General Heat Exchanger Modeling for Quasi-Steady Small Modular Reactor Heat Source Simulation

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1. Introduction

Currently many codes are being developed to simulate the integrated nuclear-renewable hybrid energy systems (NRHES) [1,2,3]. However, most codes and models that are designed to simulate NRHES system either pre-determines how much the heat is removed through the steam generator during quasi-steady simulations or just use very simplified model assuming outlet power of the nuclear plants. Analysis of NRHES may require simplified models to model the overall system to reduce simulation time. Nonetheless, when the analyzed system is off from its maximum guaranteed rating (MGR) condition, the power transferred from primary to secondary system of nuclear plant may not be equal to its 100% steady state condition and thus should be analyzed separately. The purpose of this research is to develop a general heat exchanger model for a system code used to simulate a typical small modular reactor off-design quasi-steady state, which will ultimately be used for simulating the quasi-steady small-modular reactor heat source for a typical nuclear-renewable hybrid energy system. The scope of current research is focused on analyzing the off-design quasi-steady state (thus does not include how the transient system behaves between the end states).

2. Methods

The modeled main components include a steam generator, off-design reheaters and feedwater heaters. After developing each component model, examined models are validated using the Maximum Guaranteed Rate (MGR) steady-state heat balance diagram (HBD) from the SMART100 Standard Safety Analysis Report [4]. The flow of information of one component to next are three simple coolant properties: mass flow rate, pressure, and enthalpy of the inlet water/steam. All other coolant-related properties (such as temperature and density) are calculated within each component model, from the IAPWS-IF97[5].

2.1 Off-design Quasi-Steady Heat Exchanger Modeling

For simulation of the quasi-steady heat exchanger models, effectiveness Number of Transfer Units (ε-NTU) method was used to calculate the rate of heat transfer in the counter-current heat exchangers in the quasi steady state analysis [6].

During the steam generator analysis, for example, it is not automatically assumed that all heat generated from the core is transferred from the primary to the secondary side if the secondary system runs in a condition different from the MGR condition. In this case, the amount of transferred heat is found at each iteration during the simulation.

When there is insufficient design information of the heat exchanger but MGR steady-state condition is known from heat balance diagram (HBD), its product of heat transfer coefficient (U) and the exchange area (A) may first be estimated using the LMTD method:

$$UA = \frac{Q}{LMTD} \approx \frac{Q_{MGR}}{LMTD_{MGR}} \quad (1)$$

where Q_{MGR} is the rate of heat transfer in the MGR condition and $LMTD_{MGR}$ is the logarithmic mean temperature difference [6]. Thus, the product term UA will have a unit of [W/K].

To perform ε-NTU method, one must first find the maximum possible transferrable heat which can be achieved in a counter-flow heat exchanger. If one assumes a heat-exchanger of infinite length, at least one of the fluids (hot or cold stream) will experience the maximum possible temperature difference. This fluid would have smaller value of the product of mass flow rate and the isobaric specific heat, as shown below.

$$C_{min} = \min\left(\dot{m}_H c_{p,H}, \dot{m}_C c_{p,C}\right) \quad (2)$$
$$Q_{max} = C_{min}\left(T_{H,in} - T_{C,in}\right) \quad (3)$$

The actual heat transfer rate may be found using the term effectiveness, or ε. For the countercurrent flow, following equations are valid and can be found in may heat transfer references including reference [6].

$$C_{max} = \max\left(\dot{m}_H c_{p,H}, \dot{m}_C c_{p,C}\right) \quad (4)$$
$$C_r = \frac{C_{min}}{C_{max}} \quad (5)$$
$$NTU = \frac{UA}{C_{min}} \quad (6)$$
$$\epsilon = \frac{1-\exp\left(-NTU(1-C_r)\right)}{1-C_r\exp\left(-NTU(1-C_r)\right)} \quad (7)$$
$$Q = \varepsilon Q_{max} \quad (8)$$

However, the accuracy of this process may depend on how the heat capacities $c_{p,H}$ and $c_{p,C}$ are defined. If there is no phase change for the hot/cold line, *Cp* may be found as a simple average of inlet and outlet fluid. However, for the two-phase heat-exchanger, it is also important to account for the amount of energy used in the phase change during heat transfer, which is neglected if the heat capacities C_p for the hot and cold fluids are found based only on their fluid properties (i.e. temperature, pressure, enthalpy, and/or vapor quality). To solve this problem, one method is proposed dividing sensible and latent heat and deriving pseudo-two-phase specific heat, similar to the reference [7].

Using this approach, mathematical manipulation is used to derive the terms of specific heats for both sensible and latent heat if the fluid goes through phase changes. Below equations are the derived results of the heat capacities *Cp* if the fluid changes from superheated steam to saturated or subcooled liquid (subscript *H*) or if the fluid changes from subcooled liquid to saturated or superheated steam (subscript *C*). Subscripts *S* and *L* stands for sensible and latent heat, respectively.

$$C_{p,i} = C_{p,i,S} + C_{p,i,L}, \quad i = H \text{ or } C \tag{9}$$

$$C_{p,H,S} = \frac{C_{p,H,in}\times(T_{H,in}-T_{H,sat}) + C_{p,H,out}\times(T_{H,sat}-T_{H,out})}{T_{H,in}-T_{H,out}} \tag{10}$$

$$C_{p,C,S} = \frac{C_{p,C,in}\times(T_{C,sat}-T_{C,in}) + C_{p,C,out}\times(T_{C,out}-T_{C,sat})}{T_{C,out}-T_{C,in}} \tag{11}$$

$$C_{p,H,L} = \frac{\min(h_{H,in},h_{H,g}) - \max(h_{H,out},h_{H,f})}{T_{H,in}-T_{H,out}} \tag{12}$$

$$C_{p,C,L} = \frac{\min(h_{C,out},h_{C,g}) - \max(h_{C,in},h_{C,f})}{T_{C,out}-T_{C,in}} \tag{13}$$

Once the actual transferred heat rate is found using the Equation 8, the outlet temperatures for the hot and cold streams can be calculated using the following equations below.

$$T_{H,out} = T_{H,in} - \frac{Q}{\dot{m}_H C_{p,H}} \tag{14}$$

$$T_{C,out} = T_{C,in} + \frac{Q}{\dot{m}_C C_{p,C}} \tag{15}$$

At each quasi-steady time step, Equations 1 through 15 are repeated until the transferred heat converges for the heat exchanger. The parameters Q_{MGR} and $LMTD_{MGR}$ in the Eq.1 may be replaced with the previously-iterated Q and $LMTD$, respectively at each new iteration.

2.2 Quasi-Steady Steam Generator Model

First, for a quasi-steady time step, the results are iterated for the Equations 1 through 15 until the steam-generator (SG) transferred heat converges. Then, the detailed SG model is used to simulate the steam generator component. Note that pressure drop in the orifice is not included in the pressure drop calculation due to lack of design information. The assumptions for the steam generator model are listed below.

1) No reversible flow for both primary and secondary coolants
2) No phase change in the primary side (i.e. the coolant stays subcooled)
3) No heat conduction in the SG tube axial direction
4) Homogeneous fluid in each mesh

General steps for the SG simulation are listed below.

1) Following *Section 2.1*, calculate the overall heat rate Q transferred from the primary to secondary side of the steam generator. Use the MGR steady state condition as input for the first iteration.
2) Divide the SG tube into primary coolant (shell side), tube outer surface, tube inner surface, and secondary coolant (tube side), each with *n* number of meshes.
3) For the first iteration, assume pressure and enthalpy values for primary and secondary coolant for each mesh to retrieve coolant properties required for the SG simulation. Assume pressure values for primary and secondary side based on their inlet pressures, and linearly interpolate the primary and secondary enthalpies based on inlet conditions and the amount of heat transferred.
4) Make an initial guess on inner and outer surface temperatures of the SG tube.
5) Based on the assumed pressure and enthalpy values of the coolant meshes (both primary and secondary), retrieve/calculate temperature, density, steam quality, dynamic viscosity, specific isobaric heat capacity, thermal conductivity, velocity, Reynolds number, and Prandtl number of each mesh.
6) Calculate the heat transfer coefficients for each mesh for primary and secondary sides.
   - Churchill-Berstein equation for 1-phase liquid [8]
   - Thom correlation for saturated boiling region [9]
   - Mori-Nakayama correlations for superheated region [10]
7) Calculate the conductivity of the tubes based on average temperature between inner and outer surface of the tube for each mesh. For the first iteration, use the initial guess values from Step 4. For all the other iterations, use the previous iteration's inner and outer tube surface temperatures for each mesh.
8) Based on the transferred heat balance, solve for inner and outer SG tube surface temperatures *for each mesh* using systems of equations. Derived systems of equations are then placed in a matrix form to be solved through Gaussian

elimination method. Note that following equations are solved for each SG tube mesh.

$$\mathbf{AT} = \mathbf{B} \tag{16}$$

$$\mathbf{A} = \begin{bmatrix} h_{p,j}A_{outer,j} & h_{p,j}A_{inner,j} \\ \frac{k_j A_{central,j}}{\Delta r} & -h_{s,j}A_{inner,j} - \frac{k_j A_{central,j}}{\Delta r} \end{bmatrix} \tag{17}$$

$$\mathbf{B} = \begin{bmatrix} h_{p,j}A_{outer,j}T_{p,j} + h_{s,j}A_{inner,j}T_{s,j} \\ -h_{s,j}A_{inner,j}T_{s,j} \end{bmatrix} \tag{18}$$

$$\mathbf{T} = \begin{bmatrix} T_{outer,j} \\ T_{inner,j} \end{bmatrix} \tag{19}$$

- A is the heat transfer area for the tube jth mesh. Subscripts outer, inner, and central means outer, inner, and central surface area of the tube mesh, respectively. Subscripts p and s stands for primary and secondary, and h and k are enthalpy of the coolant and thermal conductivity of the tube mesh, respectively.

9) Using the primary and secondary temperatures as well as solved SG inner and outer tube temperatures, calculate heat transferred (dQ) through each node.
10) Sum the dQ from each node. Then, find the ratio between actual transferred heat Q from Step 1 and the summed dQ. Multiply the ratio to the dQ for preserving total energy balance.
11) Calculate the gravitational pressure drop based on height.

$$\Delta P_{g,i,j} = \rho_{i,j} g \Delta L_{mesh} \times \frac{H_{tube}}{L_{tube}} \tag{20}$$

- ΔP_g is gravitational pressure loss [Pa], ρ is density of the coolant [kg/m^3], L is length of the tube, and H is height of the tube. Subscript $i = 1$ for primary and $i = 2$ for secondary. Subscript j represent jth tube mesh.

12) Calculate the frictional loss coefficient for the primary side based on Chen approximation directly [11].
13) Calculate the frictional loss coefficient for the secondary side based on Mori-Nakayama correlation for forced heat transfer in curved pipes [10].
14) Calculate the friction loss pressure drops. For the two-phase conditions at secondary, use two-phase friction multiplier φ_{lo}^2 from the Jones method or HEM method [12,13].
15) Calculate the accelerational pressure drop [11].
16) Calculate the form pressure drop from sudden contraction and expansion at the inlets and outlets of the SG tube. Assume the ratio between smaller (i.e. tube inlet/outlet) and larger (i.e. before/after the tube at SG plenum) flow areas is close to zero for pressure loss coefficients K_{sc} and K_{se} [14].
17) Update the pressure values for each SG meshes.
18) Update the enthalpy values starting from the node up top (primary inlet, secondary outlet) to the bottom (right before primary outlet, secondary inlet) using product of the heat transferred (dQ) from Step 10 and a scaling factor from previous iteration $f_{Q,k} = \left(\frac{Q_{tot,k-1}}{Q_{tot,k}}\right)$.
19) Update the temperature values for each SG meshes.
20) Repeat Steps 1~19 with updated inlet and outlet properties (e.g. inlet and outlet temperatures) to start from newly simulating the transferred heat. Repeat either 3~7 iterations or until the scaling factor from Step 18 converges to certain value (e.g. $\frac{f_{Q,k-1}}{f_{Q,k}} \leq 1.001$ for relative error of 1e-3)

Detailed SG model allows close examination at what is happening inside the SG tube during the heat transfer. Validation results are shown in **Section 3**.

3. Results

First, the results of using the heat exchanger model based on the ε-NTU method with modified heat capacities for the two-phase heat exchangers are shown in Table 1. For the validation results, the ratio of heat balance diagram (HBD) and simulated rate of heat transfer shows accurate heat transfer results of the simulated model. All iterations converged well within 3~7 iterations, when the absolute value of relative error for the calculated transferred heat between iterations reached less than 10^{-3} for k^{th} iteration.

$$error = abs\left(\frac{Q_k - Q_{k-1}}{Q_k}\right) \tag{21}$$

LPFWH, HPFWH, and RH stands for stages of low-pressure feedwater heater, high-pressure feedwater heater, and reheater, respectively.

Table 1. The Off-Design Heat Exchanger Model Simulated Results for the MGR Steady-State Operation of SMART100

Feedwater Heaters (FWH) and Reheaters (RH)	**Simulated Outlet Results**			Ratio of HBD and Simulated Rate of Heat Transfer (f_Q)
	Feedwater / Main Steam Outlet Enthalpy (kJ/kg)	Drain Outlet Enthalpy (kJ/kg)	Rate of Transferred Heat (MW)	
LPFWH1	258.1	204.1	10.4	1.0031
LPFWH2	336.4	278.1	10.8	1.0049
LPFWH3	412.2	354.0	10.5	1.0091
LPFWH4	477.2	427.0	9.0	1.013
HPFWH1	748.0	660.2	24.9	0.9607
HPFWH2	994.6	782.6	45.7	0.9966
RH1	2876.6	1033.2	17.4	0.988
RH2	2934.1	1147.4	6.9	0.997

As for the validation of the SG modeling, the results from ONCESG code in the reference [13] were used for comparison. Since the publication of the aforementioned reference, the SMART SG design has been changed. Therefore, the tested SMART SG for validation is labeled "old SMART SG" in this study to differentiate from the SMART100 SG design. Comparison with MRX and old SMART SG results in the reference are shown in Table 2 and Table 3, respectively. ONCESG Case 1 and Case 2 represent the simulated results using two different sets of empirical correlations as listed in the reference. Nominal heat capacity for code-tested MRX and old SMART SGs are 100 and 28.15 MWt, respectively [13].

The difference in T_{steam} in the Tables 2 and 3 results from difference in actual transferred heat in the SG for ONCESG and developed SG models. In the developed detailed SG model, scaling factor allows the SG outlet to have output from exact nominal transferred heat (i.e. 100 MWt and 28.25 MWt for MRX and old SMART SG, respectively). However, ONCESG in the reference may not result in the exact transferred heat value. For example, using enthalpy values for the inlet and outlet of the ONCESG results for the MRX SG, calculated transferred heat out of primary is 100.22 MWt, and heat into the secondary is 100.75 MWt (Case1) or 100.87 MWt (Case2). This difference in total transferred heat in the SG may have resulted in the differences in T_{steam} in the Tables 2 and 3 results.

Table 2. Comparison with MRX SG Results [13]

	MRX	ONCESG Case 1	ONCESG Case 2	Detailed SG Model
P_{hot} [MPa]	12	12	12	12
T_{hot} [°C]	297.5	297.5	297.5	297.5
P_{steam} [MPa]	4	4	4	4.03
T_{feed} [°C]	185	185	185	185
T_{cold} [°C]	282.5	282.5	282.5	282.7
T_{steam} [°C]	289	289	289	282
Shell-side pressure drop [MPa]	9.0E-3	1.2E-2	1.2E-2	2.23E-2
Tube-side pressure drop [MPa]	0.64	0.42	0.49	0.37

Table 3. Comparison with old SMART SG Results [13]

	SMART (old)	ONCESG Case 1	ONCESG Case 2	Detailed SG Model
P_{hot} [MPa]	15	15	15	15
T_{hot} [°C]	310	310	310	310
P_{steam} [MPa]	3.4	3.4	3.4	3.42
T_{feed} [°C]	180	180	180	180
T_{cold} [°C]	268.5	268.3	268.3	268.7
T_{steam} [°C]	300	300.9	300.9	294.4
Shell-side pressure drop [MPa]	2.57E-2	3.5E-2	3.5e-2	2.2E-2
Tube-side pressure drop [MPa]	0.3	0.34	0.35	0.28

The SG model simulated results for the SMART100 SG are shown in Figure 1, showing the expected shape of the temperature profile. Because each SG tube mesh contains simulated results of its fluid properties (pressure, enthalpy, temperature, vapor quality, mass flow rate, mass flux, etc.), the model may be further developed for future transient analysis.

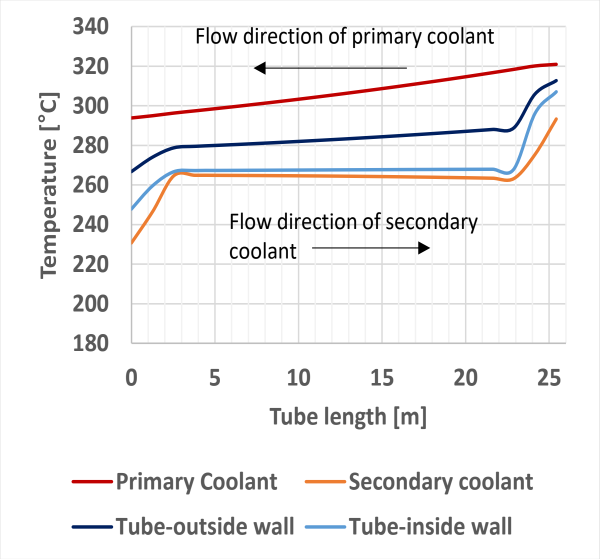

Figure 1. Simulated SG model Temperature Profiles for SMART100

4. Conclusions

In this research, models to calculate the transferred heat for quasi-steady analysis of the small modular reactors were developed. Validation with the heat balance diagram of the SMART100 Safety Analysis Report [4] and with results of the ONCESG code [13] showed good results. These models may be used in simulating the off-design quasi-steady analysis of the small modular reactor in the nuclear and renewable hybrid energy system simulations when it may not be given that the transferred heat in each heat exchangers may stay constant as the normal MGR steady state conditions.

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REFERENCES

[1] Mikkelson, Daniel, et al. "Small modular reactor modeling using modelica for nuclear-renewable hybrid energy systems applications." Transactions of the American Nuclear Society 113.INL/CON-16-39032 (2015).

[2] Cetiner, Sacit M., et al. "Nuclear Hybrid Energy Systems FY16 Modeling Efforts at ORNL." Oak Ridge National Laboratory, Oak Ridge, TN (United States) (2016).
[3] Ho, An, et al. "Dynamic simulation of a novel nuclear hybrid energy system with large-scale hydrogen storage in an underground salt cavern." International Journal of Hydrogen Energy 46.61 (2021): 31143-31157.
[4] KHNP, KAERI, KACARE, SMART100 Standard Safety Analysis Report, 2019.
[5] Wagner, Wolfgang, and Hans-Joachim Kretzschmar. "IAPWS industrial formulation 1997 for the thermodynamic properties of water and steam." International steam tables: properties of water and steam based on the industrial formulation IAPWS-IF97 (2008): 7-150.
[6] Incropera, Frank P., et al. Fundamentals of heat and mass transfer. Vol. 6. New York: Wiley, 1996.
[7] Qiao, Hongtao. "Correction of Log Mean Temperature Difference Method and Effectiveness-NTU Relations for Two-phase Heat Transfer with Pressure Drop and Temperature Glide." (2018).
[8] Hardy, J. E., J. O. Hylton, and T. E. McKnight. Empirical correlations for thermal flowmeters covering a wide range of thermal-physical properties. No. ORNL/CP-102500. Oak Ridge National Laboratory (ORNL), Oak Ridge, TN, 1999.
[9] MISHIMA, Kaichiro, et al. "Heat transfer study for thermal-hydraulic design of the solid-target of spallation neutron source." Journal of nuclear science and technology 38.10 (2001): 832-843.
[10] Mori, Yasuo, and Wataru Nakayama. "Study of forced convective heat transfer in curved pipes (2nd report, turbulent region)." International journal of heat and mass transfer 10.1 (1967): 37-59.
[11] Chen, Ning Hsing. "An explicit equation for friction factor in pipe." Industrial & Engineering Chemistry Fundamentals 18.3 (1979): 296-297.
[12] Todreas, Neil E., and Mujid S. Kazimi. Nuclear systems volume I: Thermal hydraulic fundamentals. Taylor & Francis, 1993.
[13] Yoon, Juhyeon, et al. "Development of a computer code, ONCESG, for the thermal-hydraulic design of a once-through steam generator." Journal of nuclear science and technology 37.5 (2000): 445-454.
[14] Ghiaasiaan, S. Mostafa. Two-phase flow, boiling, and condensation: in conventional and miniature systems. Cambridge University Press, 2007.