Prediction of the critical flow rate of subcooled water through the short length channel using analytical method

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1. Introduction

The penetrating cracks through the wall of steam generator tubes are encountered in the pressurized water nuclear reactor due to corrosion and mechanical damage. As shown in Fig. 1, the leakage on the SG tube occurs through the crack and the pressurized subcooled water subjected to high depressurization rate. When the pressure of water becomes lower than the vapor pressure through the crack, the water starts to flash into steam. For a given stagnation state and geometry, there is a maximum leak flow rate, i.e., critical flow rate. An accurate prediction of the critical flow rate is necessary to evaluate the transient parameters such as the reactor water level, the amount of coolant, pressure and others. Also, it plays an important role in the simulation of the Loss Of Coolant Accident.

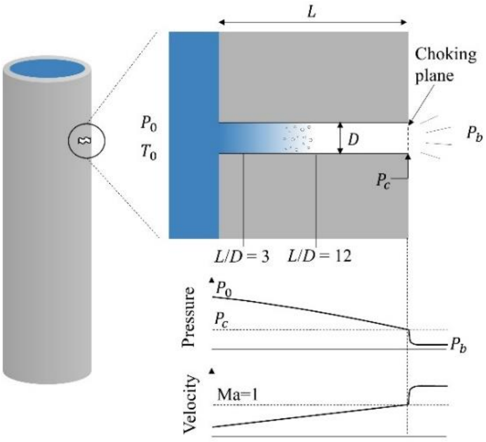


Fig. 1. Phenomenon of subcooled water critical flow through the penetrating crack on SG tube.

The ratio of channel length to diameter (L/D) has been used as an important parameter that determines the flow characteristics of critical flow. According to Henry [1], the flow is single-phase flow in the region $L/D < 3$. In the region $3 < L/D < 12$, the small amount of vapor bubbles generate in the middle of the jet. At $L/D=12$, the flow completely becomes the dispersed two-phase flow. Most of studies of critical flow are related to long channel ($L/D > 12$), which are not suitable for the SG tubes that have a wall thickness typically less than 3 mm [2].

In this study, the analytic method is developed to predict the critical mass flux of subcooled water through the short channel. The predicted results are compared with the experimental data that investigated the critical flow in slit, orifice, and short tube with L/D range in 0.37~2.11.

2. Characteristics of critical flow through short slit

As the back pressure (P_b in Fig. 1) decreases, the flow rate from the stagnation to downstream region increases. However, there is a limit to the increase in the flow rate. When the velocity of fluid reaches the speed of sound, the flow rate remains the same regardless of further reduction in the back pressure. The flow under such circumstance is called as choked. The pressure ratio of pressure at choking plane to stagnation is the critical pressure ratio and is around 0.55 for the ideal gases. However, for the two-phase flow, Xu and Wang [3] reported that the geometry has influence on the two-phase critical pressure ratio as shown in Fig. 2.

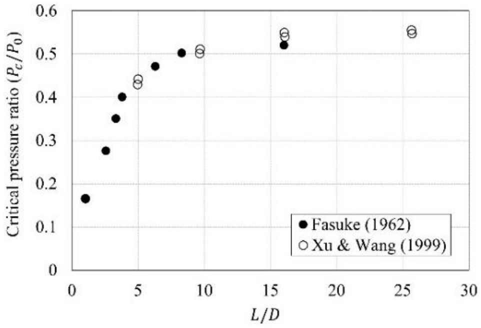


Fig. 2. Critical pressure ratio of initially saturated water flow through the nozzle [3].

The critical pressure ratio shows constant value (~0.55) in the range of $L/D >$ ~12 and it is reduced with decreasing of L/D in the rage of $L/D <$ ~12. Under the long channel, the thermal equilibrium between phases are established well and the gas flow becomes dominant over the certain point. Thus, the critical pressure ratio for the long channel shows similar results with that of ideal gases. On the other hand, there is not enough time for thermal equilibrium and flashing to be established for the short channel. The liquid phase is dominant and the speed of sound of liquid is much higher than the gases. As a results, in order to satisfy the choking condition (Ma =1), the pressure at choking plane should be decreased more than the gas phase.

3. Prediction method of critical mass flux

In general, the Henry-Fauske's Homogenoues Non-Equilibrium Model [4] has been used to predict the critical mass flux. The choking pressure should be found to calculate the Henry-Fauske model and it is determined as shown in Fig. 3.

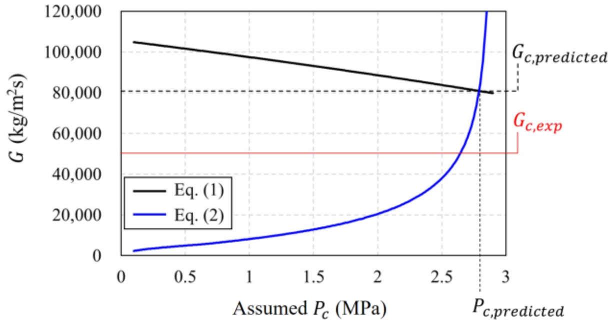


Fig. 3. Prediction of critical mass flux by Henry-Fauske Model (Calculation condition [2] : P_0=6.7 MPa, $\Delta T_{sub,0}$=50.5 K, L/D = 2.05)

$$G = \sqrt{\frac{2(P_0 - P_t)}{v_{f,0}}} \tag{1}$$

$$G = \left[\left(v_{g,eq} - v_{f,0}\right)\frac{N}{\left(s_{g,eq} - s_{f,eq}\right)}\frac{ds_{f,eq}}{dP}\right]^{-1/2} \tag{2}$$

$$N = \begin{matrix} x_{eq}/0.14 & x_{eq} \leq 0.14 \\ 1 & x_{eq} > 0.14 \end{matrix} \tag{3}$$

Eq. (1) is derive from the momentum equation without the effects of pressure loss. Eq. (2) is the Henry-Fauske model for the subcooled liquid in stagnation region. The critical mass flux and critical pressure are determined at the point where the flow rate calculated by Eq. (1) and Eq. (2) become equal. However, the predicted mass flux is much higher than the experimental result. Also, the critical pressure ratio is 0.42 higher than the previous studies that showed 0.25 at L/D = 2.05 (Fig. 2). In order to predict critical flow rate accurately, the mass flux line by Eq. (1) in Fig. 3 should be reduced and the line by Eq. (2) should be shifted to left.

The sharp edged entrance gives a significant effect on the pressure loss through the short channel. According to Henry and Fauske [3], the discharge coefficient (C_d) should be considered in the momentum equation.

$$G = C_d\sqrt{\frac{2(P_0 - P_t)}{v_{f,0}}} \tag{4}$$

Since the discharge coefficient is dependent on the various geometric parameter, it is recommended to use the experimental result for each geometry. For the short channel, the effects of phase change can be negligible and the slip ratio can be assumed as 1 because the single-phase flow is dominant. The Homogeneous Frozen Model was developed to calculate the speed of sound of two-phase flow under the homogenous condition without mass transfer between the phases.

$$a_{HFM} = \left\{\left[\alpha^2 + \alpha(1-\alpha)\frac{\rho_f}{\rho_g}\right] + \left[(1-\alpha)^2 + \cdots \alpha(1-\alpha)\frac{\rho_g}{\rho_f}\right]\frac{nP}{\rho_g a_f^2}\right\}^{-1/2} \times \frac{nP^{-1/2}}{\rho_g} \tag{5}$$

$$\alpha = \frac{1}{1 + \frac{1-x}{x}\frac{\rho_g}{\rho_f}} \tag{6}$$

$$x = N x_{eq} \tag{7}$$

$$\rho_{mix} = x\rho_g + (1-x)\rho_f \tag{8}$$

$$G = \rho_{mix} a_{HFM} \tag{9}$$

The critical mass flux can be calculated by using the two-phase speed of sound (Eq. 5) and density of mixture (Eq. 8). Fig. 4 shows the result of present method. The predicted mass flux is 4 % lower than the experimental result and the critical pressure ratio becomes 0.3.

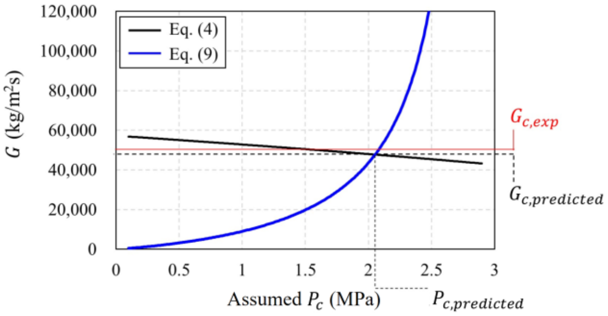


Fig. 4. Prediction of critical flow rate (Calculation condition [2] : P_0=6.7 MPa, $\Delta T_{sub,0}$=50.5 K, L/D = 2.05)

4. Results and discussions

The present prediction are compared with the experimental results that performed for the short channel geometry. Fig. 5 shows the comparison with the experimental results of Revankar et al. [2]. The critical mass flux through the various slits were measured. The stagnation and geometric condition in the range of P_0~6.8 MPa, 14 K <$\Delta T_{sub,0}$< 50 K, 1.25 < L/D < 2.11, and 0.54 < C_d < 0.8. The predicted mass flux results shows $\pm$ 12% error. The predicted critical pressure ratio is in the range of 0.3 < P_c/P_0 < 0.47.

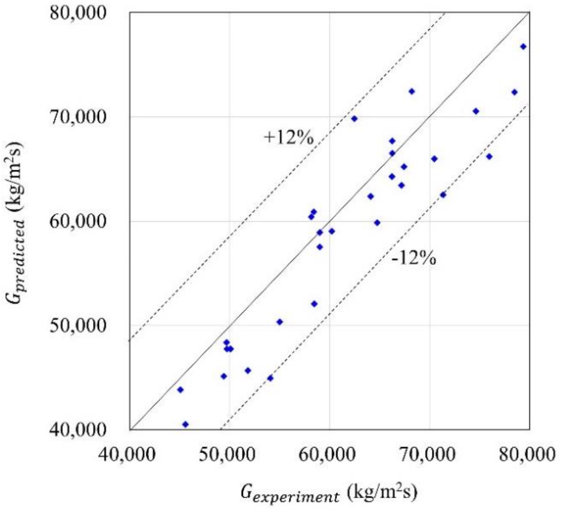


Fig. 5. Comparison of the predicted critical mass flux and experimental results [2] for the subcooled water critical flow through the slit

Fig. 6 shows the critical pressure ratio as function of L/D and inlet subcooling. As the inlet subocooling increases, the pressure ratio decreases. It increases with an increase of L/D. The Fauske's result in Fig. 2 is compared with the present results. The diameter of nozzle used in the Fasuke's experiment was 6.35 mm and the present hydraulic diameter is in the range of 0.6 ~ 1.0 mm. Thus, the present critical flow rate might be lower than the Fauske's result at the same L/D and this leads the higher critical pressure ratio.

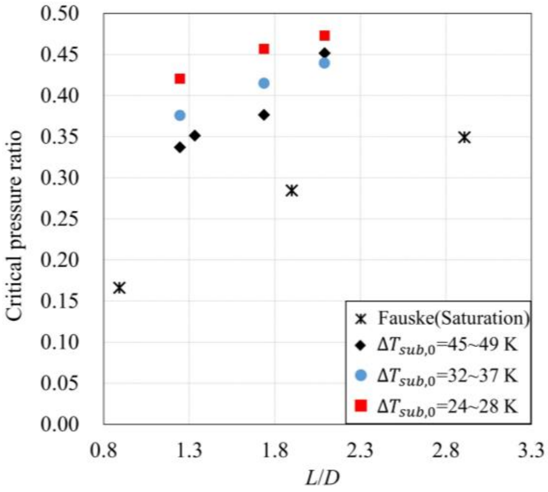
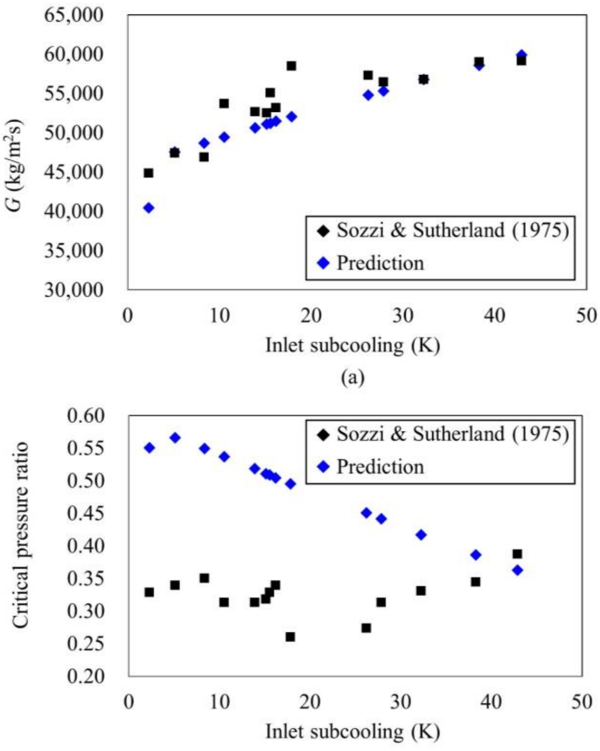


Fig. 6. The critical pressure ratio of subcooled water flow through the slits under P_0~6.8 MPa



(a)

(b)

Fig. 7. Comparison of the (a) critical mass flux and (b) critical pressure ratio between the present and experiment [5] used the orifice (P_0=6.5 MPa, L/D=0.37)

More experimental results that used different geometries are compared. Fig. 7 shows the critical mass flux and critical pressure ratio measured in the case of subcooled water flow through the orifice. The critical mass flux shows good agreement with experimental data. The predicted critical pressure ratio is around 0.55 at low subcooling. As the inlet subcooling decreases, the water state in the critical flow becomes close to the saturated state. This causes to the flashing in the flow and the choking condition becomes almost same with the ideal gas flow. With an increase of subcooling, the water state in the critical flow keeps subcooled state and the critical pressure ratio should be decreased. However, the measured pressure ratio showed almost constant value (~0.3). It is not reasonable because the measured mass flux decreases with reduction of subcooling. Additionally, the length of orifice was 4.7 mm and the critical pressure was measured by the pressure tap located at the bottom side of orifice exit. The accurate pressure measurement might have been difficult due to the short length of orifice.

Fig. 8 shows the results in the case of short tube. The predicted mass flux shows maximum ± 12% error. The measured critical mass flux in Fig. 7(a) and Fig. 8 tends to be constant when the inlet subcooling becomes higher than 40 K. It means that the critical pressure becomes same with the back pressure over a certain inlet subcooling. On the other hands, it is evaluated that the inlet subcooling that the critical and back pressure becomes equal is higher in the present method.

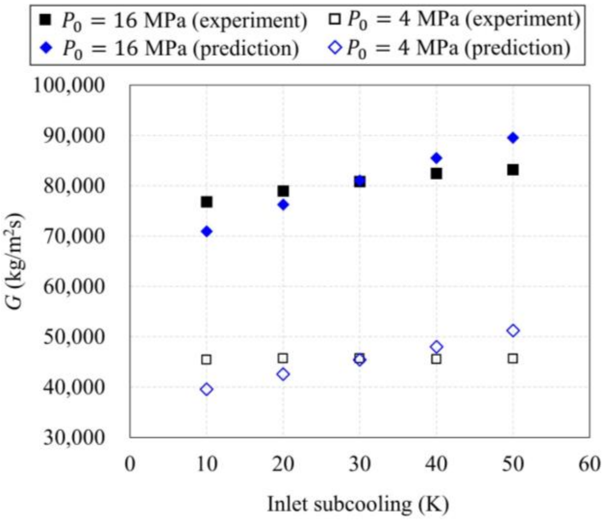


Fig. 8. Comparison of the predicted critical flow rate and experimental results [3] used the short tube (D =4.05 mm, L=4 mm, L/D=0.99)

5. Conclusions

The two-phase critical flow depends on the flow geometry condition and most of previous studies investigated the critical flow through long channel. In this study, the analytical method is presented to predict the critical mass flux of subcooled water through short channel in the range of $L/D < 3$. The determination method of the critical flow rate and critical pressure is the same with that of the Henry and Fasuke's model. The discharge coefficient is applied to the momentum equation and the Homogeneous Frozen Model is used to calculate the two-phase speed of sound.

The predicted critical mass flux is compared with the various experimental results. The compared experiments used the slit, short tube, and orifice as the flow channel and the condition are in the rage of 0.37 < L/D < 2.11, 4 MPa < P_0 < 6.8 MPa, and 2 K < $\Delta T_{sub,0}$ < 50 K. The predicted critical mass flux shows maximum $\pm$12 % error. The critical pressure ratio is evaluated as function of the inlet subcooling and L/D. The critical pressure ratio is proportional to L/D and inversely proportional to inlet subcooling.

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