Improvement of Well-Posedness of SPACE Code Using Artificial Phase Change and Viscosity

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1. Introduction

There are a number of methods to remedy the ill-posedness of the basic one-dimensional two-fluid model. The regularization of the two-fluid model includes the interfacial pressure, hydrostatic pressure, and surface tension, etc.

Holmås et al. [1] added artificial axial diffusion terms to the 1D incompressible mass and momentum equations, which resulted in well-posed two-fluid model. Fullmer et al. [2] extended that concept to create a model that prescribed the cutoff length scale precisely. Fullmer et al. [3] showed that that concept was effective to remedy the ill-posedness in the Kelvin-Helmholtz instability flow.

However, the previous works on the artificial diffusion terms has a critical problem that the total fluid mass is not conserved. This study proposes a solution to the problem.

2. Theory

2.1 Existing Model [2,3]

Artificial phase change terms are added to the 1D incompressible mass equations as follows:

$$\frac{\partial}{\partial t}(\alpha_g\rho_g)+\frac{\partial}{\partial x}(\alpha_g\rho_g u_g)=\varepsilon_g\rho_g\frac{\partial^2\alpha_g}{\partial x^2}, \qquad (1)$$

$$\frac{\partial}{\partial t}(\alpha_l\rho_l)+\frac{\partial}{\partial x}(\alpha_l\rho_l u_l)=\varepsilon_l\rho_l\frac{\partial^2\alpha_l}{\partial x^2}. \qquad (2)$$

Artificial viscosity terms are considered as follows:

$$\alpha_g\rho_g\left(\frac{\partial u_g}{\partial t}+u_g\frac{\partial u_g}{\partial x}\right)=-\alpha_g\frac{\partial p}{\partial x}+\alpha_g\rho_g\nu_g\frac{\partial}{\partial x}\left(\alpha_g\frac{\partial u_g}{\partial x}\right), \qquad (3)$$

$$\alpha_l\rho_l\left(\frac{\partial u_l}{\partial t}+u_l\frac{\partial u_l}{\partial x}\right)=-\alpha_l\frac{\partial p}{\partial x}+\alpha_l\rho_l\nu_l\frac{\partial}{\partial x}\left(\alpha_l\frac{\partial u_l}{\partial x}\right). \qquad (4)$$

Assuming $\nu=\varepsilon_g=\varepsilon_l=\nu_g=\nu_l$, the linear stability analysis yields the following dispersion relation:

$$\omega_I=-\nu k^2+\frac{\sqrt{\alpha_g\alpha_l\rho_g\rho_l}}{\bar{\rho}}|u_g-u_l|k, \qquad (5)$$

where $\bar{\rho}=\alpha_l\rho_g+\alpha_g\rho_l$. For large wavenumbers, the growth rate ω_I becomes negative, i.e., well-posed. The critical wavenumber is then given by

$$k_c=\frac{\sqrt{\alpha_g\alpha_l\rho_g\rho_l}}{\bar{\rho}\nu}|u_g-u_l|. \qquad (6)$$

However, a critical problem is non-conservation of the total mass. The sum of the right-hand sides of Eqs. (1) and (2) does not become zero, which means that the total mass is not conserved. Mathematically, no matter how the value of ν is small, the two-fluid model is well-posed. However, the value of ν should be large enough from the practical point of view (finite mesh size). At the same time, the value should not be so large that the total mass error becomes severe.

2.2 Proposed Model (Present)

To avoid the problem in the existing model, we suggest the mass equations as follows:

$$\frac{\partial}{\partial t}(\alpha_g\rho_g)+\frac{\partial}{\partial x}(\alpha_g\rho_g u_g)=\varepsilon\rho_g\frac{\partial^2\alpha_g}{\partial x^2}, \qquad (7)$$

$$\frac{\partial}{\partial t}(\alpha_l\rho_l)+\frac{\partial}{\partial x}(\alpha_l\rho_l u_l)=\varepsilon\rho_g\frac{\partial^2\alpha_l}{\partial x^2}. \qquad (8)$$

Though the mass of each phase is not conserved, the total mass is conserved. This is an obvious advantage.

Assuming $\nu=\nu_g=\nu_l=\varepsilon$, we obtain the critical wavenumber as follows:

$$k_c=\frac{\sqrt{\alpha_g\alpha_l[\alpha_g\rho_g(\rho_l+\rho_g)^2+\alpha_l\rho_l(\rho_g+\rho_g)^2]}}{\nu\sqrt{\rho_g}(\rho_g+\bar{\rho})}|u_g-u_l| \qquad (9)$$

This new model makes the two-fluid model well-posed at larger wavenumbers. This might be a disadvantage.

3. Results and discussion

In this paper, the test results for the water faucet problem in a vertical pipe with a length of 6 m and a diameter of 1 m are briefly discussed. Water is injected from the top at a velocity of 10 m/s.

3.1 Basic Two-Fluid Model

Figures 1 and 2 shows the simulation results when the artificial terms are not added to the mass and momentum equations. Figure 1 shows the water distributions at t = 0.3 s, except for the case with Δx = 6 mm (1000 cells). Figure 2 shows the total mass errors. For the case with

$\Delta x = 6$ mm, the total mass error suddenly increases at t = 0.291382 s and is unexpectedly terminated, which is due to the ill-posedness of the basic two-fluid model.

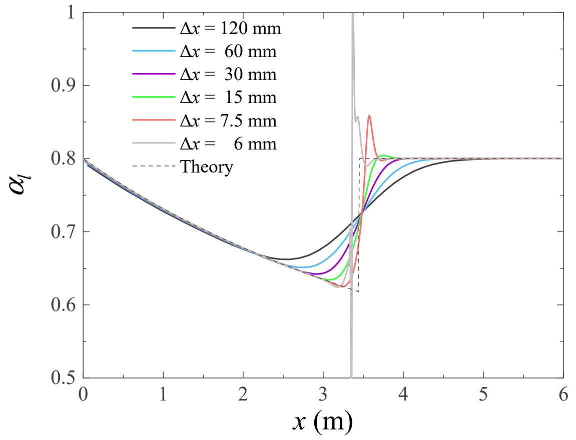

Fig. 1. Water fraction distributions at 0.3 s for the basic model

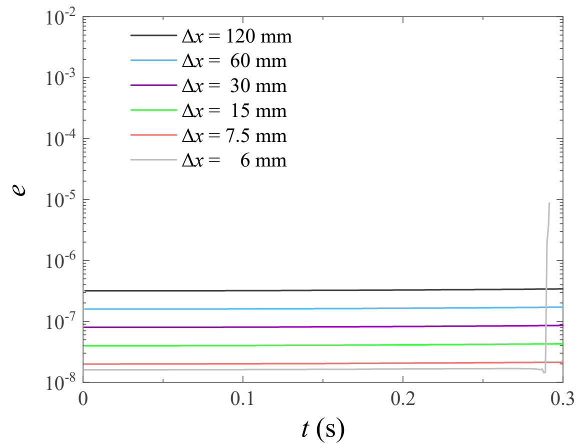

Fig. 2. Mass errors for the basic model

3.2 Existing Model

Figures 3 and 4 show the results when Eqs. (1) ~ (4) are used with $\nu = 0.25\bar{\rho}(2D)|u_g - u_l|/(2\pi)$. Even for the case with $\Delta x = 3.75$ mm (1600 cells), the water fraction is reasonably predicted. However, as shown in Fig. 4, the total mass error reaches up to 0.05%. In view of the calculation time 0.3 s, this mass error is significantly large.

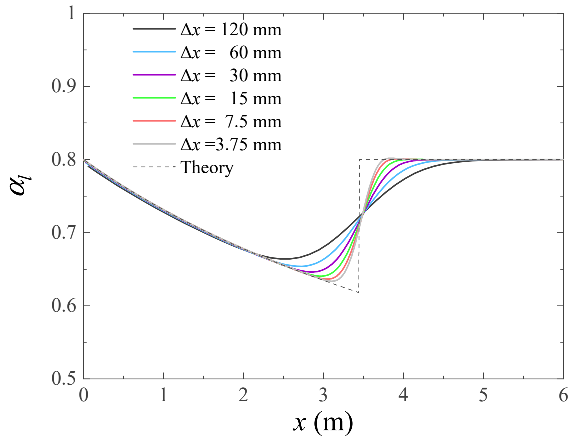

Fig. 3. Water fraction distributions at 0.3 s for the existing model

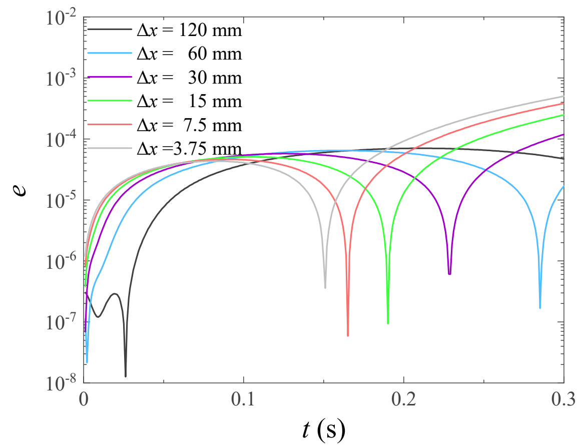

Fig. 4. Mass errors for the existing model

2.4 New Model

Figures 5 and 6 show the results when Eqs. (7), (8), (3), and (4) are used with $\nu = 0.25\bar{\rho}(2D)|u_g - u_l|/(2\pi)$. Note that the total mass errors are significantly decreased such that they are negligible.

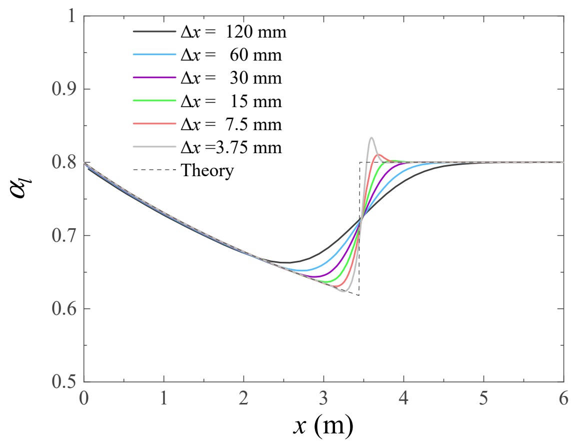

Fig. 5. Water fraction distributions at 0.3 s for the proposed model

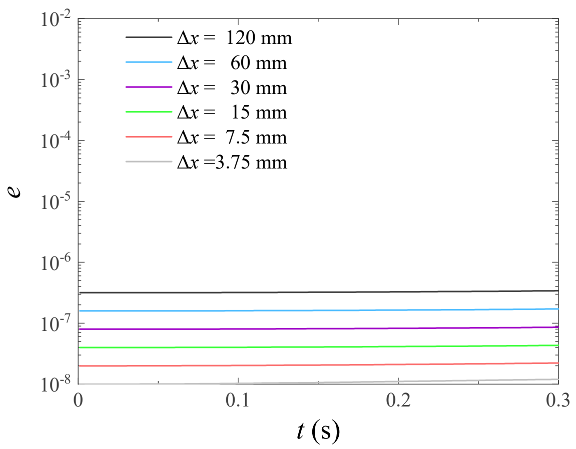

Fig. 4. Mass errors for the proposed model

3. Conclusions

In this paper, artificial phase change terms were suggested to make the two-fluid model well-posed while the total mass is conserved.

The proposed model was tested for the water faucet problem. The well-posedness was improved and the total mass error were significantly decreased.

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