Reconstruction of Beam Distribution at Phase and Coordinate Space Using Computational Tomography

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1. Introduction

A characterization of beam properties is recognized to be essential for stable operation of an accelerator. Various and specific beam diagnostic results can be utilized to improve a stability and performance of the accelerator. An emittance of the beam is a key parameter representing divergence and a size of the beam. A quadrupole scan method is widely used to measure the emittance of high energy beam. However, a detail distribution of the beam at phase space cannot be investigated using the quadrupole scan method, because the quadrupole scan method is based on the mathematical relation between a beam transfer matrix and the rms beam parameters. Thus, a computational tomography (CT) technique is introduced to analyze the beam distribution. Also, the beam distribution not only at phase space but also at coordinate space is reconstructed using the CT.

2. Methods and Results

Details of an experimental procedure and a reconstruction process are described in this section.

2.1 Experiment



Fig. 1. Layout of dump beamline for quadrupole variation experiment.

An experimental data for the reconstruction of the beam distribution at the phase space is the same as that of the quadrupole scan. That is, a set of beam profile data obtained changing a current of a quadrupole magnet (QM) is used. As shown in Fig. 1, there are six quadrupole magnets and two wire scanners at a dump beamline. The beam profiles were measured by wire scanner (WS) 1 by changing the current of the QM-3. The QM-3 can be operated in a current range from -110 A to 110 A because it is equipped with a bipolar DC power supply. The wire scanner consists of two tungsten wires placed orthogonally, as depicted in Fig. 2. This wire scanner can

cover the range of 150 mm and is driven by an electric motor and an automated system. The wire scanner is located at 1620.4 mm away from the QM-3.



Fig. 2. Pictures of wire scanner and its operation.

2.2 Method for Reconstruction of Beam Distribution in Coordinates and Phase Space

The beam distributions in two kinds of spaces were reconstructed in this study. Horizontal(x) and vertical(y) direction beam profiles obtained with a single configuration were used for the reconstruction of the beam distribution in the 2-dimensional coordinate space. Also, the sets of x or y beam profiles obtained with changing of the QM current were used for the reconstruction of the beam distribution in the 2-dimensional phase space such as x-x' or y-y', respectively.

Since only two beam profile data are used for the reconstruction in the coordinate space, the maximum entropy (MENT) algorithm is introduced with the assumption that the beam profile will be the most probable distribution. The maximum entropy algorithm for the beam diagnostics is well described in reference 1.

Meanwhile, a filtered back projection algorithm is introduced to reconstruct the beam distribution in the phase space. This algorithm is widely used in medical computational tomography. In the medical CT system, the subject keeps a position, but a detector rotates around the subject. However, in this study, the subject, i.e. the beam distribution in the phase space, rotates and the detector, i.e. wire scanner, keeps its position. In order to explain the rotation of the beam distribution in the phase space, the beam distribution at z_0 in Fig. 1 is assumed as an ellipse. Then, the beam ellipse at z_1 is determined by the beam matrix at z_0 and a transfer matrix. The transfer matrix of this experiment is a product of a thick lens Transactions of the Korean Nuclear Society Autumn Meeting Changwon, Korea, October 20-21, 2022

matrix according to the current of QM-3 and a matrix for drift motion between the QM-3 and the wire scanner. As shown in Fig. 3., let assume that the two points, $(x_{p0}, 0)$ and $(0, x'_{q0})$, rotates to (x_{p1}, x'_{p1}) and (x_{q1}, x'_{q1}) according to the transfer matrix, M.

The relation between the points defined on different planes is mathematically expressed as follows:

$$\begin{pmatrix} x_{p1} & x_{q1} \\ x'_{p1} & x'_{q1} \end{pmatrix} = M \begin{pmatrix} x_{p0} & 0 \\ 0 & x'_{q0} \end{pmatrix}$$

$$= \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} x_{p0} & 0 \\ 0 & x'_{q0} \end{pmatrix}$$

$$= \begin{pmatrix} M_{11}x_{p0} & M_{12}x'_{q0} \\ M_{21}x_{p0} & M_{22}x'_{q0} \end{pmatrix}$$
(1)

$$\tan \theta = \frac{x_{p0}}{x'_{q0}} = \frac{R_{12}}{R_{11}} \tag{2}$$

$$a = \frac{x_{p1}}{s} = \frac{R_{11}x_{p0}}{s} = \frac{R_{11}}{\cos\theta}$$
(3)



Fig. 3. Rotation and elongation of ellipse in phase space

Therefore, the beam profile measured at z_1 is equivalent to the elongated projection of the ellipse at z_0 rotated by the angle, θ . The elongation factor is *a*. Thus, the rotation angle and the elongation factor are determined by the current of the QM-3. The set of the beam profiles is modified by the elongation factors and arranged according to the rotation angles. This data set is called as a sinogram in the CT technique. The beam distribution is reconstructed by the filtered back projection algorithm and the sinogram.

Both reconstruction procedures are conducted by a MATLAB code. This MATALB based post-processing program can read and sort the wire scanner data. In addition, the Twiss parameters are evaluated by using the quadrupole scan method. A self-developed code is used for the MENT and a built-in MATLAB function is used for the filtered back projection.

2.3 Results

Fig. 4 shows the measured beam profile data while changing the QM-3 current from -110 A to 100 A at intervals of 10 A. The 2-dimensional beam profiles at the coordinate space for each current are depicted in Fig. 5. It can be found that the beam shape is changed from a horizontally focused shape to a vertically focused shape. Also, the beam center is affected by the QM-3 current. This means that the QM-3 and the beam are misaligned.



Fig. 4. (a) horizontal and (b) vertical beam profiles measured by the wire scanner with changing QM-3 current.



Fig. 5. 2-dimensional beam profiles reconstructed in coordinate space by using the maximum entropy method.

Also, the beam profiles in the 2-dimensional phase space are obtained, as shown in Fig. 6. Horizontally and vertically focused beam distribution in the phase space are confirmed. However, there are some problems in the reconstructed distribution such as artifacts and negative current near the beam ellipse. These are induced by the limited rotation angle by the configuration of the beam optics. Thus, MATALB based post-processing program evaluates a beam emittance using the Self-Consistent UnBiased Exclusion analysis (SCUBEEx) [2], to reduce effect of artifacts and negative current. The normalized rms emittance evaluated by the CT and the quadrupole scan method are compared in Table 1.



Fig. 6. Reconstructed beam distribution in (a) horizontal (b) vertical phase space.

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There is about 50% difference between the vertical emittance evaluated by the quadrupole scan method and the CT technique while the difference of the horizontal emittance is quite small. This large difference may be induced by the artifacts and the relatively small emittance despite using the SCUBEEx method.

Method	Normalized rms emittance (π mm mrad)	
	Horizontal (x)	Vertical (y)
Quadrupole scan	1.49 ± 0.037	0.49 ± 0.055
СТ	1.52 ± 0.135	0.73 ± 0.048

Table 1: Comparison of emittance evaluated by guadrupole scan and CT technique

3. Conclusions

The diagnostic method for the distribution in the 2dimensional coordinate and phase space using the computational tomography technique was studied. The beam distribution in the coordinate space is obtained by the horizontal and vertical beam profiles and using the MENT. Also, the distribution in the phase space is reconstructed by the filtered back projection algorithm and the set of the beam profiles measured by the quadrupole scan experiment. The horizontal and vertical emittance of 100 MeV proton beam evaluated from reconstructed beam distribution in phase space and the SCUBEEx method. The horizontal emittance is similar with those of the quadrupole scan method. However, the discrepancy of the vertical emittance by the artifacts should be improved.

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