# Methodology of Constitutive Equations Improvement in Safety Analysis Code using Experimental Data: MIT Pressurizer Experiment

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## 1. Introduction

Accurate safety analyses of various accident scenarios for a nuclear power plant is dependent on the accuracy of the nuclear power plant safety analysis code. Safety analysis codes are consisted of governing equations and constitutive equation, which is an empirical correlation developed from experiments. In particular, separate effect tests (SETs) greatly contribute to the performance improvement of the constitutive equations.

Integral effect tests (IETs) have been performed worldwide recently. The safety analysis code which has good accuracy for SET case sometimes shows unsatisfactory accuracy for IETs. For these cases, the error in the constitutive equations is the most influential factor other than the user effect. However, it is extremely difficult to directly use the IET data for improving the performance of the constitutive equation. Therefore, in this study, a methodology to improve the performance of the constitutive equation by directly using experiment data is suggested. If the suggested methodology can improve the code with direct application of IET data, the accuracy of a nuclear power plant safety analysis code can be significantly improved with relatively little effort.

The method starts by generating data with constitutive relation, and data is clustered using an artificial neural network. The optimal multiplier coefficient for each clustered group is obtained to improve the code accuracy. The initial point of the optimization process is calculated from the KREM method. In the previous study, data generation and clustering were conducted [1,2] and MARS-KS code was used for the demonstration example. The multiplier coefficient for each group was obtained next by comparing the code result to SUBO experiment [3]. A similar study for MIT pressurizer experiment was also conducted by Kim et al. [4] previously. In this paper, the calculated optimal multiplier coefficient for each sub-regime is presented for the MIT pressurizer experiment.

## 2. Data Clustering

In the previous study [1], data clustering was conducted. The clustered data is MARS-KS constitutive equations: wall heat transfer, wall friction, interfacial heat transfer, and interfacial friction. For calculating these equations, thermal-hydraulic conditions and geometry information have be determined. The range of input parameter is selected to include the range of nuclear power plant's design basis accident. Input parameter is randomly selected from the given range, and calculate the constitutive equations. Figure 1 shows the calculated constitutive equations which can be used for clustering.



Fig. 1. Training data (from left top: coefficient of liquid wall HTC, vapor wall HTC, liquid wall FRIC, vapor wall FRIC, liquid interfacial HTC, vapor interfacial HTC, interfacial FRIC) [1]

Self-organizing map is used for data clustering [5]. In the process of the clustering, the number of clusters should be pre-determined by the user. For determining the optimal number of cluster number, silhouette coefficient and gap coefficient are used. This process was conducted previously and the results are presented in the previous study [2]. The optimal clustering number is shown in Table I. Figure 2 shows the clustering results. By using this result, the error of the code and experiment can be decreased by calculating the multiplier coefficient for each group. The error varies according to changes in the multiplier coefficient, which infers areas of improvement for the constitutive equation.

Table I: Minimum group number of clusters [2]

	Minimum clustering number	Optimum clustering number
Wall Heat Transfer	71	109
Wall Friction	55	55
Interfacial Heat Transfer	49	83
Interfacial Friction	51	60



3. Multiplier Coefficient Optimization

### 3.1. Multiplier Coefficient Optimization Method

Multiplier coefficient optimization process can be divided into three steps: Initial point determination, gradient calculation, and step size determination. Initial point is calculated from the KREM method. Gradient is numerically calculated for each multiplier coefficient. The conjugate gradient method is used in the optimization process. Optimization ends when the size of the gradient satisfies the exit criteria or when the error is not reduced in each iteration process. During the optimization process, most of the multiplier coefficients are within the constraint range [0.8, 1.2]. The range is determined by considering the uncertainties of the constitutive equations. However, not all the sub-regime from the constitutive equations have 20% uncertainty since some sub-regime can be located outside of the experimental dataset used for developing the constitutive equations. In these cases, the range of the multiplier coefficient is expanded to the range [0.1, 10.0].

### 3.2. KREM method

KREM method is used to find the initial point for the optimization. This method was developed by Korean Nuclear Industry, which uses non-variable statistical method. From this method, the confidence limit can be calculated.

$$1 - \left(\frac{p}{100}\right)^n \ge \frac{q}{100} \tag{1}$$

Equation 1 should be satisfied in order to exceed p% of the population with q% confidence through n random extractions. In this study, since the authors assumed to have upper 95% level confidence, 59 cases were selected for determining the initial point to satisfy equation 1.

#### 3.3. MIT pressurizer experiment

In the MIT pressurizer experiment, subcooled water is injected to a pressure vessel which is partially filled with the saturated water. Detailed information about MIT experiment is in references [6].



Fig. 3. Error of the MARS-KS code according to the optimization process step.

Figure 3 shows the error for each optimization step. Step 1 in the x-axis is the default MARS-KS simulation case, where the multiplier coefficients are all unity for sub-regime. Step 2 is the case where initial conditions are determined through the KREM method. Step3 is the optimization process for each iteration. As the MIT pressurizer experiment is not steady-state experiment, MSE cannot be used as the error estimation as it was in the previous study for the SUBO experiment. Therefore, the error is calculated using a method that extends dynamic time warping [7]. The error function is defined in Equation 2. The error is estimated from the distance between the experimental data and the code results.

Error (2)  
= 
$$\sum_{i=1}^{n} \frac{|\overline{V_{i,min}}|}{n(X_{max} - X_{min})(Y_{max} - Y_{min})}$$

Figure 4 shows the experimental data, and code results during the optimization process.



Fig. 4. MIT pressurizer experiment and code simulation results

## 4. Summary and Further Works

Steady efforts are being made to improve the accuracy of nuclear power plant safety analysis codes. As a part of this effort, the constitutive equations' performance is further enhanced by accumulating more data. In this study, an artificial neural network based clustering method is used to categorize constitutive equations in finer sub-regimes. Multiplier coefficients are then applied to each sub-regime so that the safety analysis code can self-improve its accuracy from the accumulation of the data. In order to find the best set of these multiplier coefficients, an optimization method is newly developed and presented in this paper. The data required for the process was generated using MARS-KS code. The initial set of multiplier coefficient optimization is calculated using the KREM method to make the optimization process more effective. The optimal multiplier coefficients were calculated using the conjugate gradient descent method. From this method, the error between code and experiment is reduced. For the testing of the method, the MIT pressurizer experiment is used in this study. However, the error reduction was not substantial compared to the SUBO case. This can be due to the larger user effect than the constitutive equation effect in the MIT pressurizer experiment simulation. For further exploration of the suggested method IET experiments will be next selected and tested.

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