Investigation of Revised Sampling Method for on-site National Safeguards Inspection

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Introduction

- Notification (NSSC No. 2017-83) requires “on-site verification” for national safeguards inspection
- The KINAC has been developing an independent inspection supporting program, which includes sampling planning
- This research identified the limitations of IAEA sampling planning and investigated the revised sampling method for national inspection

Research Overview

- **Investigation** of the **optimized sampling method** which **satisfies** both **verification criteria** and **minimized inspection burden**

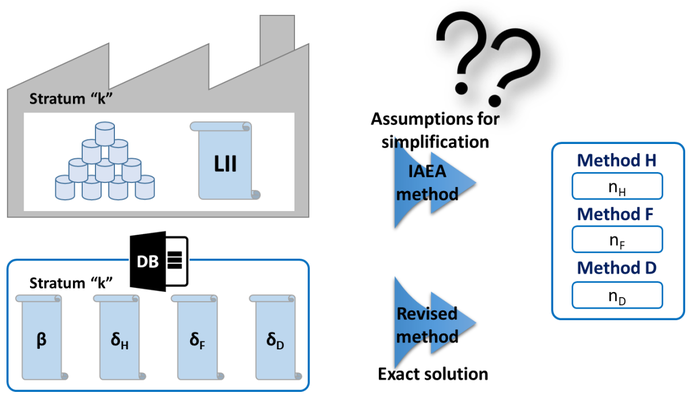


Fig. 1. Overview of the research

IAEA’s Sampling Method

- **Calculate initial sample sizes** for each stratum & verification method **using polynomial adjusted hypergeometric distribution**

$$n_H = roundup\left(N\left(1-\beta^{(1/D)}\right)\right) \qquad \beta = \frac{{}_{N-D}C_{n_H} \times {}_DC_0}{{}_NC_{n_H}} \approx (1-\frac{n_H}{N})^D$$

$$n_{F,D} = \ln(\beta)/\ln\left(1-M/\gamma'_{F,D}Nx\right) \qquad \beta = \sum_{k=0}^{n_{F,D}} \frac{{}_{m(\gamma_{F,D})}C_{k_{F,D}} \times {}_{(N-m(\gamma_{F,D}))}C_{(n_{F,D}-k_{F,D})}}{{}_NC_{n_{F,D}}}(1-p_{F,D})^{k_{F,D}} \approx (1-\frac{m(\gamma_{F,D})p_{F,D}}{N})^{n_{F,D}}$$

- **Optimize** sample size **by iteration** to converge **Qmax to be non-detection probability (β)** of a stratum

$$\mathbf{Q} = \boldsymbol{\beta_H\beta_F\beta_D}$$

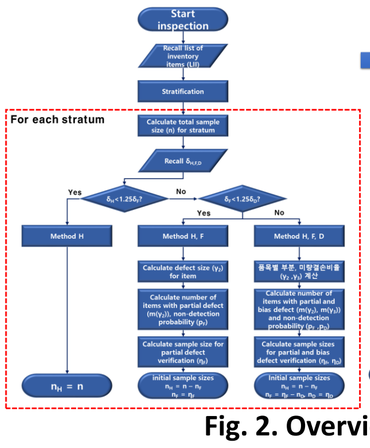
$$\beta_{H/F/D} = \begin{cases} \left[1-\dfrac{\frac{m_H}{\frac{F}{D}}q_{\frac{F}{D}}}{N_{\frac{F}{D}}-0.5\left(n_{\frac{F}{D}}-1\right)}\right]^{n_{\frac{F}{D}}} & (n \le mq) \\ \left[1-\dfrac{n_{\frac{F}{D}}}{N_{\frac{F}{D}}-0.5\left(m_{\frac{F}{D}}q_H-1\right)}\right]^{m_{\frac{F}{D}}q_{\frac{F}{D}}} & (n > mq) \end{cases}$$

Case 1. ($m_{00} \le N$)
1-a) Qmax occurs at $m = M/x$ or $m = m_{00}$
1-b) Qmax occurs at $M/x < m < m_{00}$, $Q_{max} \le \beta$
1-c) Qmax occurs at $M/x < m < m_{00}$, $Q_{max} > \beta$

Case 2. ($m_{00} > N$)
2-a) $Q_{max} > \beta$ $\quad c = \text{roundup}\left(n_F \dfrac{\ln(Q_{max})-\ln(\beta)}{\ln(\beta_F)}\right)$
2-b) $Q_{max} \le \beta$

For 1-c), 2-a) and 2-b)
$n'_{H/F} = n_{H/F} + c$
$n'_{F/D} = n_{F/D} - c$ **iterate until c = 0**

Calculate Initial Sample Sizes



Calculate Optimized Sample Sizes

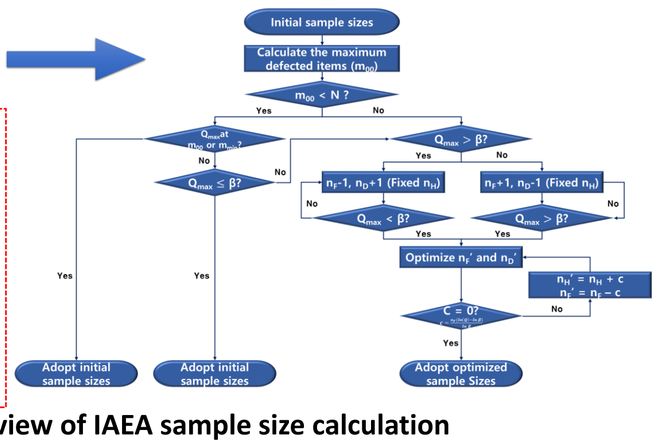


Fig. 2. Overview of IAEA sample size calculation

Revised Sampling Method

- **Calculate total sample size** for each stratum by calculating the exact solution of **hypergeometric distribution**
- **Calculate the initial sample sizes** for partial and bias defect verification **by calculating the exact solution of hypergeometric distribution using iteration**

$$\beta = \frac{{}_{(N\ -\ D)}C_n \times {}_DC_0}{{}_NC_n}$$

$$\beta_{F,D} = \sum_{k=0}^{n} \frac{{}_{(N\ -\ m(\gamma))}C_{(n\ -\ k)} \times {}_{m(\gamma)}C_k}{{}_NC_n}(1-p)^k$$

For $D \le m(\gamma) \le m_{00}$, Calculate n at $\max(\beta_{F,D})$, which satisfies $\beta_{F,D} < \beta$

$\beta = 0.8$

β_F	n_F
1	2
0.7956	33
0.7988	22
0.7947	17
0.7909	14
0.7998	12
0.7962	13

Increasing $m(\gamma)$

Fig. 3. Iteration for revised initial sampling

- **Optimize the initial sample sizes using the IAEA’s method without polynomial adjustment**

Results

- **Develop** a **stratification program** for a FFP
- **Calculate sample sizes** for each stratum and verification methods **using the IAEA and revised sampling methods**

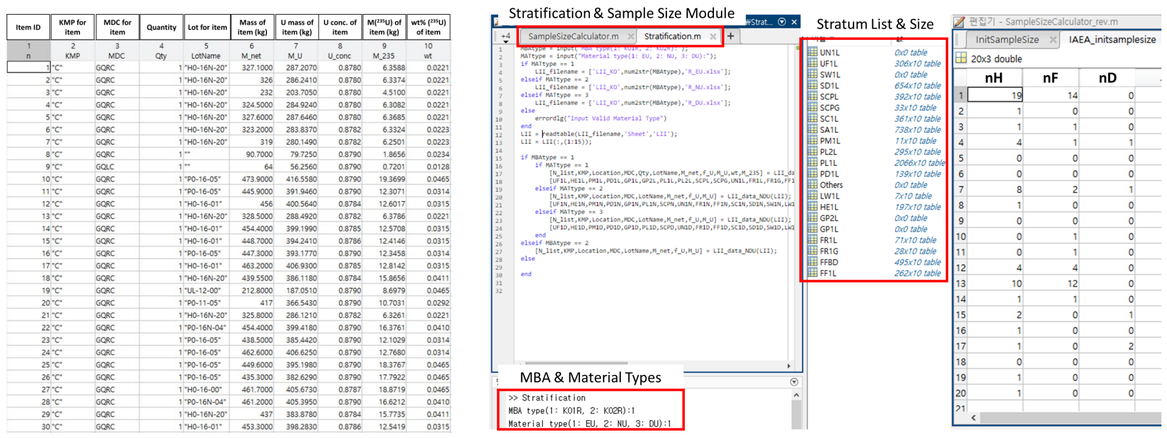


Fig. 4. List of inventory items (left) and stratification & sample size calculator (right)

- **Reduced** total **sample sizes** or sample sizes with sensitive verification methods **using revised sampling method**
  - **Reduced inspection burden for inspectors and facilities while maintaining the non-detection probability**

Stratum	IAEA’s method			Revised method		
	n_H	n_F	n_D	n_H	n_F	n_D
UF1L	19	14	0	21	12	0
HE1L	1	0	0	1	0	0
PM1L	1	1	0	1	1	0
PD1L	4	1	1	3	1	1
GP1L	0	0	0	0	0	0
GP2L	0	0	0	0	0	0
PL1L	8	2	1	9	1	1
PL2L	1	0	0	0	0	1
UN1L	0	0	0	0	0	0
FR1L	0	1	0	0	1	0
FR1G	0	1	0	0	1	0
FF1L	4	4	0	4	4	0
FFBD	10	12	0	10	12	0
SC1L	1	1	0	1	0	1
SCPL	2	0	1	2	0	1
SCPG	1	0	0	1	0	0
SD1L	1	0	2	2	0	1
SW1L	0	0	0	0	0	0
LW1L	1	0	0	1	0	0
SA1L	1	0	0	1	0	0

Fig. 5. Sample sizes (IAEA: blue, Rev: orange)

Conclusion

- The IAEA method overestimates sample size due to the polynomial adjustment of hypergeometric distribution
- The research investigated to calculate the exact solution for sampling, which reduces inspection burden
- Future work will include the optimization of the IAEA’s sample optimization process

References

[1] Subparagraph 2 of Article 4 of Regulations on the Safeguards Inspection of SNMs of the ROK, NSSC notification No. 2017-83, 2017.
[2] IAEA, Reference Manual: Statistical Concepts and Techniques for IAEA Safeguards, IAEA-SG-SCT-5, 1998.
[3] J. L. Jaech, Algorithms to calculate sample sizes for inspection sampling plans, IAEA-STR-261(Rev.0), 1990.