Transactions of the Korean Nuclear Society Spring Meeting
Jeju, Korea, May 19-20, 2022

Estimation of Linac Alignment Error using Singular Value Decomposition of the Response Matrix

Seunghyun Lee*, Hyeok-Jung Kwon, Han-Sung Kim, Jeong-Jeung Dang, Sang-Pil Yoon, Dong-Hwan Kim
Korea Multipurpose Accelerator Complex, Korea Atomic Energy Research Institute, Gyeongju 38180
**Corresponding author: shl@kaeri.re.kr*

1. Introduction

The performance of the linac greatly depends on the linac alignment. Using information on beam center at various locations in the linac, we can evaluate the quality of the linac alignment. For this, beam dynamics simulations are performed to establish the method to estimate the alignment error of the Drift Tube Linac (DTL). At Korea Multipurpose Accelerator Complex (KOMAC), we have a 100 MeV linac consisting of 11 DTLs with several beam position monitors (BPMs) as shown in Fig. 1 a) [1, 2]. Here, we intend to estimate DTL alignment errors using the technique of singular value decomposition (SVD) of the response matrix obtained from the beam dynamics simulations. The response matrix relates beam positions at the BPM locations due to the x displacement errors given to each DTL tanks, assuming that the alignment inside the DTL tank is well set. We studied and compared the actual and estimated displacement/alignment errors of DTLs with nearby BPMs.

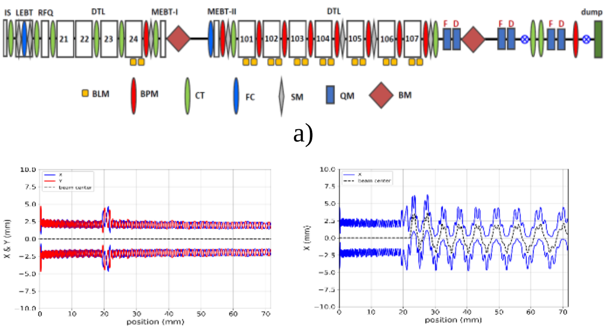


Fig. 1. Schematics of KOMAC linac layout and beam diagnostics in a), beam envelope simulation result without any DTL tank alignment error in b), and beam envelop simulation result with x displacement = 1 mm of the DTL101 tank in c).

2. Methods and Results

In this section, we will concentrate on estimation of the DTL alignment based on SVD using simulation results of the DTL alignment errors.

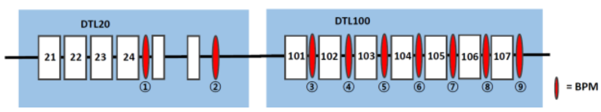


Fig. 2. Schematics of BPMs and DTL20 in a) and DTL100 in b).

The entire linac is divided into two sections, DTL20 and DTL100 as shown in Fig. 2 a) and b). In the DTL20, there are four DTL tanks and two BPMs. In the DTL100, there are seven DTL tanks and seven BPMs. The response matrix relating the beam position at the BPMs and x displacement of the DTL tank can be transformed into a diagonal matrix. The mechanism of this transformation is SVD [3, 4].

2.1 Singular Value Decomposition

We construct a response matrix, R_{ij} relating M BPMs and N DTL tanks used for the estimation of DTL alignment,

$$R_{ij} = \frac{\Delta x_i}{\Delta X_j}$$

where x_i is beam center at i BPM and ΔX_j is x displacement of j DTL. This means

$$\mathbf{R} \cdot \Delta\mathbf{X} = \Delta\mathbf{x}$$

R can be written as a product of three matrices, **U**, **Σ** and **V** as

$$\mathbf{R} = \mathbf{U} \cdot \boldsymbol{\Sigma} \cdot \mathbf{V^T}$$

where **U** and **V** are M × M and N × N unitary matrix respectively. **Σ** is an M × N diagonal matrix with diagonal elements *s*. This decomposition is unique only to a certain extent, not in every case.

To estimate the DTL x alignment error (meaning the x displacement),

$$\Delta\mathbf{X} = \boldsymbol{R^{-1}} \cdot \boldsymbol{\Delta x}$$

where $\boldsymbol{R^{-1}}$, an inverse **R** matrix,

$$\boldsymbol{R^{-1}} = \boldsymbol{V} \cdot \boldsymbol{\Sigma^{-1}} \cdot \boldsymbol{U^T}$$

and $\boldsymbol{\Sigma^{-1}}$ = all non-zero diagonal terms 1/*s*.

2.2 Simulation of Beam Shifts due to the DTL Alignment Errors

With the given *x* = 1 mm displacement of one of the DTL tanks, we simulated the beam envelop through the linac and got beam center at all the BPM locations using the TraceWin code [5] as shown in Fig. 1 b) and c). From this, we constructed the response matrices of two cases shown in Fig. 2 a) and b). The obtained response matrices, R_{DTL20} for the case a) and R_{DTL100} for the case b) are shown below, DTL tank →, BPM ↓

$$\boldsymbol{R}_{DTL20} = \begin{bmatrix} 2.9364 & -1.2022 & -0.8702 & 1.4736 \\ -0.6096 & 0.7516 & 0.3811 & -0.7323 \end{bmatrix}$$

DTL tank →, BPM ↓

$$\boldsymbol{R}_{DTL100} = \begin{bmatrix} 0.1709 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.6459 & 2.0998 & 0 & 0 & 0 & 0 & 0 \\ 1.8218 & -1.5700 & 3.0658 & 0 & 0 & 0 & 0 \\ 1.4443 & -3.0672 & 4.3256 & -1.3718 & 0 & 0 & 0 \\ 0.1528 & -1.5395 & 1.7209 & 0.2988 & -0.6023 & 0 & 0 \\ 0.0327 & -2.0125 & 2.1183 & 0.6788 & -1.1513 & -0.2646 & 0 \\ 0.4408 & -2.4986 & 2.9449 & 0.1521 & -0.5574 & -0.6992 & 0.0269 \end{bmatrix}.$$

We used the inverse of the above response matrices for the estimation of the DTL tank alignment errors.

2.3 Estimation Test of the DTL100 Alignment Errors

We first consider the case b), i.e. the DTL100 section of the linac as it has seven BPMs to give better estimation of the x displacement error. Estimation test is performed using a pre-set x displacement error such that x displacements are 0.5 mm and -0.2 mm for DTL 103 and DTL106 respectively. Beam shifts at BPMs are computed for the setting. The x displacements of the DTL tanks are back calculated and compared with the pre-set displacement errors as shown below.

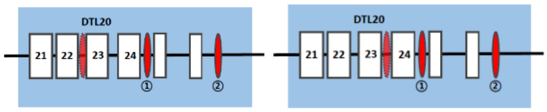
BPM readings ⇒ DTL Displacement = $R^{-1} \cdot$ BPM readings

$$\text{BPM readings} = \begin{bmatrix} 0.0000 \\ 0.0000 \\ 1.5329 \\ 2.1628 \\ 0.8604 \\ 1.1121 \\ 1.6123 \end{bmatrix} \Rightarrow \text{DTL Displacement} = \begin{bmatrix} 0.0000 \\ 0.0000 \\ 0.5000 \\ 0.0000 \\ 0.0000 \\ -0.2000 \\ 0.0000 \end{bmatrix} \begin{matrix} \leftarrow \text{DTL101} \\ \leftarrow \text{DTL102} \\ \leftarrow \text{DTL103} \\ \leftarrow \text{DTL104} \\ \leftarrow \text{DTL105} \\ \leftarrow \text{DTL106} \\ \leftarrow \text{DTL107} \end{matrix}$$

Since the BPMs are installed after every DTL tank, displacements of the DTL tanks are well estimated

2.4 Estimation Test of the DTL20 Alignment Errors

In case of the DTL20, there are only two BPMs installed after the DTL24 tank. This means the estimation of exact displacement is difficult but one might possibly guess the main cause of the alignment/displacement error. We evaluated the estimation in the given situation by back-calculating the x displacements of the DTL20 tanks when one of DTL20 tanks is shifted by 1 mm at the time, not by any combination. Here, we found that only the DTL21 tank shift can be estimated reasonably and it does not work properly for other DTL22, 23 and 24. We considered several possible locations for an extra BPM with very limited space and chose two cases to study carefully. Two cases for an extra BPM are shown in Fig. 3 a) and b). In this DTL20 section, there are no steering magnet (SM) to correct the orbit as shown in Fig. 1 a) so the estimation of the alignment error is very important.



a) b)

Fig. 3. Schematics of DTL20 with an extra BPM after the DTL22 tank in a) and after the DTL23 in b).

The response matrices of above cases a) and b) are

DTL tank →, BPM ↓

$$\boldsymbol{R}_a = \begin{bmatrix} 3.8066 & -0.9094 & 0.0000 & 0.0000 \\ 2.9364 & -1.2022 & -0.8702 & 1.4736 \\ -0.6096 & 0.7516 & 0.3811 & -0.7323 \end{bmatrix}$$

and

DTL tank →, BPM ↓

$$\boldsymbol{R}_b = \begin{bmatrix} -3.0065 & 1.6796 & 1.0701 & 0.0000 \\ 2.9364 & -1.2022 & -0.8702 & 1.4736 \\ -0.6096 & 0.7516 & 0.3811 & -0.7323 \end{bmatrix}.$$

Using the above response matrices, we again calculated the x displacements of the DTL20 tanks when one of DTL20 tanks was shifted by 1 mm at the time. In the case a), x displacement in DTL21 and DTL22 in a) are well estimated, but that of DTL23 and DTL24 are not. For the DTL23, from the calculated DTL displacement, one cannot conclude which DTL tank causes the main alignment error at all. For the DTL24, at least one can say that the DTL24 causes the main alignment error. Similarly for the case b), we studied the x displacement of the DTL20 for each 1 mm shift of the DTL20 tanks. In this case b), x displacement in DTL21 and DTL24 are well estimated, and for DTL22 and DTL23, one can distinguish the DTL tank having caused the biggest alignment error. Having an opportunity to place an extra BPM in the DTL20 section, it would be better between the DTL23 and DTL24.

3. Conclusions

Beam dynamics Simulations are performed to estimate DTL alignment errors. The technique of the SVD is applied to the response matrices obtained from simulation results. The entire KOMAC linac is divided into two sections, DTL20 and DTL100 to study and compared the actual and estimated displacement errors of DTLs with BPMs. In case of the DTL100, displacements of the DTL tanks are well estimated due to seven BPMs installed for every DTL100 tank. On the other hand, the exact displacement of the DTL20 tanks is difficult since there are only two BPMs in the DTL20 section. So for the DTL20 case, we studied and showed the possible improvement of the alignment estimation by installing one extra BPM between the DTL23 and DTL24.

REFERENCES

[1] K. Choe, Y.-G. Song, H.-S. Jeong, H.-S. Kim, and H.-J. Kwon, Preliminary Test of the KOMAC Linac BPM at the Test Bench for Upgrade, Transaction of the Korean Nuclear Society Spring Meeting 2021.

[2] H.-J. Kwon, H.-S. Kim, K.-T. Seol, J.-Y. Ryu, J.-H. Jang, and Y.-S. Cho, Design and Fabrication of the Beam Position Monitor for the PEFP Linac, Nuclear Engineering and Technology, Vol. 45, No. 4, p. 523, 2013

[3]Y. Chung, G. Decker, and K. Evans, Jr., Proceedings of International Conference on Particle Accelerators, Vol. 3, p. 2263, 1993.

[4] A. Latina, J. Pfingstner, D. Schulte, E. Adli, F. J. Decker, and N. Lipkowitz, Vo. 17, p. 042803, 2014.

[5] http://irfu.cea.fr/Sacm/logiciels/index3.php