Transactions of the Korean Nuclear Society Spring Meeting
Jeju, Korea, May 19-20, 2022

Particle motions in Magnetic Island

Sungpil Yum [a], Eisung Yoon [a*], Wonjun Tae [b], Dongkyu Kim [a], and Min Sup Hur[b]
[a]*Department of Nuclear Engineering, Ulsan National Institute of Science and Technology., 50 UNIST-gil, Eonyang-eup, Ulju-gun, Ulsan*
[b]*Department of Physics, Ulsan National Institute of Science and Technology., 50 UNIST-gil, Eonyang-eup, Ulju-gun, Ulsan*

[*]*Corresponding author: esyoon@unist.ac.kr*

1. Introduction

Tearing modes, one of obstacles toward high performance nuclear fusion reactors, manifests itself by forming magnetic island in magnetic field structure [1]. Followed by appearance of the island structure, loss of plasma energy and rapid decay of plasma currents occur during high performance discharges in general [2]. In this work, to clarify effects of presence of magnetic islands on particle trajectories, which would affect plasma confinement, we have developed a passive particle simulation code, Particles Around Magnetic Island (PAMI), that traces full orbits of electrons under magnetic islands. In this code, we use analytic expressions for both concentric circular background magnetic field and perturbed magnetic island. For the given magnetic fields, equation of motion for the particles is numerically integrated by Boris method [3]. As a result of the simulation, paths of the particles projected to a poloidal plane (R, Z) with and without magnetic island are presented in this article.

2. Methods and Results

This section illustrates the calculation of magnetic island for the magnetic fields in a tokamak geometry and Boris method for pushing a charged particle to trace its trajectory in cylindrical coordinate system (R, ϕ, Z). In the following simulation setup, we use an electron as a tracing species, and there is no applied external electric field over simulation domain.

2.1 Magnetic fields

For background magnetic field, we use an analytic expression of the concentric circular magnetic field as follows:

$$\boldsymbol{B}_0 = \frac{R_0 B_0}{R}\left(\hat{e}_\phi + \frac{r}{R_0 \bar{q}}\hat{e}_\theta\right), \tag{1}$$

where $\bar{q}$ is an effective safety factor,

$$\bar{q}(r) = q(r)\sqrt{1-\left(\frac{r}{R_0}\right)^2} \tag{2}$$

B_0 is the background magnetic field, and q is safety factor representing the number of toroidal transits per single poloidal transit along a field line on toroidal flux surface. In this research, we focus on magnetic island around $q = \frac{m}{n} = \frac{2}{1} = 2$ rational surface. R is the major radius from the center of toroidal axis, and r is the minor radius from the center of poloidal axis. Where, the R_0 is the major radius from the center of toroidal axis to the center of poloidal axis. Each unit vector $\hat{e}_\phi$, and $\hat{e}_\theta$ are the toroidal angle unit vector, and poloidal unit vector respectively. Perturbed field of magnetic island is considered in a potential form as below:

$$\boldsymbol{B}_I = \nabla \times \mathbf{A}\,, \tag{3}$$

where

$$\boldsymbol{A} = A_\parallel \hat{\boldsymbol{b}}\,, \tag{4}$$

$$A_\parallel = \tilde{A}_\parallel \exp[i(m\theta - n\phi)]\,, \tag{5}$$

$A_\parallel$ is the external perturbation designed to be resonant on the $q = 2$ rational surface so that magnetic island can be constructed.

By sum of $\boldsymbol{B}_0$, and $\boldsymbol{B}_I$,

$$\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{B}_I\,, \tag{6}$$

total magnetic field representing both equilibrium and magnetic islands is provided for the equation of motion solver.

2.2 Boris Method

Boris method is adopted as the particle pusher, which has been being widely used for full orbit calculation of plasma particle simulations. The method suggests that the equation of motion can be divided into two parts - the first part takes account of electric fields without magnetic fields, and the second part considers rotation in a plane perpendicular to the magnetic field by the Lorentz force as $\boldsymbol{F} = q(\boldsymbol{v} \times \boldsymbol{B})$, where q is the charge of the particle, $\boldsymbol{v}$ and $\boldsymbol{B}$ are vectors of velocity of the particle and the magnetic field, respectively. Following is the equation of motion,

$$\frac{\boldsymbol{v}^{n+1/2}-\boldsymbol{v}^{n-1/2}}{\Delta t}=\frac{q}{m}\left[\boldsymbol{E}+\frac{\boldsymbol{v}^{n+1/2}+\boldsymbol{v}^{n-1/2}}{2}\times\boldsymbol{B}\right], \quad (7)$$

where $\boldsymbol{v}^{n-1/2}$ and $\boldsymbol{v}^{n+1/2}$ are velocities at intermediate time steps. By introducing $\boldsymbol{v}^{-}$ and $\boldsymbol{v}^{+}$,

$$\boldsymbol{v}^{n-1/2}=\boldsymbol{v}^{-}-\frac{q\boldsymbol{E}}{m}\frac{\Delta t}{2} \text{ and} \quad (8)$$

$$\boldsymbol{v}^{n+1/2}=\boldsymbol{v}^{+}+\frac{q\boldsymbol{E}}{m}\frac{\Delta t}{2}, \quad (9)$$

the eq. (7) can be rewritten as

$$\frac{\boldsymbol{v}^{+}-\boldsymbol{v}^{-}}{\Delta t}=\frac{q}{2m}(\boldsymbol{v}^{+}+\boldsymbol{v}^{-})\times\boldsymbol{B}. \quad (10)$$

A well-known sequence to solve eq. (10) is as follows. First, we introduce an angle θ between $\boldsymbol{v}^{+}$ and $\boldsymbol{v}^{-}$. Then, we define $\boldsymbol{t}$, ratio of numerical time step to gyro frequency in terms of magnetic fields, for convenient calculation as

$$\boldsymbol{t}\equiv-\hat{b}\tan\frac{\theta}{2}=\frac{q\boldsymbol{B}}{m}\frac{\Delta t}{2}. \quad (11)$$

Subsequently, $\boldsymbol{v}^{-}$ is incremented to calculate the $\boldsymbol{v}'$ which is perpendicular to the vectors $(\boldsymbol{v}^{+}-\boldsymbol{v}^{-})$ and $\boldsymbol{B}$,

$$\boldsymbol{v}'=\boldsymbol{v}^{-}+\boldsymbol{v}^{-}\times\boldsymbol{t} \quad (12)$$

since the eq. (11) was rotated with $\theta/2$ by the definition of eq. (10). Then, the rest of $\theta/2$ is rotated as:

$$\boldsymbol{v}^{+}=\boldsymbol{v}^{-}+\boldsymbol{v}'\times\boldsymbol{s}, \quad (13)$$

where $\boldsymbol{s}$ is another vector which is a scaled version of $\boldsymbol{t}$ to meet $|\boldsymbol{v}^{+}|=|\boldsymbol{v}^{-}|$ as

$$\boldsymbol{s}=\frac{2\boldsymbol{t}}{1+t^{2}}. \quad (14)$$

As the last process for the particle push, the parallel acceleration is conducted by eq. (9).

2.3 Result

PAMI has been developed using PYTHON with multiple modules such as NumPy and standard Math libraries for calculating trajectory of a particle and structure of the Magnetic Island. In this work, we have investigated two categories of particle orbits, trapped and passing particles, in the presence or absence of magnetic islands. The different types of orbits are controlled by changes in pitch-angles defined as $v_{\perp}/v_0$ for the given same initial speed v_0 corresponding to 500eV kinetic energy. Following sections are presenting the results of the trapped particles in 2.3.1 section, and of the passing particles in 2.3.2, respectively.

2.3.1 Trapped particles

Banana shape orbit of the trapped particle in Tokamaks is originated from gradient of magnetic field strength induced by toroidicity. Our simulations with trapped particles show that location of turning point tip slightly changes for particles departing at inner/outer side of magnetic island. The path with longer time step will be tested in future work.

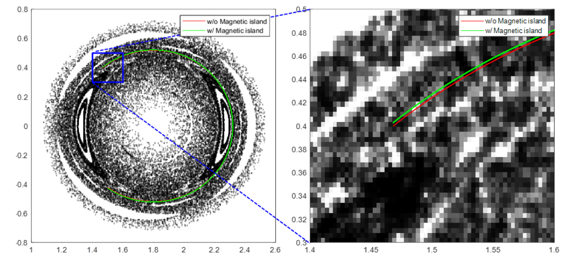

Fig. 1. Magnetic Field with/without Magnetic Island,
Initial position: outer side of magnetic island
Pitch-angle: 0.8, v_0:500eV

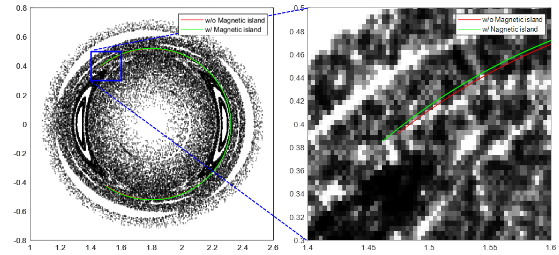

Fig. 2. Magnetic Field with/without Magnetic Island,
Initial position: inner side of magnetic island
Pitch-angle: 0.8, v_0:500eV

2.3.2 Passing particles

In case of the passing particle's trajectory, as shown in below, the Fig.3, and Fig.4, particle's orbits in the absence of Magnetic Island is orbiting on one same circular path of the (R, Z) poloidal plain without divergence as it should be. However, particle's trajectory in the presence of Magnetic Island shows decreasing radius in time. Further analysis of the phenomena will be addressed in future work.

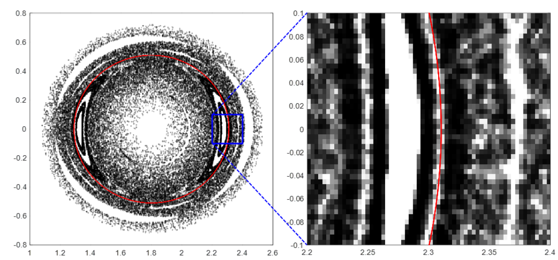

Fig. 3. Magnetic Field without Magnetic Island,
Pitch-angle: 0.7, v_0:500eV

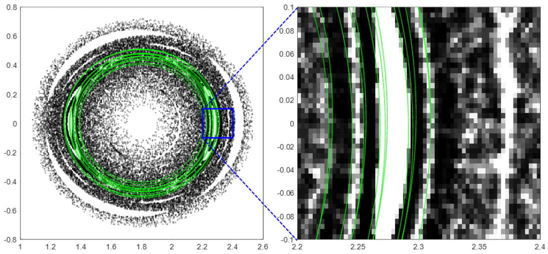

Fig. 4. Magnetic Field with Magnetic Island, Pitch-angle: 0.7, v_0:500eV

3. Conclusions

Particle Around Magnetic Island, PAMI, has been developed to investigate orbit changes due to presence of magnetic islands in Tokamaks. As expected, trajectories of both trapped and passing particles have shown different paths affected by the magnetic islands. However, we have observed nontrivial behaviors of orbits for both types of particle trajectories. Further investigation on physics underneath the results and its impact on plasma transport will be conducted with the PAMI code as a future work.

Acknowledgement

This work was supported by the Korea Institute of Energy Technology Evaluation and Planning and the Ministry of Trade, Industry & Energy (MOTIE) of the Republic of Korea (grant no. 20214000000410)

REFERENCES

[1] H. Zohm, Magnetohydrodynamic Stability of Tokamaks, Wiley, 2015.
[2] J. Wesson, Tokamaks, Clarendon Press-Oxford, p. 352-361, 1987.
[3] J. P. Boris, Relativistic Plasma Simulation-Optimization of a Hybrid Code, In: Proceeding of Fourth Conference of Numerical Simulations of Plasma, p.21-30, 1970.