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Evaluation of the Effect of Uncertainty Expression Method on Material Balance Evaluation for Nuclear Safeguards

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1. Introduction

The Material Balance Evaluation (MBE) is a process to verify the declared amount of nuclear material in nuclear facilities for nuclear safeguards. Notification No. 2017-83 of the Republic of Korea (ROK), "Regulations on the accounting and control of special nuclear material", requires to perform MBE for national inspection: verification of the uncertainty of material unaccounted for (MUF) and shipper-receiver difference (SRD). However, the ROK uses MUF results from the IAEA instead of performing an independent MBE.

The MUF, which represents the characteristics of the material accounting system of a facility, is the most frequently used statistic for the MBE; therefore, the ROK must evaluate the MUF of domestic nuclear facilities. Hypothetical testing is used to evaluate MUF using calculated MUF and MUF uncertainty. The MUF uncertainty of a facility is calculated using the "book inventory", "list of inventory item", "accounting system of a facility" and "uncertainty expression method".

The goal of this research is to evaluate the effect of the uncertainty expression method on the MUF uncertainty using a benchmark fuel fabrication plant. We applied three different uncertainty expression methods (conventional IAEA, modified IAEA and GUM) to the benchmark facility and compared the results.

Results of the IAEA methods have higher versatility for general nuclear facilities worldwide, compared to the GUM method. Results of the GUM method has higher degree of freedom for uncertainty management. It also has higher reliability for facilities which operate quality assurance program on measurement system.

2. Facility Configuration for MBE

A benchmark fuel fabrication plant (BFFP) consists of a single material balance area (MBA), which includes reconversion, pelletizing, fuel rod fabrication and assembling process. The BFFP receives enriched UF_6 cylinders. It then converts UF_6 cylinders into UO_2 powder drums. The powder drums are then pelletized into UO_2 pellets. The pellets are inserted into fuel rods and assembled into fresh fuel assemblies. The location of storages in the BFFP is depicted in Figure 1.

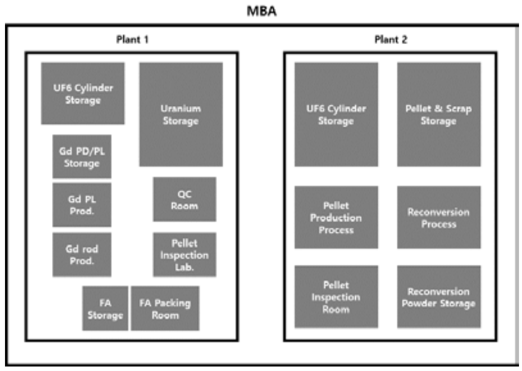

Fig. 1. Storage configuration of the BFFP.

The BFFP consists of 12 key measurement points (KMP). Table 1 lists and describes the KMPs and corresponding accounting systems.

Table 1. Lists of KMPs and accounting systems of the BFFP.

KMP	Description	Method		
		Weight	U %	wt %
KMP 1	UF_6		-	-
KMP 2	Reconversion process			
KMP 3	UO_2 powder		TGA	TIMS
KMP 4	UO_2 green pellet/scrap		-	-
KMP 5	UO_2 pellet/scrap		TGA	TIMS
KMP 6	Uranium nitrate	EBAL		
KMP 7	Fuel rod		-	-
KMP 8	Fuel assembly			
KMP 9	Scrap (clean/dirty)		TITR	TIMS
KMP 10	Solid waste		-	-
KMP 11	Liquid waste			
KMP 12	Laboratory sample		TITR	TIMS

A benchmark MBE evaluates the isotope (^{235}U) MUF of nuclear material in the BFFP using Equation (1). We assumed the uncertainty of the book inventory (PB+X-Y) to be zero for simplification, since the purpose of the benchmark MBE is to evaluate the difference between uncertainty expression methods [1].

$$MUF = PB + X - Y - PE \qquad (1)$$

where,
PB: Beginning isotope inventory,
X: Material inflow, Y: Material outflow,
PE: Ending isotope inventory.

The isotope MUF was evaluated using a hypothetical testing method [2]. Once the isotope MUF uncertainty (σ_{MUF}) is calculated and false alarm probability (α) is established, the null hypothesis (H_0) and alternative hypothesis (H_1) are established.

$$H_0: \mathrm{MUF_i} = 0,$$
$$H_1: \mathrm{MUF_i} = M$$

If the calculated isotope MUF satisfies $\mathrm{MUF_i} \leq t_\alpha \sigma_{MUF,i}$, the null hypothesis is accepted. Otherwise, the alternative hypothesis is accepted.

The quantity of ^{235}U in an item is calculated using Equation (2), which indicates measurement data of each item: the bulk weight, sampling factor, U concentration and ^{235}U enrichment.

$$X = Q \times P \times T(E) \times T(I) \qquad (2)$$

where,
X: Total ^{235}U mass(kg), Q: Net mass of an item (kg),
P: Sampling factor of an item $\left(P \sim N(1, \delta_p)\right)$,
T(E): U concentration of an item,
T(I): ^{235}U enrichment of an item.

The BFFP consists of 14,538 items with different physical and chemical characteristics. The list of inventory items (LII) includes the KMP, location, material description code (MDC), batch, lot information, net weight, U concentration and ^{235}U enrichment of each item in the facility.

Inventory items were stratified using physical and chemical characteristics. Items in a stratum were then sub-stratified based on storage location. Table 2 describes the accounting systems of each sub-stratum for each measurement results. Non-measured data was considered to be zero.

Table 2. Stratification and sub-stratification of the BFFP.

Stratum	Description	Location	Q	P	T(E)	T(I)
FF-11	Fuel Assembly	FA storage	1	0	0	0
FF-BD	Fuel Assembly (Gd)	FA storage	1	0	0	0
FR-11	Fuel Rod	QC Room	2	0	0	0
FR-11		U storage	3	0	0	0
FR-1G	Fuel rod with Gd	QC Room	2	0	0	0
FR-1G		U storage	3	0	0	0
HE-1L	UF6 Heel	UF6 Cylinder Storage - P1	4	0	0	0
UF-1L	UF6 Cylinder	UF6 Cylinder Storage - P1	4	0	0	0
PD-1L	UO2 Powder	Gd pellet/powder Storage	6	1	1	1
PD-1L		(Re)conversion Process	7	1	1	1
PD-1L		Recon Powder Storage	7	1	1	1
PL1-L	UO2 Pellet(Sintered, pure)	Gd Rod Production Process	9	0	0	0
PL1-L		U storage	3	1	1	2
PL2-L	UO2 Pellet(Sintered) (Gd)	Gd pellet/powder Storage	6	2	2	2
PL2-L		U storage	3	2	2	2
SA-1L	Lab. Sample	QC Room	2	0	3	0
SA-1L		Pellet Inspection Lab. - P1	12	0	3	0
SA-1L		U storage	15	0	3	0
SA-1L		Pellet Inspection Lab. - P2	12	0	3	0
SA-1	Lab. Sample(Fuel rod)	QC Room	2	0	0	0
PM-1L	(Re)conversion Products	U storage	3	0	0	0
PM-1L		(Re)conversion Process	7	0	0	0
SC-1L	Clean Scrap(non-pellet)	Gd Pellet Production Process	12	0	0	0
SC-1L		U storage	3	3	3	3
SC-1L		Pellet & Scrap Storage - P2	7	3	3	3
SC-1L		Pellet Production Process - P2	7	3	3	3
SC-1L		Recon Powder Storage	7	3	3	3
SC-PL	Clean Scrap(Pellet)	Gd pellet/powder Storage	6	3	3	3
SC-PL		U storage	3	3	3	3
SC-PL		Pellet Production Process - P2	7	3	3	3
SC-PL		Pellet & Scrap Storage - P2	7	3	3	3
SD-1L	Dirty Scrap	U storage	3	4	3	4
SD-1L		Recon Powder Storage	7	4	3	4
SD-1L	Liquid Waste	U storage	3	0	0	0

3. Results of Uncertainty Expression Methods

Three different uncertainty expression methods were applied to evaluate the isotope MUF uncertainty of the BFFP: conventional IAEA's method, modified IAEA's method, and guide to the expression of uncertainties in measurement (GUM).

3.1 Conventional IAEA's method

Conventional IAEA's method for uncertainty expression of isotope MUF is summarized in the literature [2, 3]. Conventional IAEA's method calculates the isotope MUF uncertainty by the following assumptions and processes:

Assumptions

1. Each item in the same stratum is homogeneous.
2. Each item with the same enrichment is measured using the same equipment.
3. Relative uncertainty of the measurement system is considered to be ITV.
4. Uncertainty for non-measured strata is considered to be zero.
5. The benchmark facility has no static material.

Processes

1. Stratification of the inventory items using stratification rules.
2. Calculation of element MUF uncertainty using Equation (3).
3. Calculation of isotope MUF uncertainty from element analysis by applying stratum averaged enrichment (Equations (4) ~ (6)).
4. Stratification of the inventory items with isotopic analysis based on ^{235}U enrichment.
5. Calculation of isotope MUF uncertainty from isotopic analysis using Equations (7) ~ (9).
6. Calculation of total isotope MUF uncertainty using Equation (10).

$$V(\mathrm{MUF}) = V_r(MUF) + V_g(MUF) + V_s(MUF) \qquad (3)$$

$$V_r(\mathrm{MUF}) = \sum_{k=1}^{K} \overline{wt}_k^2 x_k^2 \left(\frac{\delta_{rq}^2}{n_k m_k} + \frac{\delta_{rp}^2}{r_k m_k} + \frac{\delta_{rt(E)}^2}{c_k r_k m_k}\right) \qquad (4)$$

$$V_g(\mathrm{MUF}) = \sum_q \delta_{gq}^2 \sum_i \overline{wt}_{qi}^2 M_{qi}^2 + \sum_p \delta_{gp}^2 \sum_i \overline{wt}_{pi}^2 M_{pi}^2 + \sum_{t(E)} \delta_{gt(E)}^2 \sum_i \overline{wt}_{t(E)i}^2 M_{t(E)i}^2 \qquad (5)$$

$$V_s(\mathrm{MUF}) = \sum_q \delta_{sq}^2 \overline{wt}_{sq}^2 M_{sq}^2 + \sum_p \delta_{sp}^2 \overline{wt}_{sp}^2 M_{sp}^2 + \sum_{t(E)} \delta_{st(E)}^2 \overline{wt}_{st}^2 M_{st(E)}^2 \qquad (6)$$

$$V_t^*(MUF) = V_r^*(\mathrm{MUF}) + V_s^*(MUF) \qquad (7)$$

$$V_r^*(\mathrm{MUF}) = \sum_{i=1}^{G} S_i^2 \left(\frac{\delta^{*2}_{rp}}{r_i^*} + \frac{\delta^{*2}_{rt(I)}}{c_i^* r_i^*}\right) \qquad (8)$$

$$V_s^*(\mathrm{MUF}) = \sum_{i=1}^{T} T_i^2 \delta^{*2}_{st(I)} \qquad (9)$$

$$V^*(\mathrm{MUF}) = V(MUF) + V_t^*(MUF) \qquad (10)$$

where,
$V_{r/g/s}$(MUF): (element/isotope) MUF variance due to random/short-term systematic /systematic error,
x_k: Net U weight of stratum k, K: Number of strata in the facility,
$\delta_{(r/g/s)\,(q/p/t(E)/t(I))}$: Relative error of analysis method,
n_k: Item per batch in stratum k, m_k: Batch per stratum k,
$\overline{wt}_{(q/p/t(E)/t(I))}$: Average ^{235}U enrichment for each stratum and material balance,
$r_{(i/k)}^{(*)}$: Sample per batch in stratum k (isotope stratum i) for element (isotope) analysis,
$c_{(i/k)}^{(*)}$: Analysis per sample in stratum k (isotope stratum i) for element (isotope) analysis,
$M_{q/p/t(E)/t(I)} = \sum_{k=1}^{K} A_k x_{kq/p/t(E)/t(I)}$,
A_k: +1 for gain, -1 for loss.

Isotope MUF uncertainty from Processes 2 and 3 are summarized in Table 3, and from 4 and 5 are summarized in Table 4. Table 5 summarizes the results of calculated isotope MUF uncertainty and isotope MUF evaluation.

The results shown in Table 5 indicate that most of the uncertainty consists of sampling of scrap strata (SC-1L and SD-1L) and systematic error of bulk measurement of heavy strata (FF-11 and UF-1L).

To reduce the isotope MUF uncertainty, minimizing the systematic error of bulk measurement is required to reduce the isotopes because the sampling uncertainty might be overestimated due to the static material in scrap strata. However, IAEA's method to quantify systematic errors adopts a pseudo-numerical method [4].

Table 3. Isotope MUF uncertainty from element analysis.

V(MUF,Isotope)	Vr(MUF,Isotope)	Vg(MUF,Isotope)	Vs(MUF,Isotope)
1101.218	790.957	11.677	298.584

Stratum	Vr(xkq)	Vr(xkp)	Vr(xkt(E))
FF-11	0.02684	0.00000	0.00000
FF-BD	0.13767	0.00000	0.00000
FR-11	0.00000	0.00000	0.00000
FR-11	0.00000	0.00000	0.00000
FR-1G	0.00000	0.00000	0.00000
FR-1G	0.00000	0.00000	0.00000
HE-1L	0.00000	0.00000	0.00000
HE-1L	0.00000	0.00000	0.00000
UF-1L	0.66259	0.00000	0.00000
UF-1L	0.51015	0.00000	0.00000
PD-1L	0.00006	0.00721	0.00015
PD-1L	0.00000	0.00001	0.00000
PD-1L	0.00617	0.99476	0.02072
PL1-L	0.00002	0.00000	0.00000
PL1-L	0.00134	0.57481	0.01198
PL2-L	0.00000	0.00017	0.00000
PL2-L	0.00005	0.01307	0.00027
SA-1L	0.00000	0.00000	0.00000
SA-1L	0.00000	0.00000	0.00000
SA-1L	0.00000	0.00000	0.00000
SA-1L	0.00000	0.00000	0.00000
SA-1	0.00000	0.00000	0.00000
PM-1L	0.00000	0.00000	0.00000
PM-1L	0.00860	0.00000	0.00000
SC-1L	0.00001	0.00000	0.00000
SC-1L	0.00022	46.98033	0.15660
SC-1L	0.00013	7.26356	0.02421
SC-1L	0.00004	0.53546	0.00178
SC-1L	0.00001	0.06580	0.00022
SC-PL	0.00000	0.01033	0.00003
SC-PL	0.00025	31.77216	0.10591
PL1-L	0.00000	0.00458	0.00002
PL1-L	0.00012	0.01420	0.00118
SD-1L	0.00009	600.06381	0.02000
SD-1L	0.00003	100.95828	0.00337
SD-1L	0.00000	0.00000	0.00000

qpt(E)	Vg(q)	Vg(p)	Vg(t(E))
1.00000	0.000	0.000	3.543
2.00000	0.000	0.000	0.016
3.00000	3.680	0.000	3.518
4.00000	0.000	0.000	
5.00000	0.000		
6.00000	0.000		
7.00000	0.077		
8.00000	0.812		
9.00000	0.000		
10.00000	0.001		
11.00000	0.029		
12.00000	0.000		
13.00000	0.000		
14.00000	0.000		
15.00000	0.000		

qpt(E)	Vs(q)	Vs(p)	Vs(t(E))
1.00000	118.958	0.000	0.000
2.00000	0.046	0.000	0.000
3.00000	0.000	0.000	0.000
4.00000	101.505	0.000	
5.00000	78.069		
6.00000	0.000		
7.00000	0.000		
8.00000	0.000		
9.00000	0.006		
10.00000	0.000		
11.00000	0.000		
12.00000	0.000		
13.00000	0.000		
14.00000	0.000		
15.00000	0.000		

Table 4. Isotope MUF uncertainty from enrichment analysis.

V(MUF,Isotope)	Vr*(MUF)	Vs*(MUF)
795.045	762.572	32.473

ID	S_i	Vr*(MUF)
1	**42.465**	5.861E-03
2	**1.866**	1.132E-05
3	**0.720**	1.685E-06
4	**1.924**	1.203E-05
5	**5.077**	8.377E-05
6	**25.376**	2.093E-03
7	**5.554**	1.003E-04
8	**8.743**	2.484E-04
9	**21.185**	1.459E-03
10	**623.408**	1.263E+00
11	**27.593**	2.474E-03
12	**88.186**	2.527E-02
13	**645.299**	1.353E+00
14	**5.190**	8.754E-05
15	**339.788**	3.752E-01
16	**2.131**	1.476E-05
17	**1.066**	3.690E-06
18	**12.224**	4.856E-04
19	**156.238**	9.917E-03
20	**251.698**	2.059E-02
21	**6.755**	1.483E-04
22	**222.202**	3.209E-02
23	**6.656**	1.440E-04
24	**6.995**	1.590E-04
25	**453.418**	8.352E-02
26	**3.131**	3.186E-05
27	**52.183**	8.850E-03
28	**46.975**	2.391E-03
29	**5.824**	1.102E-04
30	**84.445**	7.725E-03
31	**168.711**	2.313E-02
32	**6.070**	1.197E-04
33	**125.450**	1.705E-02
34	**1670.759**	4.320E-01
35	**6.533**	1.387E-04
36	**75.919**	2.081E-03
37	**173.209**	9.750E-03
38	**685.422**	4.839E+01
39	**269.510**	7.481E+00
40	**73.175**	5.515E-01
41	**25.612**	6.757E-02
42	**10.162**	3.356E-04
43	**563.668**	1.033E+00
44	**9.574**	2.979E-04
45	**206.416**	1.385E-01
46	**244.962**	6.002E+02
47	**100.477**	1.010E+02

t	T_i	Vs*(MUF)
1	**5380.966**	2.895E+01
2	**1843.539**	3.399E+00
3	**345.439**	1.193E-01

Table 5. Results of MBE using the IAEA's conventional method.

	Meas. Inventory (kg)	Book Inventory (kg)	MUF (kg)	σ(MUF) (kg)	Significance(3σ)
U235 Weight	38,572.800	38,548.731	-24.069	43.546	No

3.2 Modified IAEA's method

Modified IAEA's method for uncertainty expression of isotope MUF is summarized in the literature [2, 3, 5]. The modified method directly calculates isotope MUF using Equation (2). The modified IAEA's method calculates the isotope MUF uncertainty by the following assumptions and processes:

Assumptions

1. Each item in the same stratum is homogeneous.
2. Each item in the same sub-stratum is measured using the same equipment.
3. Relative uncertainty of the measurement system is considered to be ITV.
4. Uncertainty for non-measured strata is considered to be zero.
5. Sampling for element analysis and enrichment analysis is consistent and single sampling is performed.
6. The benchmark facility has no static material.

Processes

1. Stratification of the inventory items using stratification rules.
2. Sub-stratification of each stratum based on item enrichment.
3. Calculation of isotope MUF uncertainty using Equations (3) and (11) ~ (13).

$$\mathrm{V(MUF)} = V_r(MUF) + V_g(MUF) + V_s(MUF) \quad (3)$$

$$\mathrm{V_r(MUF)} = \sum_{k=1}^{K} x_k^2 \left(\frac{\delta_{rq}^2}{n_k m_k} + \frac{\delta_{rp}^2}{r_k^* m_k} + \sum_{g(k)=1}^{G(k)} \left(\frac{\delta_{rt(E)}^2}{c_k r_k m_k} + \frac{\delta_{rt(I)}^2}{c_k^* r_k^* m_k} \right) \right) \quad (11)$$

$$\mathrm{V_g(MUF)} = \sum_q \delta_{gq}^2 \sum_i M_{qi}^2 + \sum_p \delta_{gp}^2 \sum_i M_{pi}^2 + \sum_{t(E)} \delta_{gt(E)}^2 \sum_i M_{t(E)i}^2 + \sum_{t(I)} \delta_{gt(I)}^2 \sum_i M_{t(I)i}^2 \quad (12)$$

$$\mathrm{V_s(MUF)} = \sum_q \delta_{gq}^2 M_q^2 + \sum_p \delta_{gp}^2 M_p^2 + \sum_{t(E)} \delta_{gt(E)}^2 M_{t(E)}^2 + \sum_{t(I)} \delta_{gt(I)}^2 M_{t(I)}^2 \quad (13)$$

where,

$\mathrm{V}_{r/g/s}(\mathrm{MUF})$: Isotope MUF variance due to random/short-term systematic/systematic error,

x_k: Net ^{235}U weight of stratum k, K: Number of strata in the facility,

$\delta_{(r/g/s)\,(q/p/t(E)/t(I))}$: Relative error of analysis method,

n_k: Item per batch in stratum k, m_k: Batch per stratum k,

$\overline{wt}_{(q/p/t(E)/t(I))}$: Average ^{235}U enrichment for each stratum and material balance,

$r_{(i/k)}^{(*)}$: Sample per batch in stratum k (isotope stratum i) for element (isotope) analysis,

$c_{(i/k)}^{(*)}$: Analysis per sample in stratum k (isotope stratum i) for element (isotope) analysis,

$\mathrm{M}_{q/p/t(E)/t(I)} = \sum_{k=1}^{K} A_k x_{kq/p/t(E)/t(I)}$,

A_k: +1 for gain, -1 for loss.

Table 6 describes the sub-stratification of a single stratum based on item enrichment. Table 7 summarizes the results of calculating the isotope MUF variance of each source and total isotope MUF variance. Table 8 summarizes the results of calculated isotope MUF uncertainty and isotope MUF evaluation.

The results shown in Table 8 indicate that most of the uncertainty consists of the sampling of scrap strata (SC-

1L and SD-1L) and the systematic error of bulk measurement of heavy strata (FF-11 and UF-1L). The characteristics of the modified method are similar to the conventional method; however, the size of sampling variance of the modified method is reduced compared to the conventional method due to the simplified sampling process in Assumption 5.

Table 6. Sub-stratification based on item enrichment.

Stratum	Description	Location(wt%)	q	q(i)	t_E	t_E(i)	p_I	p_I(i)	t_I	t_I(i)
PD-1L	UO2 Powder	PD Storage(1.28)	8	1	1	1	1	1	1	1
		1.6					1	3		
		2					1	5		
		2.2					1	6		
		2.3					1	7		
		2.4					1	8		
		2.9					1	10		
		3.15					1	12		
		3.45					1	13		
		4					1	17		
		4.1					1	18		
		4.5					1	20		
		4.65					1	21		

Table 7. Isotope MUF uncertainty using the modified IAEA's method.

u(MUF,Element) (kg)	V(MUF,Element) (kg2)	Vr(MUF)	Vg(MUF)	Vs(MUF)
33.773	1140.622	793.839	29.349	317.433

Stratum	Vr(xkq)	Vr(xkp)	Vr(xkt(E))	qpt(E)	Vg(q)	Vg(p)	Vg(t(E))	qpt(E)	Vs(q)	Vs(p)	Vs(t(E))
FF-11	0.02684	0.00000	0.00000	1.00000	0.000	0.000	3.543	1.00000	118.958	0.000	0.000
FF-BD	0.13767	0.00000	0.00000	2.00000	0.000	0.000	0.016	2.00000	0.046	0.000	0.000
FR-11	0.00000	0.00000	0.00000	3.00000	3.680	0.000	3.518	3.00000	0.000	0.000	0.000
FR-11	0.00000	0.00000	0.00000	4.00000	0.000	0.000		4.00000	101.505	0.000	
FR-1G	0.00000	0.00000	0.00000	5.00000	0.000			5.00000	78.069		
FR-1G	0.00000	0.00000	0.00000	6.00000	0.000			6.00000	0.000		
HE-1L	0.00000	0.00000	0.00000	7.00000	0.077			7.00000	0.000		
HE-1L	0.00000	0.00000	0.00000	8.00000	0.812			8.00000	0.000		
UF-1L	0.66259	0.00000	0.00000	9.00000	0.000			9.00000	0.006		
UF-1L	0.51015	0.00000	0.00000	10.00000	0.001			10.00000	0.000		
PD-1L	0.00006	0.00721	0.00015	11.00000	0.029			11.00000	0.000		
PD-1L	0.00000	0.00001	0.00000	12.00000	0.000			12.00000	0.000		
PD-1L	0.00617	0.99476	0.02072	13.00000	0.000			13.00000	0.000		
PL1-L	0.00002	0.00000	0.00000	14.00000	0.000			14.00000	0.000		
PL1-L	0.00134	0.57481	0.01198	15.00000	0.000			15.00000	0.000		
PL2-L	0.00000	0.00017	0.00000								
PL2-L	0.00005	0.01307	0.00027								
SA-1L	0.00000	0.00000	0.00000								
SA-1L	0.00000	0.00000	0.00000								
SA-1L	0.00000	0.00000	0.00000								
SA-1L	0.00000	0.00000	0.00000								
SA-1	0.00000	0.00000	0.00000								
PM-1L	0.00000	0.00000	0.00000								
PM-1L	0.00860	0.00000	0.00000								
SC-1L	0.00001	0.00000	0.00000								
SC-1L	0.00022	46.98033	0.15660								
SC-1L	0.00013	7.26356	0.02421								
SC-1L	0.00004	0.53546	0.00178								
SC-1L	0.00001	0.06560	0.00022								
SC-PL	0.00000	0.01033	0.00003								
SC-PL	0.00025	31.77216	0.10591								
PL1-L	0.00000	0.00458	0.00002								
PL1-L	0.00012	0.01420	0.00118								
SD-1L	0.00009	600.06381	0.02000								
SD-1L	0.00003	100.95828	0.00337								
SD-1L	0.00000	0.00000	0.00000								

Table 8. Results of MBE using the modified IAEA's method.

	Meas. Inventory (kg)	Book Inventory (kg)	MUF (kg)	σ(MUF) (kg)	Significance(3σ)
U235 Weight	38,572.800	38,548.731	-24.069	33.7730958	No

3.3 GUM method

While the conventional uncertainty expression methods present random and systematic errors, the guide to the expression of uncertainties in measurement (GUM) has been developed to overcome these limitations. GUM insists the quantification of systematic errors using a mathematical basis is impossible since achieving the true value is impossible. As a result, the GUM method insists a systematic error is quantified based on a non-mathematical process or assumption.

The GUM method quantifies the uncertainty of observation by following three processes. First, it identifies the measurements which contribute to the observation. Then, it quantifies the uncertainty of each measurement by combining the sources of uncertainty. Finally, it propagates the uncertainty of measurements. The following assumptions and processes were used to calculate the isotope MUF uncertainty of the BFFP.

Assumptions

1. Bulk weight is measured for each item and U concentration and isotope are analyzed for each lot.
2. The relative uncertainty of individual measurement is considered to be ITV because the purpose of benchmark MBE is to compare the difference between each method.
3. Uncertainty for non-measured strata is considered to be zero.
4. The facility operates a quality assurance program for its measurement system.
5. The covariance between the individual measurement result is zero due to the independence of each measurement.
6. The benchmark facility has no static material.

Processes

1. Stratification of inventory items based on the measurement system.
2. Identification of the equation to calculate the amount of ^{235}U for each item or stratum using Equation (14).
3. Identification of measurement (measured value) within the equation in Process 1.
4. Identification and calculation of uncertainty sources of each measurement
5. Calculation of uncertainty of each item, stratum and the BFFP using Equations (15) and (16).

The LII of the BFFP was re-organized based on the stratum. The example of inventory items in the single stratum (PD-1L(Gd), Pure UO_2 powder includes Gd poison) is depicted in Table 9.

Table 9. Example of re-organization of inventory items in PD-1L(Gd).

Q	P	T(E)	T(I)	Total U235	V(235U)
6	1	1	1	42.465	0.007

KMP	Plant	Location	MDC	Batch Name	Co nt. No	Qty	ULot No	Gross (KG)	Net (KG)	Uran (%)	Uran (KG)	U235 (%)	U235 (KG)
C	Plant 1	Powder Sto	GQRC	L16	8LM220	1	HO-16N20	474.9	327.1	87.804	287.207	2.214	6.359
C	Plant 1	Powder Sto	GQRC	L16	8LM220	1	HO-16N20	476	326	87.804	286.241	2.214	6.337
C	Plant 1	Powder Sto	GQRC	L16	8LM220	1	HO-16N20	381	232	87.804	203.705	2.214	4.510
C	Plant 1	Powder Sto	GQRC	L16	8LM220	1	HO-16N20	476.7	324.5	87.804	284.924	2.214	6.308
C	Plant 1	Powder Sto	GQRC	L16	8LM220	1	HO-16N20	477.6	327.6	87.804	287.646	2.214	6.368
C	Plant 1	Powder Sto	GQRC	L16	8LM220	1	HO-16N20	473.2	323.2	87.804	283.837	2.231	6.332
C	Plant 1	Powder Sto	GQRC	L16	8LM220	1	HO-16N20	471	319	87.804	280.149	2.231	6.250

The amount of ^{235}U in the BFFP was calculated using Equation (14). Since the size of relative uncertainty of the measurement system is assumed to be equivalent to the ITV, Process 3 and 4 were neglected. Once the relative uncertainty of each measurement has been set, the uncertainty of isotope MUF for each lot inside the stratum was calculated using Equation (15). The total isotope MUF uncertainty was then calculated using Equation (16). The equation (16) indicates the GUM method has much higher degree of freedom for stratification compared to the IAEA's methods.

$$X = \sum_{k=1}^{K}\left(\sum_{j(k)=1}^{J(k)}\left(\sum_{i(jk)=1}^{I(jk)} Q_{ijk}\right) \times P_{jk} \times T(E)_{jk} \times T(I)_{jk}\right) \quad (14)$$

$$\mathrm{u}(\mathrm{x}_{\mathrm{jk}})^2 = \left[\sum_{i(jk)=1}^{I(jk)}\left(\frac{\partial x_{jk}}{\partial Q_{i(jk)}}\right)^2 Q_{i(jk)}^2\right]\delta_q^2 + \left(\frac{\partial x_{jk}}{\partial P_{jk}}\right)^2 P_{jk}^2\delta_p^2 + \left(\frac{\partial x_{jk}}{\partial T(E)_{jk}}\right)^2 T(E)_{jk}^2\delta_{t(E)}^2 + \left(\frac{\partial x_{jk}}{\partial T(I)_{jk}}\right)^2 T(I)_{jk}^2\delta_{T(I)}^2 \quad (15)$$

$$\mathrm{u}(X)^2 = \sum_{k=1}^{K}\sum_{j(k)=1}^{J(k)}\left(u(x_{jk})^2\right) \quad (16)$$

where,
X: Isotope weight inside an MBE (BFFP),
u(X): Uncertainty of isotope
k: stratum k,
K: Number of strata in the facility,
$j(k)$: lot j of stratum k,
J(k): Number of lots in stratum k,
$i(jk)$: item i of lot k of stratum k,
I(jk): Number of items in lot j of stratum k,
$Q/P/T(E)/T(I)$: Measurement results
$\delta_{(q/p/t(E)/t(I))}$: Relative error of analysis method.

Table 10 summarizes the calculated isotope MUF uncertainty for each stratum. The results of isotope MUF evaluation using the GUM method are described in Table 11.

The results shown in Table 10 indicate the isotope MUF uncertainty mainly consists of dirty scrap (SD-1L) and pure UO_2 pellet (PL-1L) strata. The characteristics of the GUM method are the removal of systematic errors which results in the reduced uncertainty of heavy strata (FF-11 and UF-1L).

However, since the assumed uncertainties using the BFFP were significantly underestimated, the uncertainty of DA sampling strata will increase drastically. Future works will include realistic uncertainty for measurement systems and demonstrate the feasibility of applying the GUM method for the MBE for national inspection.

The isotope MUF uncertainty can be reduced by reducing the static material and sampling uncertainty of the scrap strata (SC-1L and SD-1L). The uncertainty can also be reduced by improving the bulk, sampling, element analysis and enrichment analysis of the measurement systems.

Table 10. Results of isotope MUF uncertainty for each stratum.

Stratum	V*(MUF) (kg^2)
FF-11	0.062
FF-BD	0.282
FR-11	0.014
FR-1G	0.002
HE-1L	0.000
UF-1L	2.434
PD-1L	6.067
PL1-L	20.724
PL2-L	0.233
SA-1L	0.000
SA-1	0.000
PM-1L	0.017
SC-1L	5.439
SC-PL	9.591
SD-1L	143.284

Table 11. Results of MBE using the GUM method.

	Meas Inventory (kg)	Book Inventory (kg)	MUF (kg)	σ(MUF) (kg)	Significance(3σ)
U235 Weight (KG):	38,572.800	38,548.731	-24.069	13.717	No

4. Results

Three different uncertainty expression methods were applied to evaluate the isotope MUF of the BFFP. The characteristics of the three methods are summarized in Table 12. The results shown in Table 12 indicate the GUM method is the most appropriate method for national inspection, once the quality of a target facility's measurement system was verified.

Table 12. Characteristics of uncertainty expression methods.

<table>
<tr><th></th><th>Conventional IAEA's method</th><th>Modified IAEA's method</th><th>GUM method</th></tr>
<tr><td rowspan="3">Assumptions</td><td>- Each item with the same enrichment is measured with the same system</td><td>- Each item in a sub-stratum is homogeneous
- Sampling for element and isotopic analysis is consistent</td><td rowspan="2">- Bulk weight is measured for each item, element and isotope are analyzed for each lot
- Facility operates quality assurance program for its measurement system
- The covariance between individual measurement is zero</td></tr>
<tr><td colspan="2">- Each item in a stratum is homogeneous</td></tr>
<tr><td colspan="3">- Relative uncertainty (error) of measurement system is ITV
- Uncertainty of non-measured strata is zero
- The BFFP has no static material</td></tr>
<tr><td>Advantages</td><td colspan="2">- Once design information is provided, the method can be applied all facilities
- The method can simplify the list of inventories based on stratum</td><td>- The uncertainty expression method is mathematically supported
- Management of uncertainty is possible</td></tr>
<tr><td>Disadvantages</td><td colspan="2">- ITV value cannot be achieved once the sample preparation of DA is included
- Management of uncertainty is impossible</td><td>- Realistic uncertainty using the GUM method is much bigger than IAEA's method</td></tr>
</table>

5. Conclusions

A notification (NSSC No. 2017-83) by the ROK government requires domestic MBE to be performed. The uncertainty expression method is the most important factor to quantify the uncertainty of nuclear material within a facility.

We examined the effect of the uncertainty expression method on the MBE by comparing the isotope MUF uncertainty of the BFFP between three different methods.

Our results indicate the conventional and modified IAEA's method can be easily applied to general nuclear facilities due to its simplified assumptions. However, the reliability of uncertainty quantification results was challenging. Also, uncertainty management was impossible due to inherent problems.

The GUM method overcame the limitations of the IAEA's method. However, it requires a qualified measurement system. Since the measurement uncertainties used for the MBE of the BFFP were assumed to be ITV, realistic uncertainty for measurements also must be quantified.

Future work will include the quantification of measurement uncertainties using the GUM method and demonstrate the feasibility of applying the GUM method as an uncertainty expression method for domestic MBE.

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