*Transactions of the Korean Nuclear Society Spring Meeting*
*Jeju, Korea, May 19-20, 2022*

# Improvement of crossflow model of MARS-KS by introducing inter-channel turbulent mixing model

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## 1. Introduction

As of state-of-the-art thermal-hydraulic system analysis codes, MARS-KS features a multi-dimensional analysis capability based on own multi-dimensional component, namely MULTID [1]. The multi-dimensional capability of MULTID is attributed from three-dimensional convection and diffusion. Also, it models additional diffusion of momentum and energy based on the simple turbulent mixing length model. With this, MULTID is capable of implementing the diversion and turbulent crossflows either, but the mixing through the turbulence model is confined to the diffusion transfer. When analyzing the bundle geometry, it is well known that the convective transfer is dominant compared to the diffusion transfer as the motion of the large eddies nearby the rod gaps yields subsidiary convection between subchannels [2]. Due to this, state-of-the-art subchannel analysis codes generally adopt the mixing model based on the convective transfer, which postulates direct inter-channel mixing [3].

In this study, the crossflow model of MARS-KS has been improved by introducing the inter-channel mixing model. Not only for MULTID, the one-dimensional component, which has no turbulence model, has been also improved by introducing the model. As the form of balance equation utilized in both components is the same except for the convection and diffusion, the modification of the field equation has been made based on the same methodology. The performance of the introduced model has been evaluated based on PSBT bundle experiment [4]. For the evaluation, the improved results of both one- and multi-dimensional models have been compared with the previous results without the inter-channel mixing model.

## 2. Code improvement

### *2.1 Inter-channel mixing model*

The inter-channel mixing model consists of two-different types of methodologies: equal-mass exchange (EM) and equal-volume exchange (EV). The EM model literally postulates equivalent mass exchange between channels. Through this, the model implements net momentum and energy transfers whereas none of net mass transfer is implemented. In general, this model is appropriate for single-phase mixing problem where net mass transfer is negligible. Meanwhile, the EV model postulates equivalent volume exchange, and thus, the model could implement net mass transfer once the densities of exchanging volumes are different each other. Such a case corresponds to two-phase flow condition where the globes of liquid and vapor are exchanged. The mass, momentum, and energy transfers postulated by both models are formulated as below.

- Mass exchange

$$w'_{A\leftrightarrow B} = \frac{\varepsilon}{l} A_{gap}(\rho_B - \rho_A) = 0 \tag{1}$$

$$w'_{A\leftrightarrow B} = \theta \left(\frac{\varepsilon}{l}\right)_{SP} A_{gap}\Big[(\hat{\rho})_B - (\hat{\rho})_A - K_{VD}\{(\hat{\rho})_B - (\hat{\rho})_A\}_{EQ}\Big] \tag{2}$$

where, $w'_{A\leftrightarrow B}$ represents net mass transfer between channels. The term $\frac{\varepsilon}{l}$ represents mixing velocity, and it is correlated by eddy diffusivity $\varepsilon$ and mixing length $l$. The term $\hat{\rho}$ stands for mixture density defined as $\hat{\rho} = \alpha_g\rho_g + \alpha_f\rho_f$.

- Momentum exchange

$$M'_{A\leftrightarrow B} = w'(v_B - v_A) \tag{3}$$

$$M'_{A\leftrightarrow B} = \theta \left(\frac{\varepsilon}{l}\right)_{SP} A_{gap}\Big[(G)_B - (G)_A - K_{VD}\{(G)_B - (G)_A\}_{EQ}\Big] \tag{4}$$

where, $M'_{A\leftrightarrow B}$ represents net momentum transfer between channels. $v$ and $G$ stand for fluid velocity and mass flux, respectively.

- Energy exchange

$$Q'_{A\leftrightarrow B} = w'(h_B - h_A) \tag{5}$$

$$Q'_{A\leftrightarrow B} = \theta \left(\frac{\varepsilon}{l}\right)_{SP} A_{gap}\left[\left(\widehat{\rho h}\right)_B - \left(\widehat{\rho h}\right)_A - K_{VD}\left\{\left(\widehat{\rho h}\right)_B - \left(\widehat{\rho h}\right)_A\right\}_{EQ}\right] \tag{6}$$

where, $Q'_{A\leftrightarrow B}$ represents net energy transfer between channels. The term $\widehat{\rho h}$ stands for mixture enthalpy per unit volume.

Eq. (1), Eq. (3), and Eq. (5) represent net mass, momentum, and energy transfers by EM model. Basically, EM model postulates the mixing based on the mixing rate of mass $w'$. The mixing rate is generally correlated by non-dimensional parameter, namely mixing coefficient $\beta$, as $w' = \beta A_{gap} \bar{G}$. Meanwhile, Eq. (2), Eq. (4), and Eq. (6) represent net mass, momentum, and energy transfers by EV model. The EV model basically postulates net mixing by the difference in mixture density, and thereby the mixing occurs if the void difference exists between channels. The term $\theta\left(\frac{\varepsilon}{l}\right)_{SP}$ in each equation stands for two-phase mixing velocity, and it is correlated by the single-phase mixing velocity, which is the same in EM model, multiplied by two-phase turbulent intensity multiplier $\theta$ from Beus correlation [5]. The terms enclosed by subscript 'EQ' stands for void drift terms, and they implement net mixing not to make uniform void distribution within bundle. The void drift is correlated proportional to the difference of mass flux between channels, and its magnitude is adjusted by the multiplier, namely void drift coefficient $K_{VD}$.

### *2.2 Modification of field equation*

In order to introduce the inter-channel mixing model, the field equation of MARS-KS has been modified. The modification has been made by defining the mixing term as an additional source in each balance equation. The equation derived below is the general form of each balance equation with respect to specific phase '$k$' of working fluid within control volume '$V$'.

- Mass balance equation

$$\frac{\partial}{\partial t}(\alpha_k \rho_k V) + \nabla \cdot (\alpha_k \overline{\rho_k}\, \overline{v_k}^{\rho}) = \Gamma_k + (w_k')''' \qquad (7)$$

where, the first and second terms on the left-hand side (LHS) represent net change of mass and transport by convection, respectively. $\Gamma_k$ on the right-hand side (RHS) stands for net mass transfer due to phase change, and $(w_k')'''$ is the derived mixing of mass by inter-channel mixing model per unit volume.

- Momentum balance equation

$$\frac{\partial}{\partial t}(\alpha_k \overline{\rho_k}\, \overline{v_k}^{\rho}) + \nabla \cdot (\alpha_k \overline{\rho_k v_k}^{\rho} \overline{v_k}^{\rho}) = -\alpha_k \nabla \overline{P_k} + \alpha_k \overline{\rho_k}\, \mathrm{g} + \Gamma_k\, \overline{v_k}^{\Gamma} + F_{\sigma k} + F_{wk} + (M_k')''' \qquad (8)$$

where, the terms on the LHS represent net change of momentum and convective transport of momentum. The first and second terms on RHS stand for the forces induced by press gradient and gravity. The third term represents the momentum transfer due to phase change. The fourth and fifth terms are frictional forces with respect to phasic interface and wall. The last term $(M_k')'''$ is the derived momentum mixing by inter-channel mixing model per unit volume.

- Energy balance equation

$$\frac{\partial}{\partial t}(\alpha_k \overline{\rho_k u_k}^{\rho}) + \nabla \cdot [\alpha_k \overline{\rho_k}\, \overline{u_k}^{\rho} \overline{v_k}^{\rho}] = Q_{\sigma k} + Q_{wk} - \overline{P_k}\left[\frac{\partial \alpha_k}{\partial t} + \nabla \cdot (\alpha_k \overline{v_k})\right] + \Gamma_k h_k + \alpha_k \overline{Q_k} + \alpha_k \Phi_k^2 + (Q_k')''' \qquad (9)$$

where, the terms on the LHS represent net change of internal energy and energy transport by convection. The first and second terms on RHS represent heat transfer by phasic interface and wall surface. The third term stands for pressure work, and the fourth term represents heat transfer induced by phase change. The fifth and sixth terms are the generation and dissipation of heat with respect to phase '$k$'. The last term $(Q_k')'''$ is the derived energy mixing by inter-channel mixing model per unit volume.

## 3. Performance evaluation

### *3.1 PSBT benchmark evaluation*

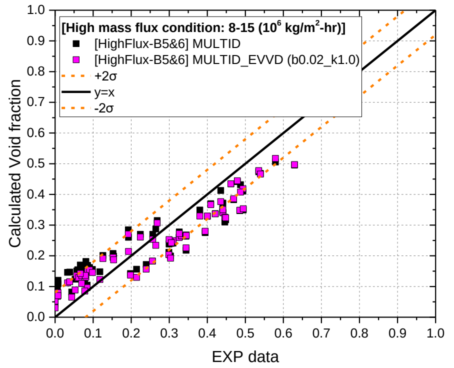
For brevity, the model description will be replaced by the previous work of this study [6]. As the mixing rate of introduced model is proportional to the channel mass flux, the data selection has been made for evaluating the model performance according to the mass flow condition as listed in Table I. For the evaluation, the results of improved one- and multi-dimensional models have been compared with the previous results without the inter-channel mixing model. For the inter-channel mixing calculation, the mixing and void drift coefficients have been given as 0.02 and 1.0, respectively.

Fig. 1 shows the predicted void fraction by improved and original 3D models. The comparison of the results clearly show that the general performance gets improved by applying the inter-channel mixing model. Especially, the low void region (void.lt.30%) has been remarkably improved as the overpredicted results change to be lowered according to the derived mixings between channels. Similar results are also captured in the prediction of 1D model as depicted in Fig. 2.

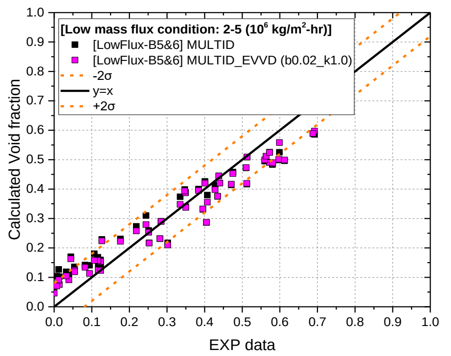
When comparing the root mean square errors (RMSE) listed in Table II and Table III, the results reveal that the high mass flux cases get influenced more than the low mass flux cases as expected. However, when comparing the improved results of 1D and 3D models, the listed RMSEs indicate that the 1D model results in better performance compared to the 3D model. Especially, the results of higher mass flux cases show the remarkable changes in higher void region (void.gt.30%). These changes are made from the modeling characteristic of 1D model where the test section has been modeled with two-lumped channels: central and peripheral channels. As depicted in Fig. 3, the lumping of peripheral channels yields larger estimation of difference in mass flux between channels compared to the 3D model. Due to this, the estimated mixing by void drift gets more remarkable, and thereby the resulting void distribution of 1D model changes as the vapor in the higher void region moves toward the central channel as depicted in Fig. 4. From this, the results of 1D model show more concentrated void predictions in the higher void region.

Table I: Selected cases for the evaluation

| Test series | Case number | Test conditions |
|---|---|---|
| B5 | 5.1122, 5.1232, 5.1341, 5.2131, 5.2241, 5.2332, 5.3112, 5.3222, 5.3331, 5.4212, 5.4321 | [High mass flux]<br>Pressure: 4.8~16.4 MPa<br>Inlet temperature: 436~595 K<br>Mass flux: 7~15 $10^6 kg/m^2hr$ |
| | 5.1452, 5.2452, 5.3442, 5.4432, 5.4562, 5.5431, 5.5551, 5.6441, 5.6551 | [Low mass flux]<br>Pressure: 4.8~16.6 MPa<br>Inlet temperature: 422~595 K<br>Mass flux: 2~5 $10^6 kg/m^2hr$ |
| B6 | 6.1122, 6.1231, 6.1342, 6.2132, 6.2242, 6.2342, 6.3122, 6.3232, 6.3332, 6.4222, 6.4332, 6.5211, 6.5332, 6.6321, 6.6331 | [High mass flux]<br>Pressure: 4.8~16.5 MPa<br>Inlet temperature: 426~585 K<br>Mass flux: 8~15 $10^6 kg/m^2hr$ |
| | 6.1452, 6.2461, 6.3451, 6.4452, 6.4562, 6.5442, 6.5562, 6.6451, 6.6561 | [Low mass flux]<br>Pressure: 4.9~16.6 MPa<br>Inlet temperature: 417~585 K<br>Mass flux: 2~5 $10^6 kg/m^2hr$ |

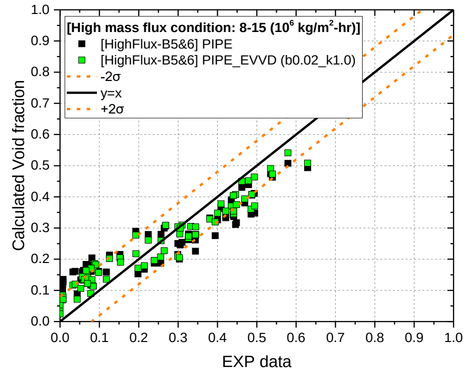



(a) high mass flux condition

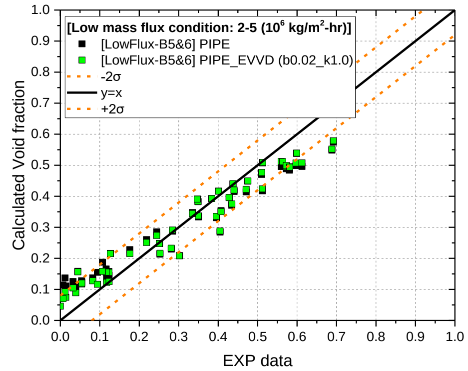



(b) low mass flux condition

Fig. 1 Void prediction of MULTID (3D component)




(a) high mass flux condition




(b) low mass flux condition

Fig. 2 Void prediction of PIPE (1D component)

Table II: Calculated root mean square errors of 3D model

| Case classification | Test conditions | RMSE | |
|---|---|---|---|
| | | MULTID (Original) | MULTID (EVVD) |
| Void < 30% | High mass flux | 0.07605 | 0.05826 |
| Void > 30% | | 0.08749 | 0.08681 |
| All | | 0.08094 | 0.07136 |
| Void < 30% | Low mass flux | 0.06938 | 0.05540 |
| Void > 30% | | 0.06707 | 0.06506 |
| All | | 0.06811 | 0.06096 |

Table III: Calculated root mean square errors of 1D model

| Case classification | Test conditions | RMSE | |
|---|---|---|---|
| | | PIPE (Original) | PIPE (EVVD) |
| Void < 30% | High mass flux | 0.08389 | 0.06196 |
| Void > 30% | | 0.08864 | 0.06621 |
| All | | 0.08587 | 0.06374 |
| Void < 30% | Low mass flux | 0.06904 | 0.05311 |
| Void > 30% | | 0.07126 | 0.06711 |
| All | | 0.07028 | 0.06128 |

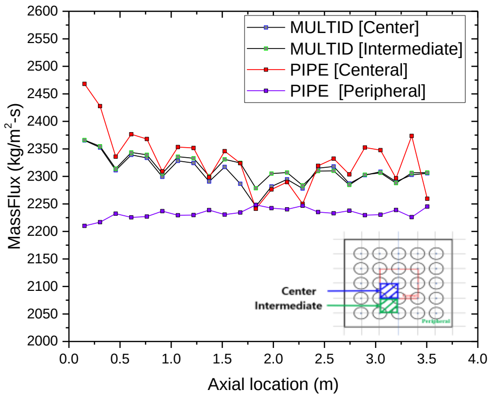



Fig. 3. Axial mass flux distribution (Case No. 5.4321)

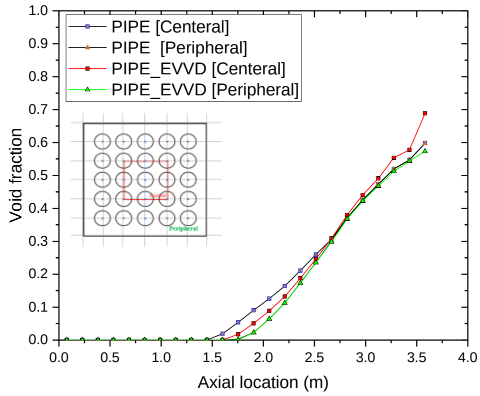



Fig. 4. Axial void distribution of 1D model (Case No. 5.4321)

### 3. Conclusion

In this study, the crossflow model of MARS-KS has been improved by introducing the inter-channel turbulent mixing model utilized in subchannel analysis. For the improvement, the field equation of MARS-KS has been modified by defining the mixing term as an additional source of each balance equation. As the general form of balance equation was the same between one- and multi-dimensional components, the modified equation has been applied in both models. Through the evaluation based on PSBT bundle experiment, the improved models showed great performance compared to the previous ones as expected. In addition, the improved results of 1D model revealed better performance compared to the 3D model. Especially, the void prediction in the higher void region has been remarkably improved in 1D model, as the implemented mixing by void drift gets dominant compared to 3D model. Nevertheless, it is obvious that the introduced inter-channel mixing model improves the general performance of 1D and 3D models. This indicates that the implementation of additional mixing has a great influence on the void prediction in bundle, and thus, this should be considered for the better estimation of thermal hydraulics in bundle. But further evaluation is still necessary. Therefore, various bundle experimental data such as GE 3X3 and RPI 2X2 will be employed for the evaluation of both improved 1D and 3D models.

### ACKNOWLEDGEMENT

This work was supported by the Nuclear Safety Research Program through the Korea Foundation Of Nuclear Safety(KoFONS) using the financial resource granted by the Nuclear Safety and Security Commission(NSSC) of the Republic of Korea. (No. 2003002)

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