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Evaluation of Correlation Between Engineering Demand Parameters for Accurate Seismic System Reliability Analysis

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1. Introduction

Seismic system reliability analysis needs to be performed with proper consideration of various uncertainties in seismic motions and structural responses of complex systems. For accurate system reliability analysis, it is essential to incorporate the correlation between the seismic demands into component and system reliability analysis. To this end, this paper presents a new theoretical framework and estimation methods recently proposed by the authors to evaluate the correlation between the engineering demand parameters (EDPs) of structures in the system [1, 2].

2. Estimation of Correlation Between Engineering Demand Parameters (EDPs)

2.1 Uncertainties in Engineering Demand Parameters

Engineering Demand Parameters (EDPs) are adopted to estimate structural damage of components and systems as structural response quantities. The mean of EDP (D) is estimated by a regression function on the selected IM, while its variability can be described by the residual term, i.e., [3]

$$D = S(\mathrm{IM})\Psi(\mathrm{IM}) \tag{1}$$

where $S(\cdot)$ denotes the deterministic mean of EDP for a given IM and $\Psi(\cdot)$ represents the variability of EDP for a given IM. Assuming that the natural logarithm of IM, $\widehat{\mathrm{IM}}$ is introduced, the natural logarithm of an EDP, $\widehat{D}$ can be expressed as

$$\widehat{D} = s(\widehat{\mathrm{IM}}) + \psi(\widehat{\mathrm{IM}}) \tag{2}$$

where $s(\cdot)$ represents the regression function of $\widehat{D}$ on $\widehat{\mathrm{IM}}$ and $\psi(\cdot)$ denotes the uncertain residual of $\widehat{D}$ for given $\widehat{\mathrm{IM}}$, termed "EDP residual." It is assumed that $\psi(\widehat{\mathrm{IM}})$ follows a zero-mean Gaussian distribution.

When the power-law model [4], i.e., $D = a \cdot (\mathrm{IM})^b$ is adopted as the regression function $s(\cdot)$, the $\widehat{D}$ can be expressed as

$$\widehat{D} = \ln a + b \cdot \widehat{\mathrm{IM}} + \psi(\widehat{\mathrm{IM}}) \tag{3}$$

where a and b represent the parameters of power-law model, determined by structural analyses.

$\widehat{\mathrm{IM}}$ at the site of ith structure given an earthquake scenario is predicted using a ground motion prediction equation (GMPE) as

$$\ln Y_i = f(M, R_i, \lambda_i) + \eta + \varepsilon_i \tag{4}$$

where $\ln Y_i$ denotes the natural logarithm of the selected IM such as spectral acceleration (S_a), peak ground acceleration (PGA), and peak ground velocity (PGV); $f(M, R_i, \lambda_i)$ represents the attenuation relation; and η and ε_i respectively denote the inter-event residual representing the earthquake-to-earthquake variability and intra-event residual describing site-to-site variability [5].

2.2 Fragility Analysis Considering Correlation Between EDPs of Structures

Seismic fragility is defined as the conditional probability that a structure exceeds a certain limit state for a given IM. The failure of the ith structure can be described as the exceedance of the EDP (D_i) over the limit state d_i. Assuming that EDP follows Lognormal distribution, the safety factor $F_i = \ln d_i - \ln D_i = \hat{d}_i - \widehat{D}_i$ follows a Gaussian distribution. Then the fragility given $\widehat{\mathrm{IM}}_i = x$ is derived as

$$P\left(F_i \le 0 \middle| \widehat{\mathrm{IM}}_i = x\right) = 1 - \Phi\left(\frac{\hat{d}_i - s_i(x)}{\sigma_{\psi_i(x)}}\right) \tag{5}$$

where $\Phi(\cdot)$ denotes the cumulative distribution function (CDF) of the standard Gaussian distribution. Taking into account the uncertainty of IM for a given earthquake scenario, the failure probability is derived as

$$P_{f_i} = \int_{-\infty}^{\infty} P\left(F_i \le 0 \middle| \widehat{\mathrm{IM}}_i = x\right) f_{\widehat{\mathrm{IM}}_i}(x)\,dx \tag{6}$$

where $f_{\widehat{\mathrm{IM}}_i}(\cdot)$ is the probability density function (PDF) of $\widehat{\mathrm{IM}}_i$ given an earthquake scenario, which was assumed to follow a Gaussian distribution.

Based on Eq. 6, the joint failure probability of the ith and jth structures given an earthquake scenario can be computed by a single-fold integration [6], i.e.,

$$P_{f_{ij}} = P_{f_i} P_{f_j} + \int_0^{\rho_{F_i F_j}} \varphi_2(-\beta_i, -\beta_j; \rho)\,d\rho \tag{7}$$

For efficient calculation Eq. 7, the correlation coefficient $\rho_{F_iF_j}$ is derived by Kang et al. (2021) as follows:

$$\rho_{F_iF_j} = \frac{b_ib_j\sigma_{\widehat{\mathrm{IM}}_i}\sigma_{\widehat{\mathrm{IM}}_j}}{\sqrt{b_i^2\sigma_{\widehat{\mathrm{IM}}_i}^2+\sigma_{\psi_i}^2}\cdot\sqrt{b_j^2\sigma_{\widehat{\mathrm{IM}}_j}^2+\sigma_{\psi_j}^2}}\rho_{\widehat{\mathrm{IM}}_i\widehat{\mathrm{IM}}_j} + \frac{\sigma_{\psi_i}\sigma_{\psi_j}}{\sqrt{b_i^2\sigma_{\widehat{\mathrm{IM}}_i}^2+\sigma_{\psi_i}^2}\cdot\sqrt{b_j^2\sigma_{\widehat{\mathrm{IM}}_j}^2+\sigma_{\psi_j}^2}}\rho_{\psi_i\psi_j} \tag{8}$$

where b_i and b_j are the power-law model parameters; $\sigma_{\widehat{\mathrm{IM}}_i}$, $\sigma_{\widehat{\mathrm{IM}}_j}$, σ_{ψ_i}, and σ_{ψ_j} are the standard deviations of $\widehat{\mathrm{IM}}_i$, $\widehat{\mathrm{IM}}_j$, ψ_i, and ψ_j, respectively; and $\rho_{\widehat{\mathrm{IM}}_i\widehat{\mathrm{IM}}_j}$ denotes the correlation coefficient between $\widehat{\mathrm{IM}}_i$ and $\widehat{\mathrm{IM}}_j$, termed "IM correlation"; and $\rho_{\psi_i\psi_j}$ denotes the correlation coefficient between the EDP residuals, termed "EDP residual correlation."

First, the standard deviations $\sigma_{\widehat{\mathrm{IM}}}$ and correlation coefficients $\rho_{\widehat{\mathrm{IM}}\widehat{\mathrm{IM}}}$ of IMs can be predicted by existing GMPE studies [7, 8] respectively as

$$\sigma_{\widehat{\mathrm{IM}}} = \sqrt{{\sigma_\eta}^2 + {\sigma_\varepsilon}^2} \tag{9}$$

$$\rho_{\widehat{\mathrm{IM}}_i\widehat{\mathrm{IM}}_j} = \rho_{\eta\eta}\frac{\sigma_\eta\sigma_\eta\left(T_n^j\right)}{\sigma_{\widehat{\mathrm{IM}}_i}\sigma_{\widehat{\mathrm{IM}}_j}} + \rho_{\varepsilon_i\varepsilon_j}\left(\Delta_{ij}\right)\frac{\sigma_{\varepsilon_i}\sigma_{\varepsilon_j}}{\sigma_{\widehat{\mathrm{IM}}_i}\sigma_{\widehat{\mathrm{IM}}_j}} \tag{10}$$

where σ_η and σ_ε represent the standard deviations of the inter- and intra-event residuals, respectively; $\rho_{\eta\eta}$ and $\rho_{\varepsilon_i\varepsilon_j}\left(\Delta_{ij}\right)$ respectively are the correlation coefficients of inter-event and intra-event residuals at two sites i and j with distance Δ_{ij} (km).

In a recent study, the authors proposed an Incremental Dynamic Analysis (IDA)-based method for estimating the EDP residuals of structures. From the IDA curves of each structure using a set of ground motions, the standard deviations σ_ψ and correlation coefficients $\rho_{\psi\psi}$ of EDP residuals corresponding to the given IMs can be estimated through the procedure illustrated in Fig. 1.

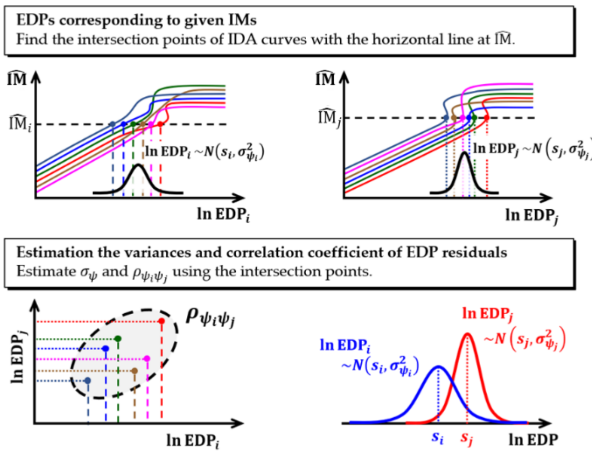


Fig. 1. Overall procedure of IDA-based method correlation.

However, the IDA-based method has a limitation in that the estimation results may vary depending on the selected ground motions and requires high computational costs. Therefore, the authors also employed an IM-invariant method that estimates the EDP residual of structures based on the elastic-range responses for more efficient estimation. By using the proposed two estimation methods according to a given structural system, the EDP residuals can be accurately and efficiently estimated.

2.3 Contributions of IM and EDP Residual Correlation to Correlation between EDPs

The safety factor correlation in Eq. 8 can be expressed as the contributions of IM and EDP residual correlation as follows.

$$\rho_{F_iF_j} = A_S\rho_{\widehat{\mathrm{IM}}_i\widehat{\mathrm{IM}}_j} + A_\psi\rho_{\psi_i\psi_j} \tag{11}$$

where A_S and A_ψ respectively denote the contributions of the IM correlation and the EDP residual correlation to the correlation coefficient between safety factors. The contributions of the safety factor coefficient (A_S, A_ψ) depend on the power-law model parameters (b_i, b_j) and the standard deviations of IM and EDP residuals.

In order to investigate the effects of these parameters on A_S and A_ψ, let us consider the case in which $b_i = b_j = b$, $\sigma_{\widehat{\mathrm{IM}}_i} = \sigma_{\widehat{\mathrm{IM}}_j} = \sigma_{\widehat{\mathrm{IM}}}$, and $\sigma_{\psi_i} = \sigma_{\psi_j} = \sigma_\psi$. The contributions can be expressed as $A_S = r^2/(r^2+1)$, and $A_\psi = 1/(r^2+1)$ where $r = b \cdot \sigma_{\widehat{S_a}} / \sigma_\psi$. Fig 2 shows the relationship between r and the contributions. As the ratio r decreases, the contribution of EDP residual correlation becomes larger while that of IM correlation decreases.

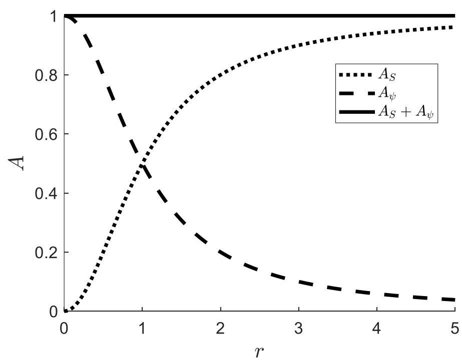
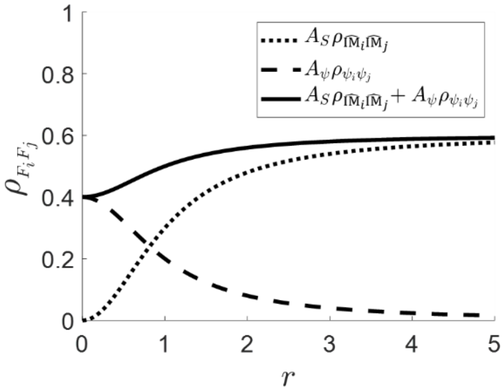
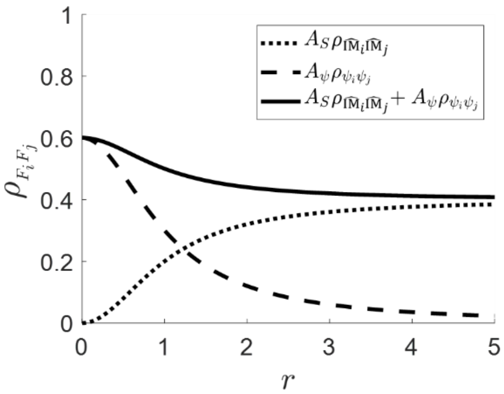


Fig. 2. Contributions of IM and EDP residual correlation to the safety factor correlation.

Using Eq. 11, the correlation coefficients $\rho_{F_iF_j}$ for two cases of $\rho_{\widehat{\mathrm{IM}}_i\widehat{\mathrm{IM}}_j}$ and $\rho_{\psi_i\psi_j}$ are shown in Fig 3. It is confirmed that, if a system consists of pairs of structures with small r values, ignoring the EDP residual correlation can lead to the significant underestimation of total EDP correlations in seismic system reliability analysis.



(a)



(b)

Fig. 3. Correlation coefficient between the safety factors according to the ratio r: (a) $\rho_{\widehat{\mathrm{IM}}_i \widehat{\mathrm{IM}}_j} = 0.6$, $\rho_{\psi_i \psi_j} = 0.4$; and (b) $\rho_{\widehat{\mathrm{IM}}_i \widehat{\mathrm{IM}}_j} = 0.4$, $\rho_{\psi_i \psi_j} = 0.6$.

3. Conclusions

Based on the authors' recent studies on evaluating the EDP correlations, a new theoretical framework was represented to properly consider the correlations between engineering demand parameters (EDPs) of structures in seismic system reliability analysis. To this end, the correlation between the safety factors of two different structures that can consider both the IM correlation and EDP residual correlation was newly derived. In addition, the IDA-based method and IM-invariant methods were introduced to accurately and efficiently estimate the standard deviations and correlation coefficients of EDP residuals. Furthermore, a comprehensive investigation was performed to quantify the contributions of IM correlation and EDP residual correlation based on several assumptions. It is demonstrated that the standard deviation of each correlation has a significant influence on the contribution to the safety factor correlation. The proposed theoretical framework and two estimation methods are expected to facilitate accurate and efficient seismic reliability analysis.

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