Evaluation of Correlation Between Engineering Demand Parameters for Accurate Seismic System Reliability Analysis

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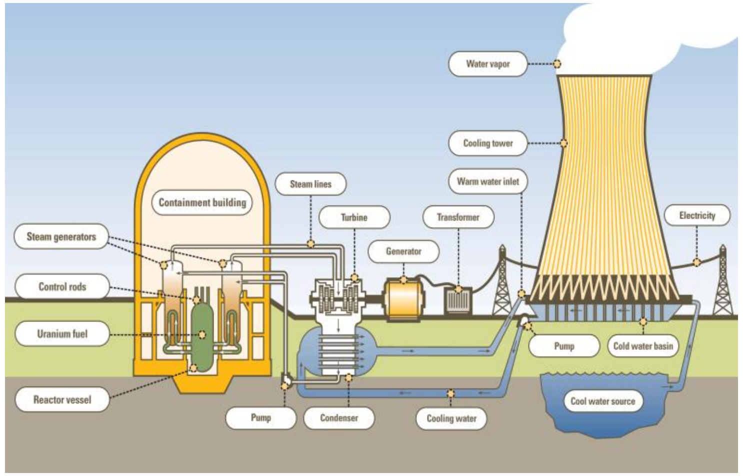
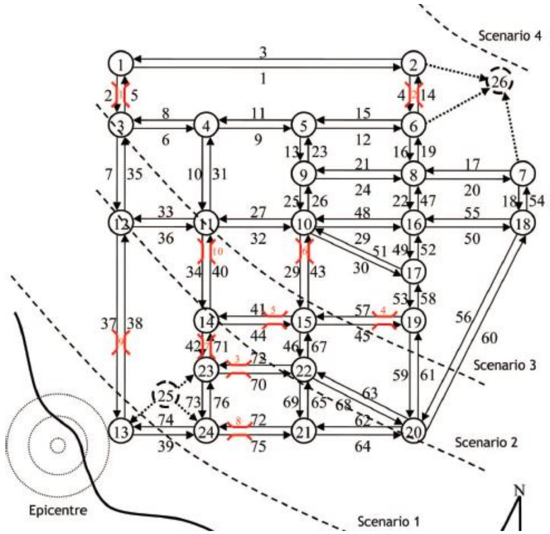
Seismic system reliability analysis

Seismic reliability analysis in complex systems

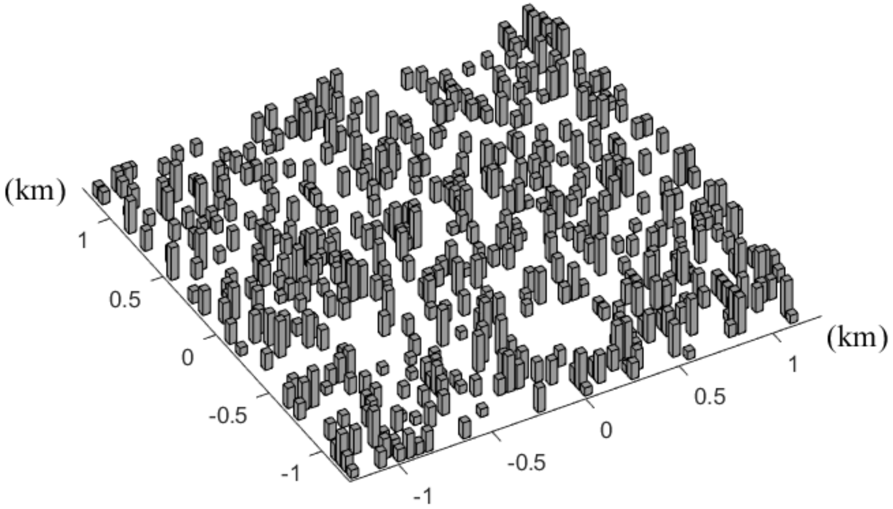
- Complex systems such as nuclear power plants (NPPs), lifeline networks, and building inventories are subject to various types of uncertainties.
- Due to these uncertainties, components in the complex system such as equipment of NPPs, network components, buildings in a region are dependent on each other, thus the seismic reliability analysis needs to be performed at system-level.



Nuclear power plant



Traffic network in Sioux Falls

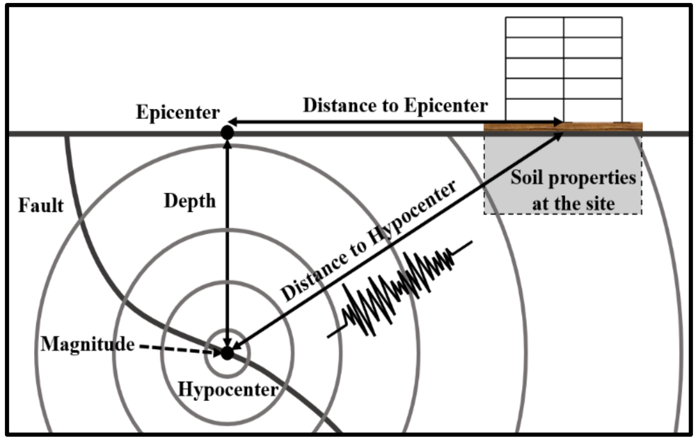


Spatially distributed buildings

Seismic system reliability analysis

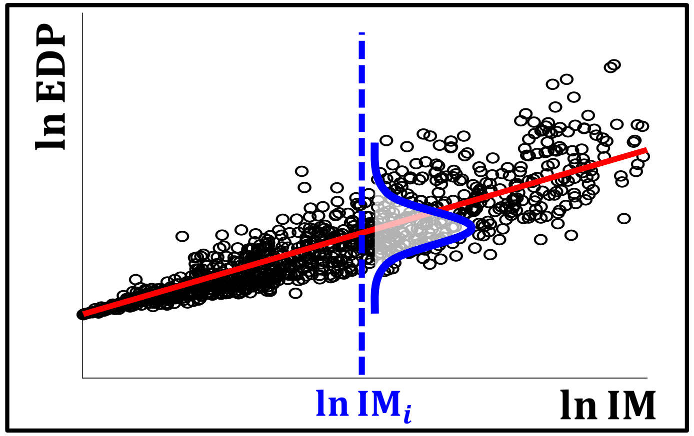
Uncertainties in seismic system reliability analysis

- Uncertainties of intensity measures (IMs)

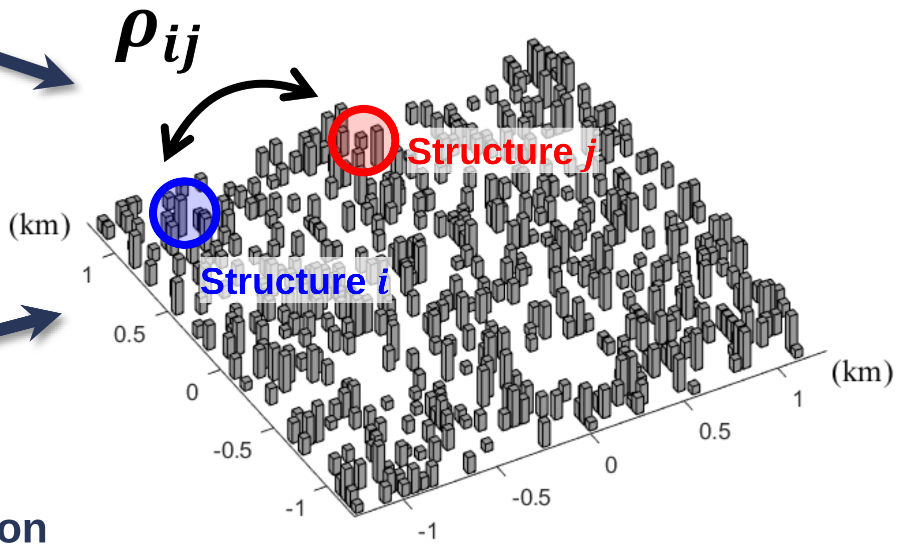


Effects of magnitude, distance, and site characteristics

- Uncertainties of residuals of engineering demand parameters (EDP residuals)



Effects of ground motion characteristics



Spatially distributed buildings

Theoretical Framework

Proposed framework

- The mean EDP of a structure is predicted by a regression function of the selected intensity measure (IM), while its uncertainty can expressed by the residual term, "EDP residual".
  - $\mathrm{EDP}_i|\mathrm{IM}_i = S_i(\mathrm{IM}_i)\Psi_i(\mathrm{IM}_i)$
  - $\ln \mathrm{EDP}_i \,|\, \ln \mathrm{IM}_i = s_i(\ln \mathrm{IM}_i) + \psi_i(\ln \mathrm{IM}_i)$

    Mean | Variability — EDP residual
  - $\widehat{D_i} = s_i(\widehat{\mathrm{IM}}_i) + \psi_i(\widehat{\mathrm{IM}}_i)$
- By using the power-law, $D_i = a \cdot \mathrm{IM}^b$, the relationship can be defined as $s_i(\widehat{\mathrm{IM}}_i) = \ln a_i + b_i\widehat{\mathrm{IM}}_i$.
  - $\widehat{D_i} = \ln a_i + b_i\widehat{\mathrm{IM}}_i + \psi_i(\widehat{\mathrm{IM}}_i)$

Seismic fragility

- Fragility is defined as the conditional probability that the selected EDP ($\widehat{D_i}$) exceeds a specified limit state ($\widehat{d_i}$) given a value of IM.
- Assuming that D_i follows a Lognormal distribution, the safety factor F_i follows a Gaussian distribution.
  - Safety factor $F_i = \widehat{d_i} - \widehat{D_i}$

    $= \ln d_i - \ln a_i - b_i\widehat{\mathrm{IM}}_i - \psi_i(\widehat{\mathrm{IM}}_i) < 0$ **(Failure)**

Theoretical framework

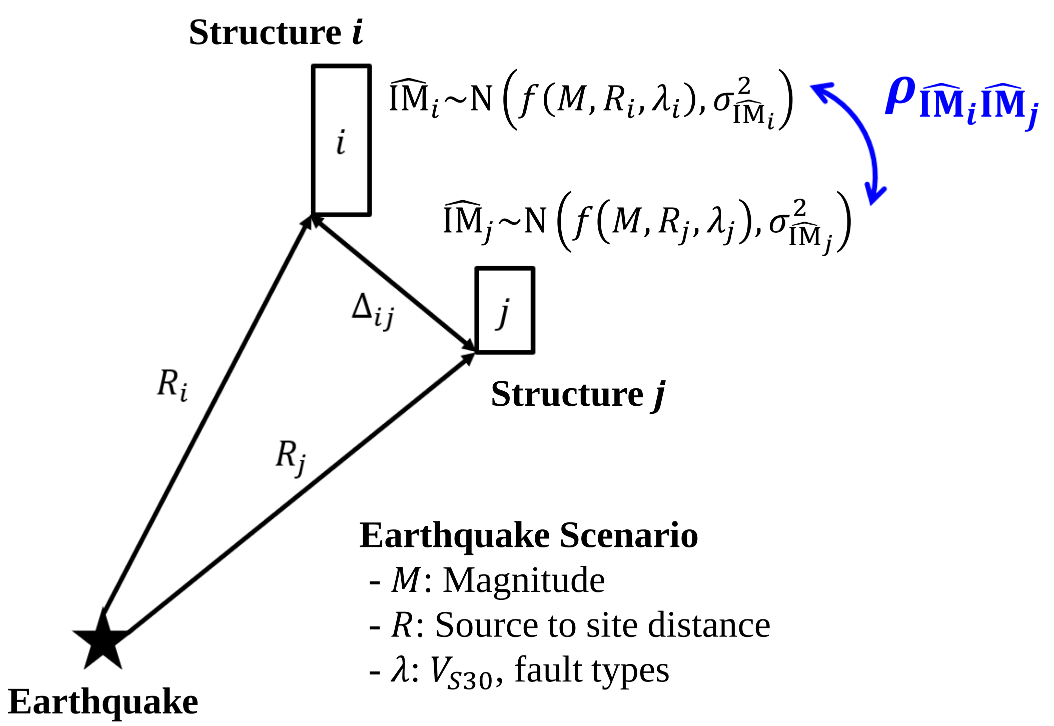
Correlation between EDPs

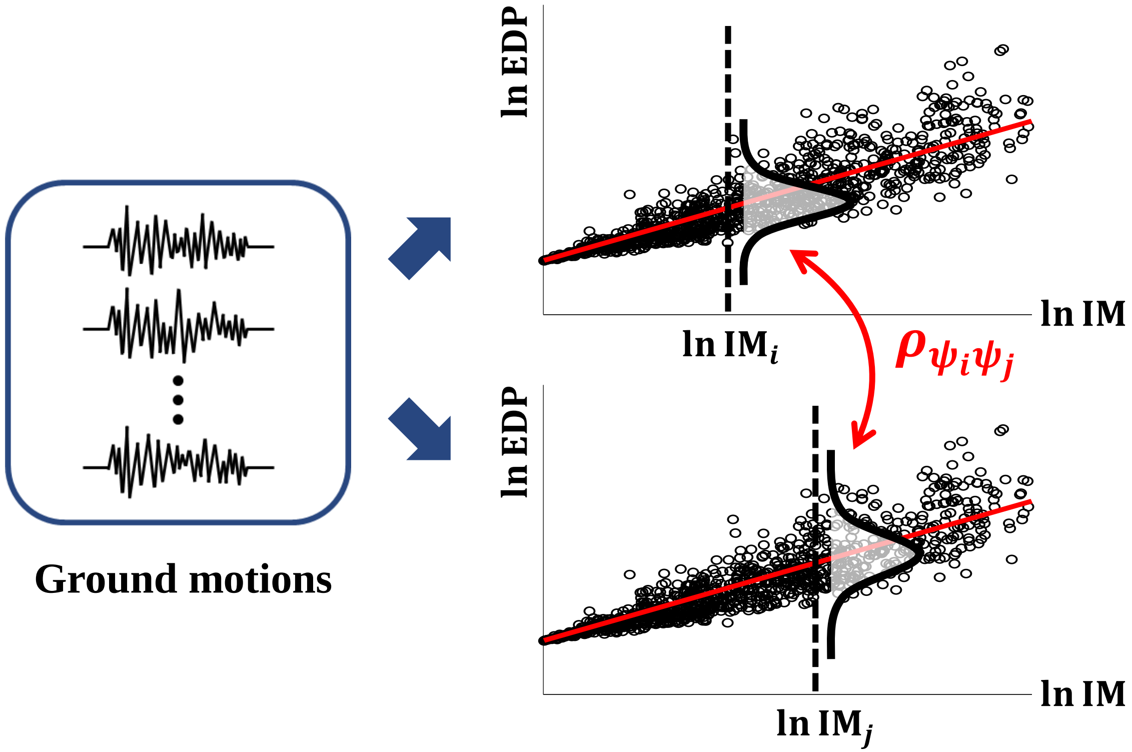
- Safety factor correlation
  - $F_i = \ln d_i - \ln a_i - b_i \widehat{\mathrm{IM}}_i - \psi_i(\widehat{\mathrm{IM}}_i)$

$\rho_{\widehat{\mathrm{IM}}_i \widehat{\mathrm{IM}}_j}$: IM correlation

$\rho_{\psi_i \psi_j}$: EDP residual correlation

  - $F_j = \ln d_j - \ln a_j - b_i \widehat{\mathrm{IM}}_j - \psi_i(\widehat{\mathrm{IM}}_j)$



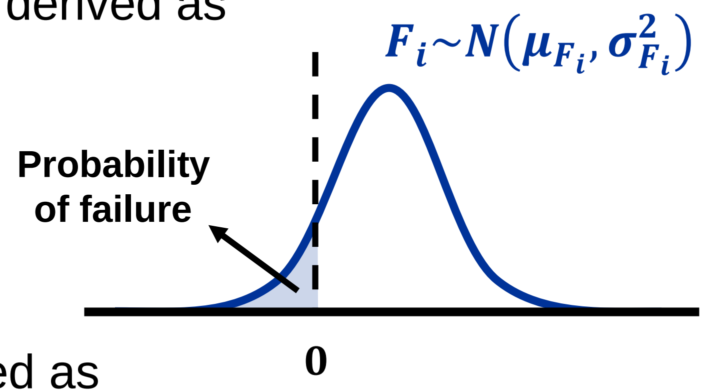


Theoretical framework

Failure probability of structure *i*

- The fragility, the conditional failure probability <u>given IM value</u>, $\widehat{\mathrm{IM}}_i = x$, is derived as
  - $P(F_i \leq 0 | \widehat{\mathrm{IM}}_i = x) = P\left(\psi_i(x) \geq \hat{d}_i - s_i(x)\right)$

$$= 1 - \Phi\left(\frac{\hat{d}_i - s_i(x)}{\boldsymbol{\sigma_{\psi_i(x)}}}\right)$$



- The failure probability for a <u>given earthquake scenario</u> can be represented as
  - $p_{f_i} = P(F_i \leq 0) = \int_{-\infty}^{\infty} P(F_i \leq 0 | \widehat{\mathrm{IM}}_i = x) \boldsymbol{f_{\widehat{\mathrm{IM}}_i}}(x) dx$

- The joint failure probability of structure i and j
  - $p_{f_{ij}} = P(F_i \leq 0 \cap F_j \leq 0) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P\left(F_i \leq 0 \cap F_j \leq 0 | \widehat{\mathrm{IM}}_i = x_i, \widehat{S_a}_j = x_j\right) f_{\widehat{\mathrm{IM}}_i\widehat{\mathrm{IM}}_j}(x_i, x_j) dx_i dx_j$

$$= \boldsymbol{p_{f_i}} \cdot \boldsymbol{p_{f_j}} + \int_0^{\boldsymbol{\rho_{F_iF_j}}} \varphi_2(-\beta_i, -\beta_j, \rho) d\rho$$

By using single-fold integration, $p_{f_{ij}}$ can be calculated by $\boldsymbol{p_{f_i}}$, $\boldsymbol{p_{f_j}}$, and $\boldsymbol{\rho_{F_iF_j}}$.

Theoretical framework

Equation for incorporating both IM and EDP residual correlation

- Derived the correlation coefficient between safety factors
  - $$\rho_{F_iF_j} = \frac{b_i b_j \sigma_{\widehat{\mathrm{IM}}_i}\sigma_{\widehat{\mathrm{IM}}_j}}{\sqrt{b_i^2\sigma_{\widehat{\mathrm{IM}}_i}^2+\boldsymbol{\sigma_{\psi_i}^2}}\cdot\sqrt{b_i^2\sigma_{\widehat{\mathrm{IM}}_j}^2+\boldsymbol{\sigma_{\psi_j}^2}}}\rho_{\widehat{\mathrm{IM}}_i\widehat{\mathrm{IM}}_j} + \frac{\boldsymbol{\sigma_{\psi_i}\sigma_{\psi_j}}}{\sqrt{b_i^2\sigma_{\widehat{\mathrm{IM}}_i}^2+\boldsymbol{\sigma_{\psi_i}^2}}\cdot\sqrt{b_i^2\sigma_{\widehat{\mathrm{IM}}_j}^2+\boldsymbol{\sigma_{\psi_j}^2}}}\boldsymbol{\rho_{\psi_i\psi_j}}$$
    $$= A_S\rho_{\widehat{\mathrm{IM}}_i\widehat{\mathrm{IM}}_j} + A_\psi\rho_{\psi_i\psi_j}$$

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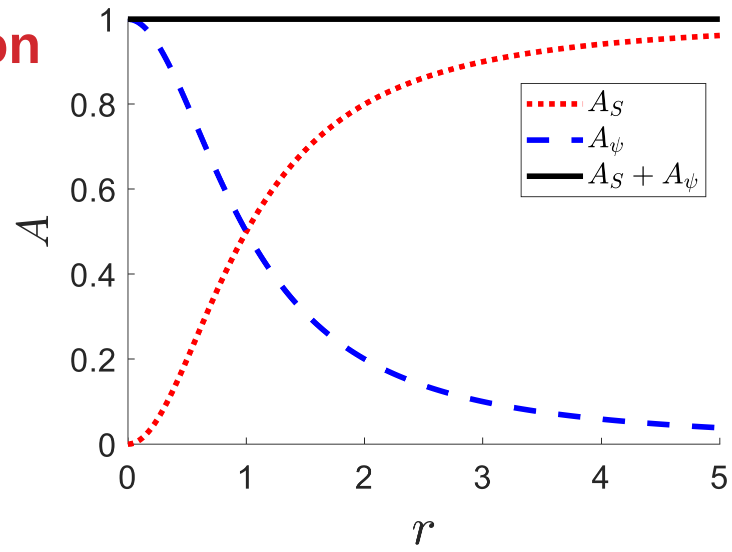
Theoretical framework

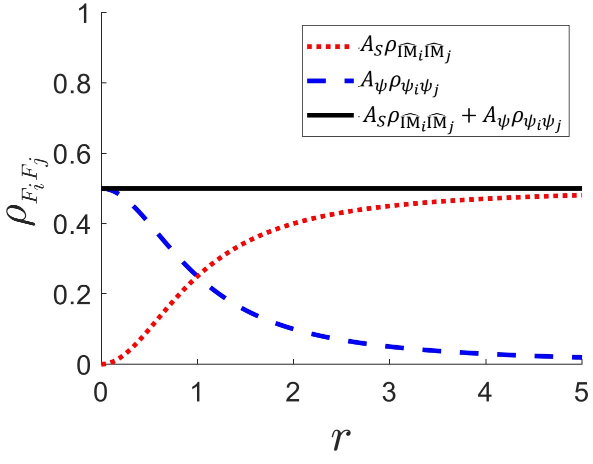
Equation for incorporating both IM and EDP residual correlation

❖ When $b_i = b_j = b$, $\sigma_{\widehat{\mathrm{IM}}_i} = \sigma_{\widehat{\mathrm{IM}}_j} = \sigma_{\widehat{\mathrm{IM}}}$, and $\sigma_{\psi_i} = \sigma_{\psi_j} = \sigma_\psi$

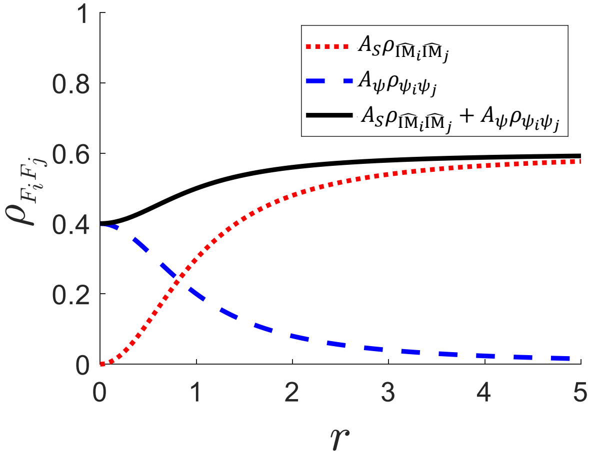
- $$\rho_{F_iF_j} = \frac{b_i b_j \sigma_{\widehat{\mathrm{IM}}_i}\sigma_{\widehat{\mathrm{IM}}_j}}{\sqrt{b_i^2\sigma_{\widehat{\mathrm{IM}}_i}^2+\boldsymbol{\sigma_{\psi_i}^2}}\cdot\sqrt{b_i^2\sigma_{\widehat{\mathrm{IM}}_j}^2+\boldsymbol{\sigma_{\psi_j}^2}}}\rho_{\widehat{\mathrm{IM}}_i\widehat{\mathrm{IM}}_j} + \frac{\boldsymbol{\sigma_{\psi_i}\sigma_{\psi_j}}}{\sqrt{b_i^2\sigma_{\widehat{\mathrm{IM}}_i}^2+\boldsymbol{\sigma_{\psi_i}^2}}\cdot\sqrt{b_i^2\sigma_{\widehat{\mathrm{IM}}_j}^2+\boldsymbol{\sigma_{\psi_j}^2}}}\boldsymbol{\rho_{\psi_i\psi_j}}$$
$$= A_S\rho_{\widehat{\mathrm{IM}}_i\widehat{\mathrm{IM}}_j} + A_\psi\rho_{\psi_i\psi_j}$$
$$= \frac{r^2}{r^2+1}\rho_{\widehat{\mathrm{IM}}_i\widehat{\mathrm{IM}}_j} + \frac{1}{r^2+1}\rho_{\psi_i\psi_j} \quad \left(\text{where } r = \frac{b\cdot\sigma_{\widehat{\mathrm{IM}}}}{\sigma_\psi}\right)$$



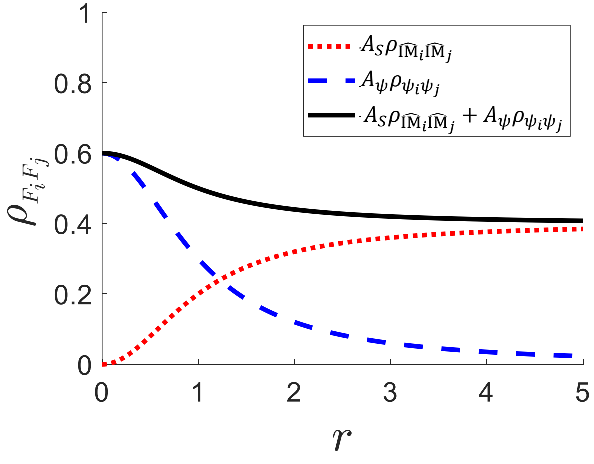
- $\rho_{\widehat{\mathrm{IM}}_i\widehat{\mathrm{IM}}_j} = \rho_{\psi_i\psi_j} = 0.5$



- $\rho_{\widehat{\mathrm{IM}}_i\widehat{\mathrm{IM}}_j} = 0.6$, $\rho_{\psi_i\psi_j} = 0.4$



- $\rho_{\widehat{\mathrm{IM}}_i\widehat{\mathrm{IM}}_j} = 0.4$, $\rho_{\psi_i\psi_j} = 0.6$



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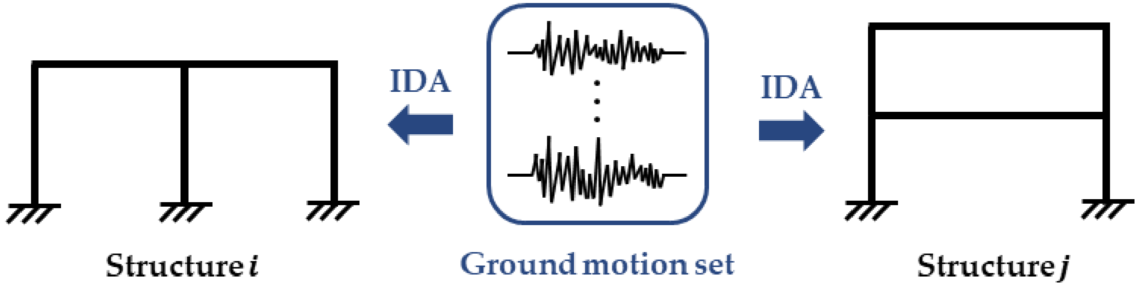
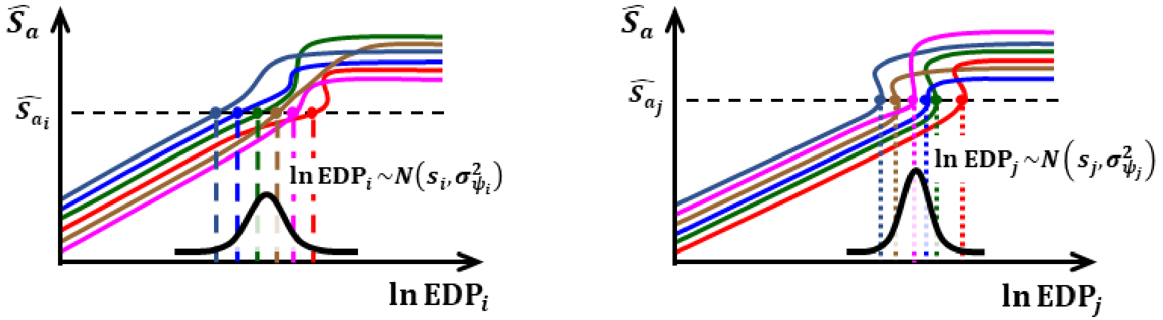
Method for estimating the EDP residual

Incremental Dynamic Analysis (IDA)-based method

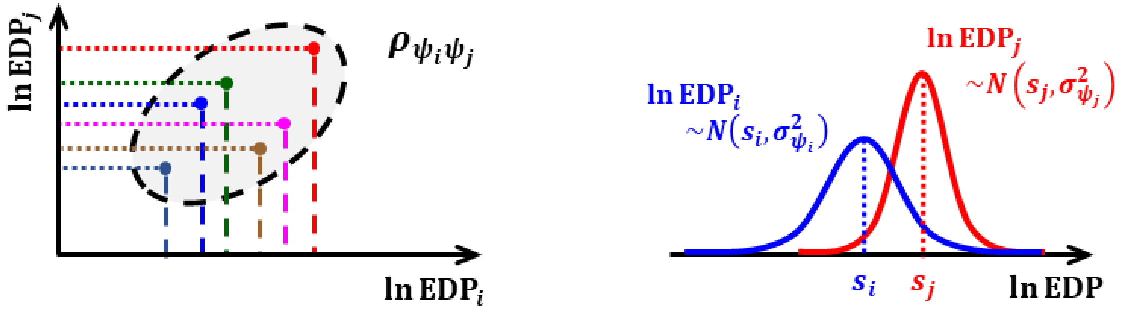
Step 1
Perform the IDAs on each structure to determine the relationship between EDP and IM.



Step 2
Plot the IDA curves and estimate the variance of the natural logarithm of EDP given IM for each structure.



Step 3
Estimate the correlation coefficient $\rho_{\psi_i \psi_j}$ between $\ln \text{EDP}_i$ and $\ln \text{EDP}_j$.



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Application to seismic system reliability analysis

Probabilistic regional loss estimation considering EDP correlation

- Loss of the building
  - $\mu_{L_i} = \sum_{k=1}^{m} l_i^{DS_k} p_{f_i}^{DS_k}$
  - $\sigma_{L_i}^2 = E[L_i^2] - \mu_{L_i}^2 = \sum_{k=1}^{m} \left(l_i^{DS_k}\right)^2 p_{f_i}^{DS_k} - \mu_{L_i}^2$
  - $\rho_{L_iL_j} = \frac{E[L_iL_j]-\mu_{L_i}\mu_{L_j}}{\sigma_{L_i}\sigma_{L_j}} = \frac{\sum_{k=1}^{m}\sum_{l=1}^{m} l_i^{DS_k} l_j^{DS_l} \boldsymbol{P\left(L_i=l_i^{DS_k} \cap L_j=l_j^{DS_l}\right)} - \mu_{L_i}\mu_{L_j}}{\sigma_{L_i}\sigma_{L_j}}$

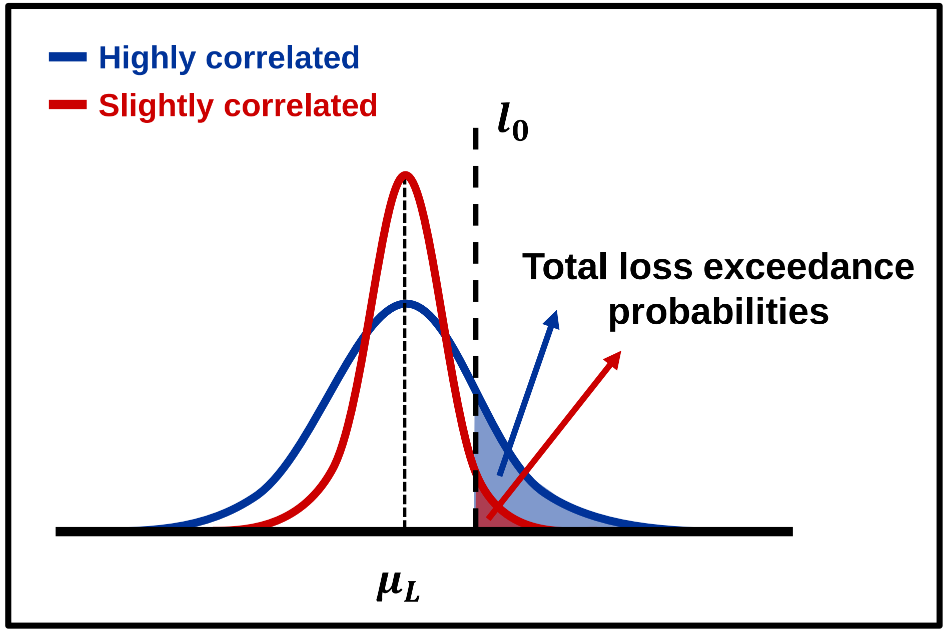
$$p_{f_{ij}} = \boldsymbol{p_{f_i}} \cdot \boldsymbol{p_{f_j}} + \int_0^{\rho_{F_iF_j}} \varphi_2(-\beta_i, -\beta_j, \rho)\, d\rho$$

Joint failure probability of structure *i* and *j*

Effect of safety factor correlation on total loss of a region

- Total loss of N buildings in a region
  - $\mu_L = \sum_{i=1}^{N} \mu_{L_i}$
  - $\sigma_L^2 = \sum_{i=1}^{N} \alpha_i^2 \cdot \sigma_{L_i}^2 + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} \alpha_i \cdot \alpha_j \cdot \sigma_{L_i}\sigma_{L_j}\boldsymbol{\rho_{L_iL_j}}$
  - Total loss exceedance probability

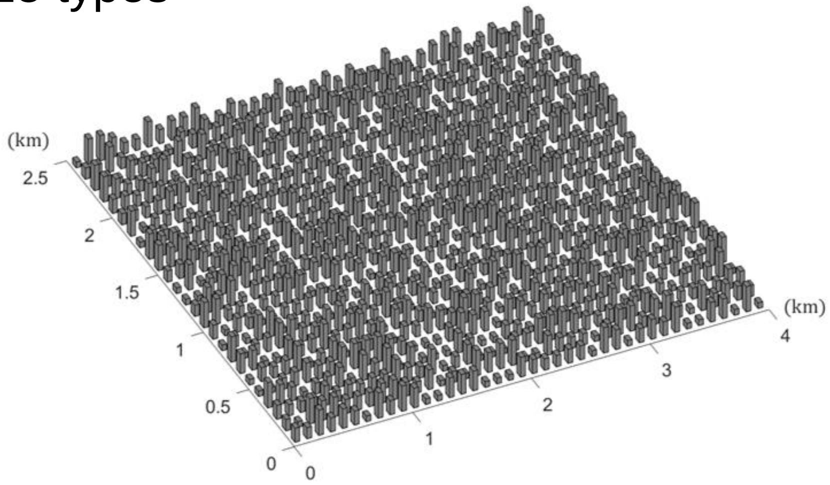
$$P(L > l_0) = 1 - \Phi\left(\frac{\ln l_0 - \lambda_L}{\zeta_L}\right)$$



Numerical example

Regional seismic loss assessment

- Building types: (2, 4, 8, 12, 20) story buildings × 3 SCWB ratio = 15 types
- Number of buildings: 40 × 25 = 1,000
- Area: 4.0 km × 2.5 km
- Randomly generated following uniform distribution
- Region: Virtual city in California



1,000 hypothetical buildings

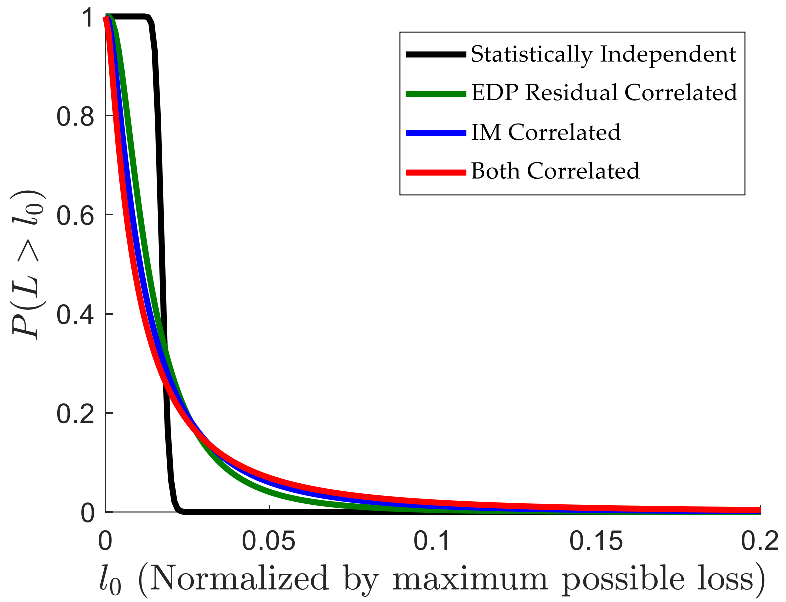
Given an earthquake scenario

- GMPE: Boore & Atkinson (2008)
- Spatial correlation models: Goda & Hong (2008), Baker & Cornell (2006)
- Earthquake scenario
  - $M = 5\sim8 \quad \rightarrow \quad \boldsymbol{M = 7.0}$
  - $R_{jb} < 200\text{ km} \quad \rightarrow \quad \boldsymbol{R_{jb} \approx 66.2\text{ km}}$
  - $V_{S30} = 180\sim1300\text{ m/s} \quad \rightarrow \quad \boldsymbol{V_{S30} = 760\text{ m/s}}$

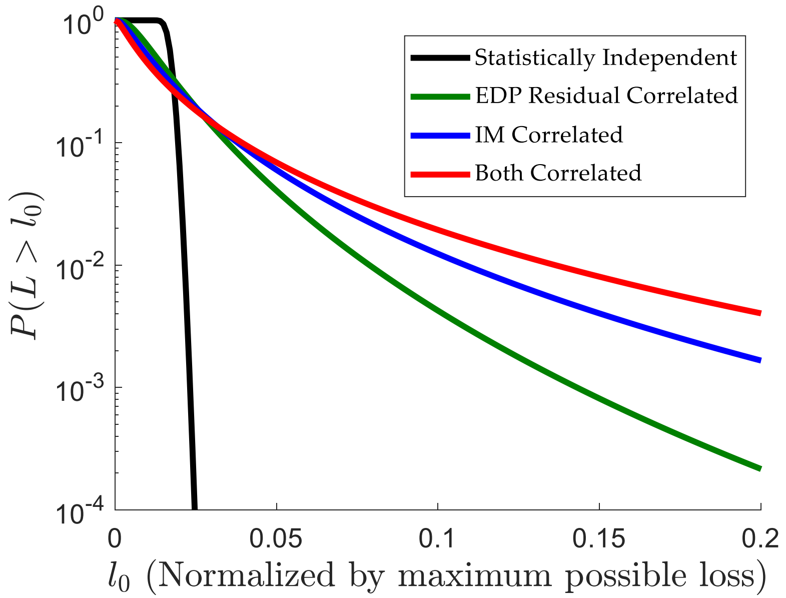
Regional seismic loss assessment

Total loss exceedance probabilities

- Linear-Linear scale



- Linear-Log scale

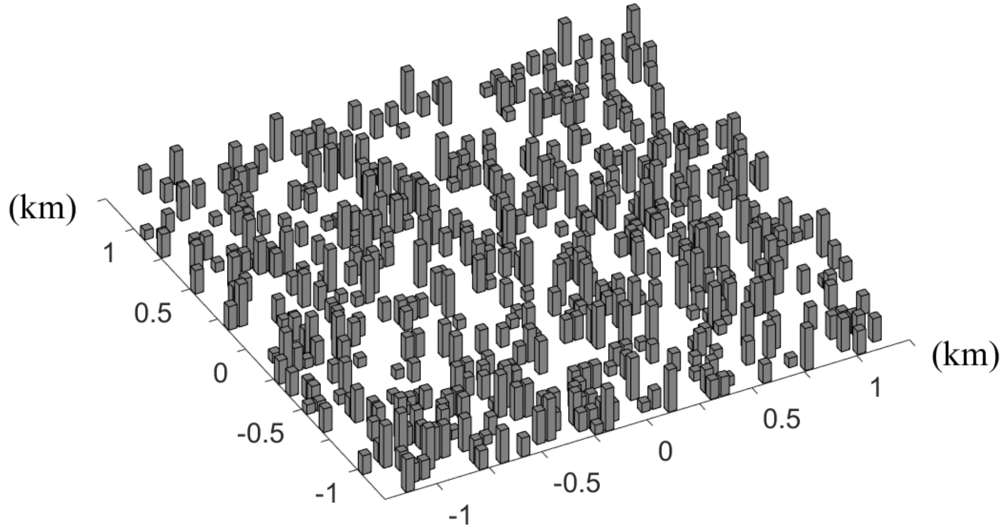


Probability of total loss	Both correlated	IM correlated	EDP residual correlated	Uncorrelated
P(5% total loss)	0.0690	0.0600	0.0406	0
P(10% total loss)	0.0194	0.0124	0.0042	0
P(15% total loss)	0.0080	0.0040	0.0008	0
P(20% total loss)	**0.0040**	**0.0017**	0.0002	0

Numerical example

Regional seismic loss assessment

- Building types: 44 archetype buildings
- Number of buildings: 500
- Area: 2.5 km × 2.5 km
- Randomly generated following uniform distribution
- Region: Four virtual cities in California



500 hypothetical buildings

- Four virtual cities
  - City A: Steel and wood buildings
  - City B: Concrete and masonry buildings
  - City C: High-rise buildings (≥ 8 stories)
  - City D: Low-rise buildings (< 8 stories)

- Given an earthquake scenario
  - $M = 7.5$
  - $R_{jb} \approx 35.4$ km
  - $V_{S30} = 760$ m/s

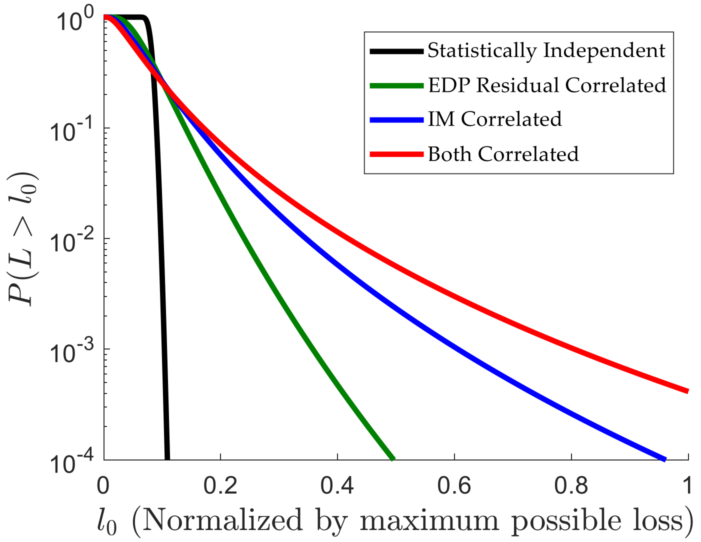
Building types	Number of archetype buildings			
	City A	City B	City C	City D
S1L	46	-	-	56
S1M	92	-	-	111
S1H	294	-	250	-
C1L	-	46	-	55
C1M	-	91	-	111
C1H	-	295	250	-
URML	-	23	-	28
URMM	-	45	-	56
W1	22	-	-	27
W2	46	-	-	56

Regional seismic loss assessment

Total loss exceedance probabilities

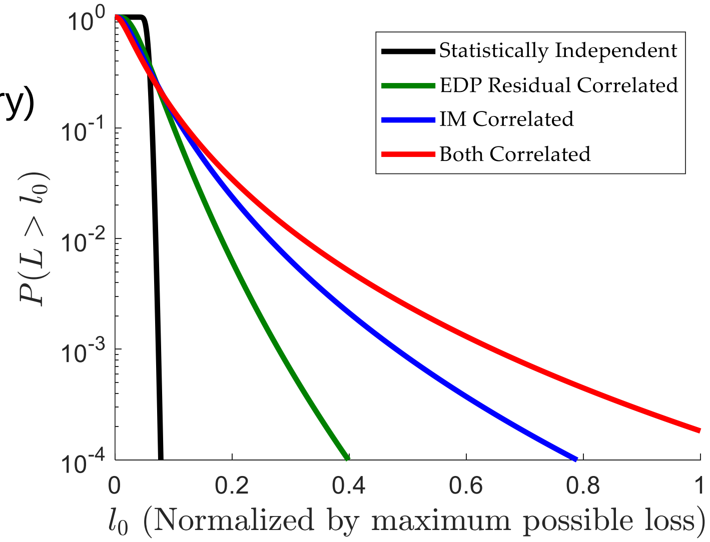
- City A

  (Steel & Wood)



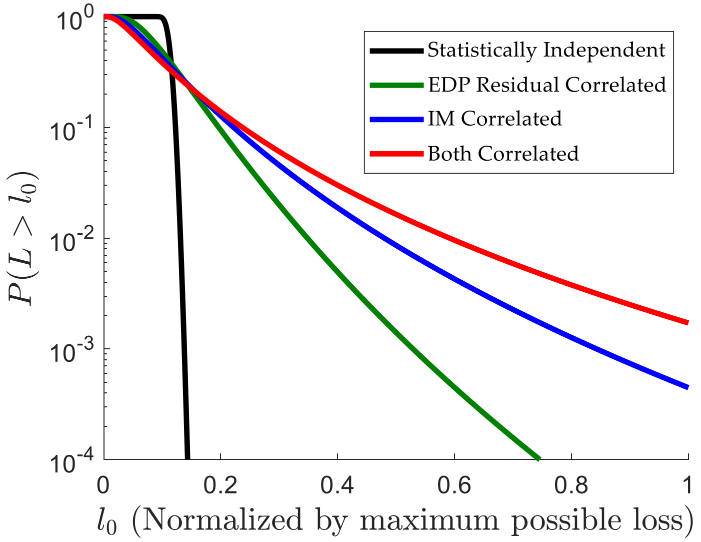
- City B

  (Concrete & Masonry)



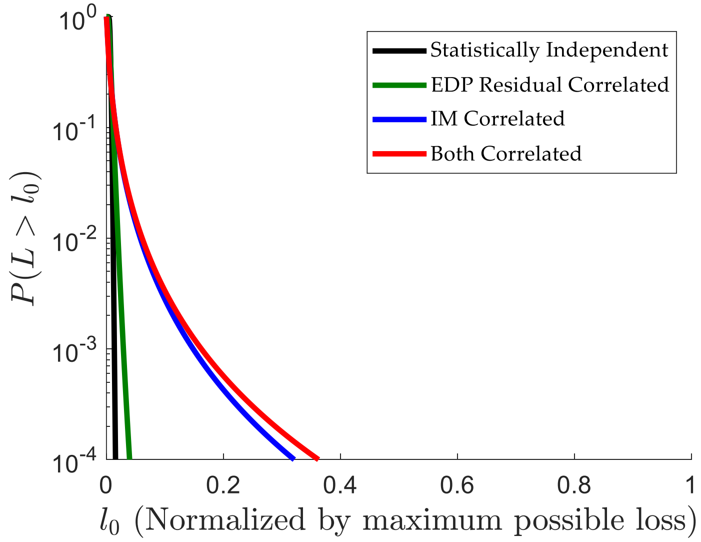
- City C

  (High-rise)



- City D

  (Low-rise)



Concluding remarks

Summary

- New seismic reliability analysis framework is developed for considering both the IM correlation and EDP residual correlation in a complex system.
- This research was proposed to estimate the variance and correlation of EDP residual correlation by using the analysis results.
- To verify the developed method, probabilistic seismic loss was estimated for a virtual region. It was shown that negligence of the EDP residual correlation underestimated the probability of total loss.

Further research topics

- Apply the proposed framework to other system reliability analysis with various disasters and risks.
- Extend the developed methods for estimating EDP residuals to other types of IMs and EDPs depending on the structural failure of interest.

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Thank you.

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