can be modified as:

$$F(t) = 1 - \left(\frac{n}{n-1}\right)^{\left\{-\left(\frac{t}{t_{1st}}\right)^{\beta}\right\}} \quad (4)$$

$$f(t) = \frac{\beta}{t_{1st}}(t)^{\beta-1}\left(\frac{n}{n-1}\right)^{\left\{-\left(\frac{t}{t_{1st}}\right)^{\beta}\right\}} ln\left(\frac{n}{n-1}\right) \quad (5)$$

where n is the number of the observed data. The values of the model parameters t_{1st}, η and β are unknown. In the current work, Bayesian approach will be used to estimate the values of the model parameters.

3.2. Bayesian approach

In Bayesian approach, the model parameters (i.e. η and β) are treated as random variables that have certain probability distributions. The distribution or mostly called as posterior distribution of a model parameter is determined by updating the prior belief (i.e. distribution) of the parameter given the observed data (e.g. time to PWSCC). Let θ be the parameter vector containing the model parameters. Suppose that $g(t|\theta)$ is the likelihood function of the observed data t given the parameter θ, $h(\theta)$ is the prior distribution of the parameter θ. The posterior distribution of θ given t, $f(\theta|t)$, can be obtained by:

$$f(\theta|t) = \frac{g(t|\theta)h(\theta)}{\int g(t|\theta)h(\theta)d\theta} \quad (6)$$

The likelihood function can generally be given by:

$$g(t|\theta) = \prod_{i=1}^{r} f(t_i) \prod_{j=1}^{n-r}\left\{1 - F(t_j)\right\} \quad (7)$$

where r is the number of failures, $t_i = t_1, t_2, \ldots, t_r$ denotes the uncensored (complete) time to failure data, and $t_j = t_1, t_2, \ldots, t_{n-r}$ denotes the censored time to failure data. In the case of Alloy 600 MA tubing, both uncensored and censored data exist. On the other hand, only censored data exist in the case of Alloy 690 TT tubing since none of the tubing had developed PWSCC.

The prior distributions of the model parameters used for the analysis in the current work include gamma distribution, normal distribution, and beta distribution, whose distribution functions are respectively given by Equations (8), (9), and (10).

$$h(\theta) = \frac{b^a \theta^{a-1}}{\Gamma(a)} e^{-\theta b} \quad (8)$$

$$h(\theta) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(\theta-\mu)^2/2\sigma^2} \quad (9)$$

$$h(\theta) = \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)(u-l)} \left(\frac{\theta - l}{u-l}\right)^{p-1} \left(\frac{u-\theta}{u-l}\right)^{q-1} \quad (10)$$

where a, b, μ, σ, p, q, l, and u are called as the hyper-parameters.

4. Numerical Analysis and Results

4.1. Prior distributions of the model parameters

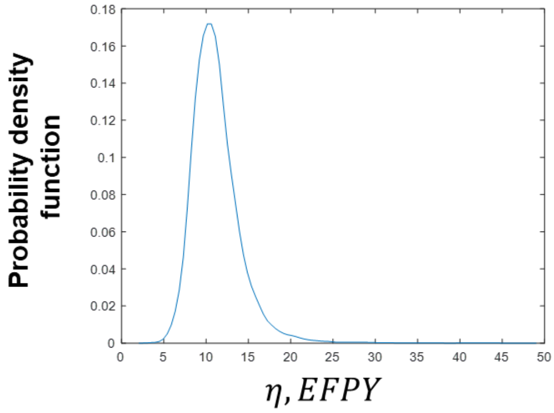
For the analysis of Alloy 600 MA tubing, the prior distribution of β is a beta distribution with $p = q = 1$, $l = 0$, and $u = 5$. Thus, the prior assumption is that the β is uniformly distributed from 0 to 5 [2]. The prior distribution of η is a gamma distribution with $a = 1/2$ and $b = 0$, which makes the distribution become a Jeffrey prior [3].

The posterior distribution of β for Alloy 600 MA tubing can serve as a prior distribution for the analysis Alloy 690 TT tubing. The normal distribution with $\mu = 18.91$ and $\sigma = 5$ as the prior distribution of the earliest failure of Alloy 690 TT tubing. It is initially assumed that the earliest failure would likely happen immediately after the plant with the longest service time was examined.

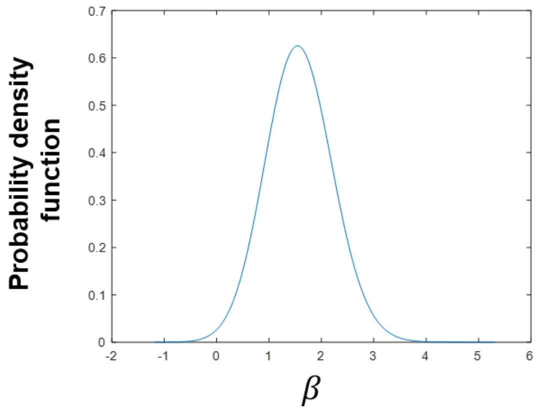
4.2. Results

The posterior distributions of η and β for Alloy 600 MA are shown in Fig. 1. The posterior mean of η is 10.77 EFPYs while the mean of the β posterior is 1.54. The various credible interval with the corresponding credible levels from the obtained posterior distributions are shown in Table I. As a comparison, the results obtained from maximum likelihood estimation (MLE) [1] are written in Table II.

The posterior distributions of t_{1st} and β for Alloy 690 TT are shown in Fig. 2. The posterior mean of t_{1st} is 19.80 EFPYs while the mean of the β posterior is 1.58. The earliest failure time t_{1st} can be transformed into the Weibull scale parameter η via Equations (3). The distributions of η for Alloy 690 tubing, having a mean of 243.30 EFPYs, is shown in Fig. 3. The various credible interval with various credible levels from the obtained posterior distributions are shown in Table III. As a comparison, Weibayes analysis yields values as listed in Table IV.



(a)



(b)

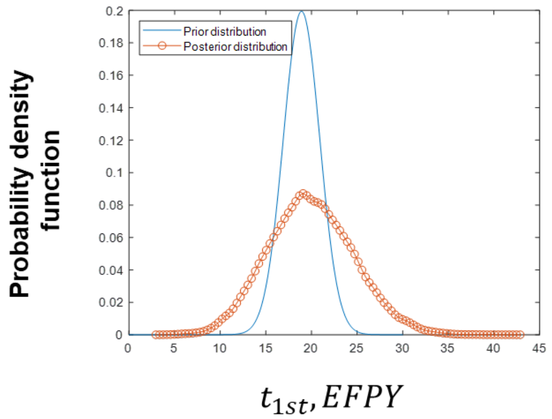
Fig. 1. The posterior densities of (a) $\boldsymbol{\eta}$ and (b) $\boldsymbol{\beta}$ for Alloy 600 MA tubing

Table I. Credible levels and intervals of Fig. 1

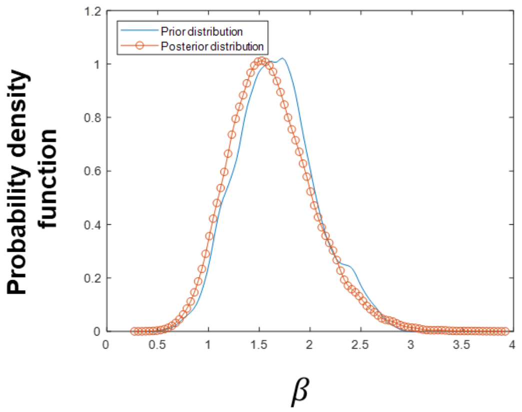
Parameter	Credible level	Credible interval (min. value – max. value)
$\boldsymbol{\eta}$	50%	9.38 – 12.51 EFPYs
	75%	8.56 – 14.07 EFPYs
	95%	7.18 – 18.45 EFPYs
$\boldsymbol{\beta}$	50%	1.29 – 1.85
	75%	1.13 – 2.06
	95%	0.87 – 2.44

Table II. Parameter estimation from MLE [1]

Parameter	Value
$\boldsymbol{\eta}$	10.33 EFPYs
$\boldsymbol{\beta}$	1.61



(a)



(b)

Fig. 2. The prior and posterior densities of (a) $\boldsymbol{t_{1st}}$ and (b) $\boldsymbol{\beta}$ for Alloy 690 TT tubing

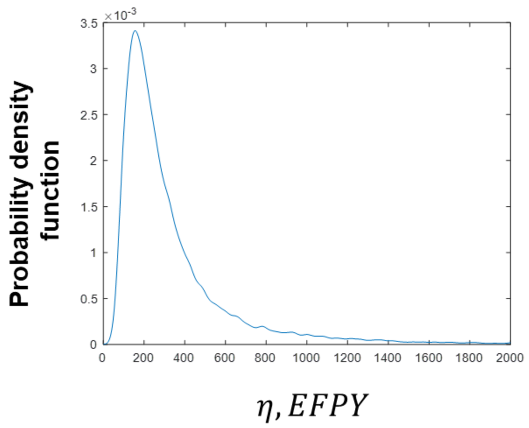


Fig. 3. The probability density of $\boldsymbol{\eta}$ for Alloy 690 TT tubing

Table III. Credible levels and intervals of Fig. 2 and 3

Parameter	Credible level	Credible interval (min. value – max. value)
$\boldsymbol{\eta}$	50%	160.39 – 400.03 EFPYs
	75%	123.68 – 616.62 EFPYs
	95%	83.34 – 1557.9 EFPYs
$\boldsymbol{t_{1st}}$	50%	16.71 – 23.01 EFPYs
	75%	14.56 – 25.34 EFPYs
	95%	11.23 – 29.42 EFPYs
$\boldsymbol{\beta}$	50%	1.33 – 1.87
	75%	1.16 – 2.09
	95%	0.91 – 2.52

Table IV. Parameter estimation from Weibayes [1]

Parameter	Value
$\boldsymbol{\eta}$	129.26 EFPYs
$\boldsymbol{\beta}$	1.61

5. Conclusion

model is developed by adopting Bayesian approach to predict the time to PWSCC in Alloy 690 tubing. The prior distributions of the model parameters are constructed based on solid reasoning. The results obtained from A Bayesian approach are compared to those yielded by more frequently used approaches, e.g. MLE and Weibayes. Since no Alloy 690 tubing had experienced PWSCC, there are no data to confirm which approach giving the best estimation.

The fact that the model parameters are treated as random variables, thereby reducing the uncertainty, is the main advantage of Bayesian approach. Moreover, Bayesian approach allows probabilistic interpretation about the parameter value located within a certain credible interval based on the posterior distribution of the parameter. For the future work, multiple prior distributions of the model parameters will be examined for the sensitivity assessment.

ACKNOWLEDGEMENT

This work was supported by KINS (Korean Institute of Nuclear Safety).

REFERENCES

[1] Steam Generator Management Program: Improvement Factors for Pressurized Water Reactor Steam Generator Tube Materials. EPRI, Palo Alto, CA: 2009. 1019044.
[2] Yuan, X.X., Mao, D. and Pandey, M.D., 2009. A Bayesian approach to modeling and predicting pitting flaws in steam generator tubes. Reliability Engineering & System Safety, 94(11), pp.1838-1847.
[3] Cannarile, F., Compare, M., Mattafirri, S., Carlevaro, F. and Zio, E., 2015, September. Comparison of Weibayes and Markov Chain Monte Carlo methods for the reliability analysis of turbine nozzle components with right censored data only. In Safety and Reliability of Complex Engineered Systems-Proceedings of the 25th European Safety and Reliability Conference, ESREL (pp. 1937-1944).