Study on Virtual Thermometry used in Small Modular Reactor Using Dynamic Data Reconciliation.

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1. Introduction

Process variables include not only the uncertainty of an inherent of the instrument, but also the uncertainty occurred due to the surrounding environment or instrument failure [1,2,3]. In particular, the uncertainty of process variables used for small modular reactors (SMRs) is likely to be more increased due to the compact system size, the geometry of changed system, and the harsh environment by the operating nature. Also, these conditions are led to a limitation of diversity and redundancy for installing process instruments. Therefore, the estimation of accurate state for process variables used in the SMR should be backup as a problem to be overcome in the future [4,5] The data reconciliation (DR) technique is widely used for this purpose. The DR is a methodology for estimating an accurate state by re-distributing the uncertainty of measured variables such that first principles are satisfied. This can be considered a potential method for minimizing the uncertainty of measured variables occurred by the nature of SMR. Furthermore, it is necessary to estimate the accurate state of time dependent parameters on the basis of the dynamic data reconciliation (DDR) technique.

With sharing this motivation, this study was investigated how the impoverished quality in measurement affects the state estimation to apply the DDR technique for estimating the state. The Resistance Temperature Detector (RTD) and Thermocouple (TC), which is used in SMR as a thermometry, were selected for comparative analysis since the temperature variable is significant for maintaining the robustness of nuclear fuel [6]. Also, the characteristics of each instrument are suitable for comparative analysis because they have opposite characteristics.

2. Methods and Results

This section describes the characteristics of RTD and TC and the DDR technique.

2.1 Characteristic of RTD and TC

According to International Electrotechnical Commission (IEC) 60751 and ASTM E644 standards, the specifications of RTD and TC are shown in Table 1.

Table 1. Characteristic of RTD and T.C

Type	Response time(τ)	Tolerance
RTD	6 sec	Class A: ± (0.15 + 0.002*t)℃
		Class B: ± (0.3 + 0.005*t)℃
TC	2 sec	Class A: ± 1.5℃ (or ±0.4%)
		Class B: ± 2.5℃ (or ±0.75%)

Each of instruments have opposite characteristics. RTD has a slow response time, while the accuracy is high. In the case of TC, the response time is high, but the uncertainty of the measured value is high. In addition, since TC is more available under harsh environmental conditions than RTD, it is widely used as measuring the inlet and outlet temperature of the reactor core. Due to the contradictory characteristics and usage environment, it is not easy to find optimal usage in the SMR design process [7].

2.2 Steady State Data Reconciliation

DR is a constraint least square objective function to obtain an optimal solution based on first principles such as mass balance and heat balance. If the system or process does not change for a certain period of time, the steady state DR that uses the average value of the measured values is useful and is expressed as Equation (1) [8].

$$min(\hat{y},\hat{z}) = (y-\hat{y})^T V(y-\hat{y}) \quad (1)$$
$$\text{Subject to} \quad f(\hat{y},\hat{z}) = 0$$
$$g(\hat{y},\hat{z}) \geq 0$$

Where $\boldsymbol{y}$ is measured value, $\boldsymbol{\hat{y}}$ is reconciled value, $\boldsymbol{\hat{z}}$ is unmeasured value and $\boldsymbol{V}$ is covariance matrix. And $\boldsymbol{f}$ and $\boldsymbol{g}$ is first principle model.

If the redundancy of measurement points and valid physical model are guaranteed, it is possible to minimize random errors such as incorrect installation of the instrument, noise caused by the surrounding environment, and fluctuations. In addition, it has the advantage of detecting gross errors caused by instrument failure and miscalibration.

2.3 Dynamic Data Reconciliation

DDR is used when input parameters and system model changes in real time. DDR, which is frequently used in discrete time models, is expressed as Equation (2) [8].

$$x_k = A_k x_{k-1} + B_k u_{k-1} + w_{k-1} \qquad (2)$$
$$y_k = H_k x_k + v_k$$

Where $\boldsymbol{x_k}$ state variables, $\boldsymbol{u_k}$ is adjusted input, $\boldsymbol{w_k}$ is system model disturbance, $\boldsymbol{y_k}$ is measured values, and $\boldsymbol{v_k}$ is measurement error. And, $\boldsymbol{A}$ is equation of system model and $\boldsymbol{B}$ is equation of optional control input and $\boldsymbol{H}$ is state equation related measurement $\boldsymbol{y_k}$. The subscript $\boldsymbol{k}$ is time.

A is a system model for estimating the state of step at $\boldsymbol{k}$ in $\boldsymbol{k\text{-}1}$ state and it changes with each sampling time due to a time dependent function.

In Equation (2), system model disturbances express system state following the normal distribution having a zero mean in Equation (3). And they are related with covariance matrices $\boldsymbol{R_k}$ and $\boldsymbol{Q_k}$.

$$w \sim N(0, Q) \qquad (3)$$
$$v \sim N(0, R)$$

As shown in Equation (2), due to the random disturbance of the system model, the true value of variables at each sampling time always contains the random error. Therefore, in order to derive the optimal state estimation at time $\boldsymbol{k}$, the least squared function at time $\boldsymbol{k}$ is applied as in Equation (4).

$$min(\hat{x}_k, \hat{y}_k) = \sum_{t=0}^{k} (x_k - \hat{x}_k)^T V (x_k - \hat{x}_k) \qquad (4)$$

Here, it should be noted that there is $\boldsymbol{u_k}$, which is applied as a weight by calculating the deviation between the measured value and the estimated value every $\boldsymbol{k}$ time. This determines how much measurement value is reflected in the result value.

Since DDR is a Bayesian recursive estimation model that estimates the information of $\boldsymbol{k}$ time using the information of $\boldsymbol{k\text{-}1}$ time, therefore, the above series of processes is calculated recursively. In other words, DDR is a filtering method for estimating the state using data which is acquired during all sampling time.

2.4 Physical Model

In this study, the one-dimensional heat conduction equation of Eq. (8) was used as the physical model to demonstrate the DDR.

$$\frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} \qquad (8)$$

Where $\boldsymbol{T}$ is temperature, $\boldsymbol{t}$ is time, $\boldsymbol{x}$ is length and $\boldsymbol{k}$ is thermal conductivity.

In addition, the finite-difference method was applied as shown in Equation (9) to calculate the node-wise temperature distribution by applying the boundary condition of the heat conduction problem [9].

$$T_i^{k+1} = k \frac{\Delta t}{\Delta x^2} T_{i-1}^k \left(1 - 2k \frac{\Delta t}{\Delta x^2}\right) T_i^k + k \frac{\Delta t}{\Delta x^2} T_{i+1}^k \qquad (9)$$

2.5 Reference Data Set

First, numerical solutions were obtained as a reference data set for analysis by applying the initial conditions to the one-dimensional heat conduction equation mentioned in Section 2.3 below.

T=0 at x=0, for t>0
T = 45℃ for t=0, x>0
*α = 4.7*E-7 m²/s*
r = αΔt/Δx²
Final time is 200sec

And, the measured value $\boldsymbol{y_k}$ was produced by applying the standard deviation of instrument and a normally distributed random number to the mean value.

3. Result

In this study, the sensitivity evaluation was performed in consideration of the various form of impoverished variable such as a measured variable with high uncertainty of RTD occurred by the slow response time and inaccurate measured variable of TC occurred due to the inaccurate system model of instrument.

First of all, the DDR was conducted assuming that all variables were perfectly acquired in a normal condition, which is case 1. The case 2 is consideration of a measured variable with high uncertainty at a specific sampling time due to a long response time like RTD. The sensitivity analysis about the measured variables with high uncertainty can be performed by adjusting the factor $\boldsymbol{H}$, which is a value representing the state of the measured value in Equation (2). The case 3 is tried to the sensitivity evaluation in consideration of an inaccurate measured variable such as such as TC, which is a fast response time but inaccurate. It was analyzed by adjusting the factor $\boldsymbol{A}$, which represents the accuracy of the measurement system in Equation (2). And, the standard deviation for a covariance error was assumed to be 2℃.

3.1 Result of Case 1.

($\boldsymbol{\sigma_s = 0.1, \sigma_y = 2, \sigma_i = 0.8 + rand, H = 1}$, *where* $\boldsymbol{\sigma_s}$ *is Standard Deviation of System Model,* $\boldsymbol{\sigma_y}$ *is Standard Deviation of Measured Value,* $\boldsymbol{\sigma_i}$ *is Standard*

Deviation for initial value with random value following the Gaussian distribution.)

Figure 1 is showing the result of case 1. Measured variables $\boldsymbol{y_k}$ are more sporadically distributed than the numerical solution due to the uncertainty of the measured variable. In other words, it cannot be regarded as an accurate state estimation due to the uncertainty of the measured variable. On the other hand, the result derived from DDR was approached almost consistent with the numerical solution.

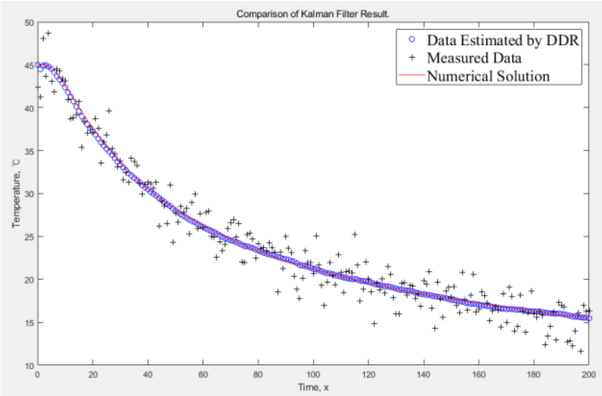

Figure 1. the result of case 1

3.2 Result of Case 2.
$(\boldsymbol{\sigma_s = 0.1, \sigma_y = 2, \sigma_i = 0.8 + rand, H = 0.5})$

In case 2, sensitivity analysis was performed considering when the factor $\boldsymbol{H}$ was 0.5, that is, only the measured variable is 50% saturated. As shown in Figure 2, measured variable with high uncertainty show that accurate state estimation was impossible. This indicates that the data having a high uncertainty cannot eventually be used for state estimation.

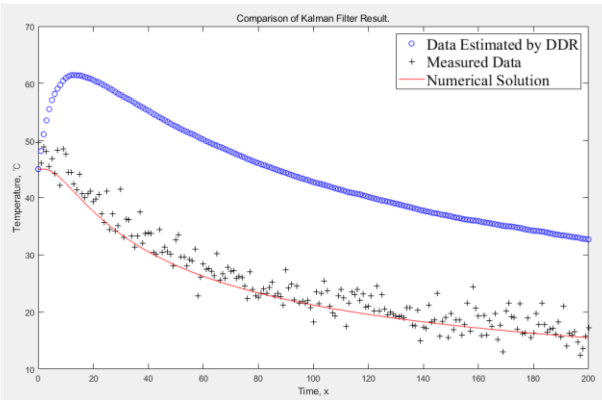

Figure 2. the result of case 2

3.3 Result of Case 3.
$(\boldsymbol{\sigma_s = 5, \sigma_y = 2, \sigma_i = 0.8 + rand, H = 1})$

In Case 3, sensitivity analysis was performed by adjusting the factor $\boldsymbol{A}$, which is representing the accuracy of system model. Figure 3 shows the DDR results using inaccurate measured variables. According to the result of case 2 and 3, inaccurate measured variables are more accurate than variables with high uncertainty when perform the DDR. However, this result eventually shows that it is difficult to estimate the accurate state estimation when the inaccuracy of the measured variable is increased.

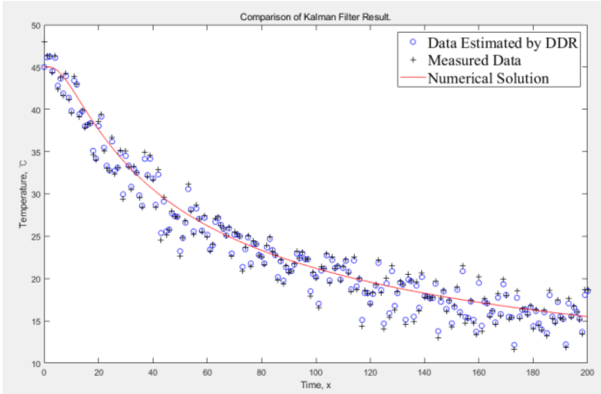

Figure 3. the result of case 3

4. Conclusions

In this study, the dynamic data reconciliation technique was applied to perform the sensitivity analysis for observing how the uncertainty of measured variable and the inaccuracy of system affect the results of the DDR. Eventually, it was found that the measured variable with high uncertainty such as RTD is more influencing to the state estimation process of DDR than variables by inaccurate system such as TC. As a further study, we will apply the DDR to some of actual system models of the SMR. In addition, as the results discussed on above, we will propose a method to improve the impoverished data quality occurred by sampling time using a data preprocessing.

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