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Hydraulic Resistances for Inverse Problems of Two-phase Flow Networks Using MARS-KS Code

Young Seok Bang [a*], Jungjin Bang[a], Seong-Su Jeon [a], Bub Dong Chung [a], Youngsuk Bang [a]
[a]*Future and Challenge Tech. Co. Ltd., Heungdeok1ro 13, Yeongdeok-dong, Giheung-gu, Yongin- si, Gyeonggi-do*
[*]*Corresponding author: ysbang@fnctech.com*

1. Introduction

Recently, there is an increasing need to analyze performance and thermal-hydraulic behavior of a complex two-phase flow network involving a phase change such as a feedwater system of a nuclear power plants (NPP), which aims to develop a nuclear renewable hybrid energy system (NRHES) [1]. Most analysis so far is a so-called "direct problem" in which the configuration and arrangement of equipment and piping, detailed geometries and flow conditions in the network are provided and to calculate the performance parameters such as pressure, flow rates, etc. However, sometimes, given the performance parameters at the important points in the network, it may be necessary to determine the arrangement and sizing of the system satisfying the given constraints. It implies a solution to the "inverse problem". In the detailed design process of NRHES, how to obtain a solution to the inverse problem can be a very important part.

In general, the analysis of inverse problems can be regarded as a repetitive solution process of direct problem, starting from the assumptions of the overall configurations and dimensions of the system, calculating the performance parameters, changing the assumed configurations and recalculating. Therefore, for effective inverse problem analysis, how to properly change the configuration and dimension is an important point.

Hydraulic resistance in the flow path is generally an important factor in determining the relationship between the performance parameters and the configuration and dimensions of the system [2]. The present study proposes a method to determine hydraulic resistance that can satisfy the given constraints. A system thermal-hydraulic code, MARS-KS [3], is used for this inverse problem. Since the feedwater heaters are typical examples of a two-phase flow network with phase change [4], we discuss a method considering network in two-phase flow as well as one in single-phase flow.

2. Definition of Problems

The secondary system of the steam generator of NPP consists of main steam system to high and low-pressure steam turbines, condensate water system from turbines to condenser, feedwater system to steam generator by heating the condensed water, etc. [5]. The configuration and shape of the overall system may be characterized as follows.

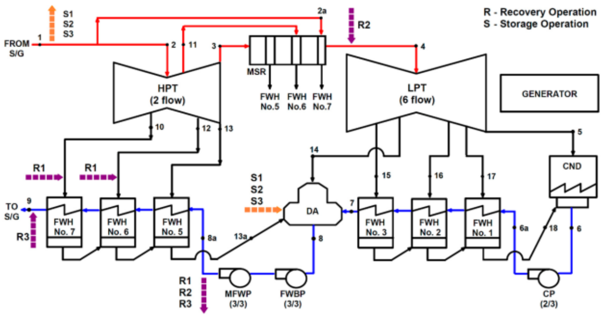


Fig. 1. Concept of secondary systems with NRHES [6]

(1) A variety of pipes having various dimensions are divided from one equipment or merged into one equipment.
(2) When the two flow paths are merged, two-phase flow may be involved in which steam and water are injected into each flow path as in the deaerator.
(3) The merged water flow is further cooled-down by heat transfer through tubes as in the feedwater heaters and is drained to a single outlet.
(4) Most pumps are arranged on a single flow path.
(5) In the case of a steam turbine, it generally has one inlet flow path and several steam extraction flow paths and outlet flow paths

From this example of actual design [5], it can be stated that no matter how complex the flow network is, it can be described as a combination of a single flow path, a dividing flow path, and a merging flow path. In addition, it can be said that even for heat exchangers with merged and divided flow paths, actual heat transfer can be described as occurring in a single flow path after the merger. Therefore, the problem we need to solve can be defined in three ways.
(1) Hydraulic resistances at dividing and merging flow paths in a single-phase flow
(2) Hydraulic resistances at merging flow paths with steam and water inflow
(3) Hydraulic resistances at a merged flow path in two-phase flow with an external heat transfer.
Upstream and downstream flow conditions should be satisfied for all the cases.

Hydraulic resistances to networks including turbines and pumps accompanied by mechanical energy transfer will be addressed as a follow-up study in this paper.

Considering these three components and their connectivity within the entire network, the hydraulic resistance of the network can be defined. The integration method for the entire flow network will be addressed with the evaluation of the hydraulic resistances of the turbines and pumps.

3. Theoretical Background

Although the MARS-KS code does not solve the Bernoulli equation, the concept of hydraulic resistance is derived from this equation, so we develop the theory based on the Bernoulli equation in single phase flow. However, for two-phase flow, the same theoretical development as the single-phase flow is difficult due to the high complexity of the two-fluid model, and we choose to use the code calculation results.

3.1 Single phase flow

Figure 1 shows an example of the dividing and merging of the flow paths, and the figure on the right shows how to model it in the system code. In general, the properties of flow at the dividing/merging point, *a*, is considered unknown. The governing Bernoulli equations along each flow path in dividing case are as follows:

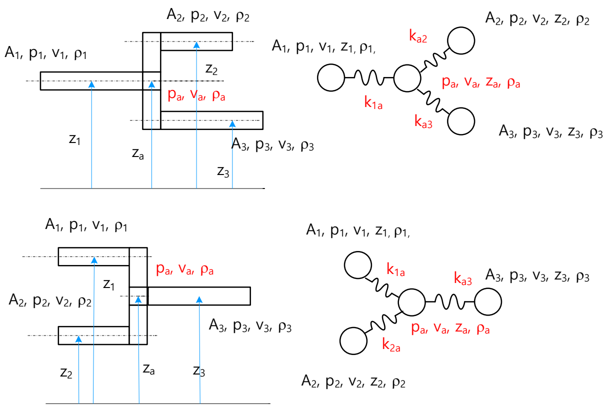


Fig. 2. Branch flow network and merging flow network and their modeling

$$P_1 + \frac{1}{2}\rho_1 v_1{}^2 = P_a + \frac{1}{2}\rho_a v_a{}^2 + K_{1a}\frac{1}{2}\rho_{1a} v_{1a}{}^2 \quad (1a)$$

$$P_a + \frac{1}{2}\rho_a v_a{}^2 = P_2 + \frac{1}{2}\rho_2 v_2{}^2 + K_{a2}\frac{1}{2}\rho_{a2} v_{a2}{}^2 \quad (1b)$$

$$P_a + \frac{1}{2}\rho_a v_a{}^2 = P_3 + \frac{1}{2}\rho_3 v_3{}^2 + K_{a3}\frac{1}{2}\rho_{a3} v_{a3}{}^2 \quad (1c)$$

where,

$$P_i = p_i + \rho_i g z_i \quad (2)$$

$$K_{1a} = \left(f_{1a}\frac{L_{1a}}{D_{1a}} + k_{1a}\right) \quad (3)$$

And f_{1a} and k_{1a} mean a friction factor and form loss factor over the flow path, respectively. Defining a flow dividing ratio, φ_1, and applying to continuity equation,

$$\rho_2 v_2 A_2 + \rho_3 v_3 A_3 = \varphi_1 \rho_1 v_1 A_1 + (1-\varphi_1)\rho_1 v_1 A_1 \quad (4)$$

From the Bernoulli equations, deriving expressions of velocities and substituting to the continuity equation, the hydraulic resistances at each dividing path can be obtained.

$$K_{a2} = \frac{1}{\varphi_1{}^2} M_{12}\{\delta P_{12} - p_{d1}(K_{1a} - 1)\} - 1 \quad (5a)$$

$$K_{a3} = \frac{1}{(1-\varphi_1)^2} M_{13}\{\delta P_{13} - p_{d1}(K_{1a} - 1)\} - 1 \quad (5b)$$

where,

$$M_{12} = \frac{2\rho_2 A_2{}^2}{(\rho_1 v_1 A_1)^2}, M_{13} = \frac{2\rho_3 A_3{}^2}{(\rho_1 v_1 A_1)^2},$$
$$\delta P_{12} = P_1 - P_2, \delta P_{13} = P_1 - P_3, p_{d1} = \frac{1}{2}\rho_1 v_1{}^2 \quad (6)$$

The above equations imply that when the flow properties at points 1, 2, and 3 and the flow dividing ratio are determined, hydraulic resistances satisfying them can be theoretically obtained. Here,

(1) the hydraulic resistance of the main flow path, K_{1a}, may be selected by the user within a range in which all hydraulic resistance values are positive.
(2) the pressure distribution from the inlet to the outlet calculated by the code with the combination of selected K_{1a} and K_{a2} and K_{a3} from Eq (5a, 5b) as inputs shall have a monotonous trend.

The hydraulic resistances in the case of the merged flow path can be derived through a similar process as follows.

$$K_{1a} = \frac{1}{\varphi_1{}^2} M_{13}\{\delta P_{13} - p_{d3}(K_{a3} + 1)\} + 1 \quad (7a)$$

$$K_{2a} = \frac{1}{(1-\varphi_1)^2} M_{23}\{\delta P_{23} - p_{d3}(K_{a3} + 1)\} + 1 \quad (7b)$$

where,

$$M_{13} = \frac{2\rho_1 A_1{}^2}{(\rho_3 v_3 A_3)^2}, M_{23} = \frac{2\rho_2 A_2{}^2}{(\rho_3 v_3 A_3)^2},$$
$$\delta P_{13} = P_1 - P_3, \delta P_{23} = P_2 - P_3, p_{d3} = \frac{1}{2}\rho_3 v_3{}^2 \quad (8)$$

Those equations can be easily derived even when dividing or merging from or to two or more flow paths.

3.2 Non-dimensional form

In order to confirm whether the aforementioned hydraulic resistance equations are applicable to various geometric and flow conditions, it is necessary to evaluate the dimensionless form and similarity of the equations. Introducing non-dimensional variables as follows:

$$\rho^* = \frac{\rho}{\rho_1}, v^* = \frac{v}{v_1}, A^* = \frac{A}{A_1}, p^* = \frac{p}{p_1}, z^* = \frac{z}{z_1}, g^* = \frac{g}{g_1} \quad (9)$$

Application of those equation to the Eq.(5a) and (5b) lead to the final dimensionless form.

$$K_{a2} = \frac{2\rho_2{}^* A_2{}^{*2}}{\varphi_1{}^2}\Big\{Eu_1(1 - p_2{}^*) + Nz_1(1 - \rho_2{}^* z_2{}^*) - \frac{1}{2}(K_{1a} - 1)\Big\} - 1 \quad (10a)$$

$$K_{a3} = \frac{2\rho_3^* A_3^{*2}}{(1-\varphi_1)^2}\left\{Eu_1(1-p_3^*) + Nz_1(1 - \rho_3^* z_3^*) - \frac{1}{2}(K_{1a}-1)\right\} - 1 \quad (10b)$$

where,

$$Eu_1 = \frac{p_1}{\rho_1 v_1^2}, Nz_1 = \frac{\rho_1 g z_1}{\rho_1 v_1^2} \quad (11)$$

From those equations, it can be said that the relationship between the hydraulic resistance at the upstream flow path and one at the downstream divided flow paths can be expressed in non-dimensional form, and those equations have a similarity for all the geometric and flow conditions if the Euler number ($\boldsymbol{Eu_1}$) and the elevation related non-dimensional number ($\boldsymbol{Nz_1}$) at the inlet are the same.

Fig. 3 shows the hydraulic resistances for the dividing flow path calculated by Eq.(9), in which inlet and outlet condition are ideally selected and several dividing ratios are applied. Each curve in the figure represents a combination of $\boldsymbol{K_{a2}}$ and $\boldsymbol{K_{a3}}$ that leads to the same dividing ratio for a certain range of $\boldsymbol{K_{1a}}$. From those curves, one can determine the hydraulic resistances satisfying the dividing ratio under the given condition of upstream and downstream. The uncovered region in the figure means that at least one of the hydraulic resistance values is negative.

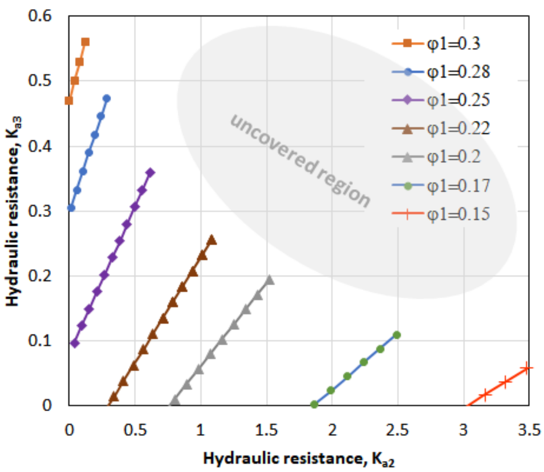


Fig. 3. Calculated hydraulic resistances for the selected flow dividing ratio

3.3 Two-phase flow merging

To calculate the case where steam and water are introduced from the separated points and condensation occurs in a merged flow path, the change in pressure caused by condensation at the interface between water and steam should be considered.

In the MARS-KS code, the heat and mass transfer at the interface at the steam-water mixing volume is calculated at the pressure calculated by momentum equation which the user-specified hydraulic resistance was implemented, and, thus, the calculation iterated. However, in those process, there may be a significant difference in densities and velocities at the junctions between the assumption and the calculation, thus, the proposed model using Eq.(7) cannot be applied. For this problem, this study presents a method to obtain hydraulic resistances that start with the assumption that all hydraulic resistances are zero and then adjust them to get the desired or possible pressure distribution. Let the calculated properties at the point '*a*' under zero hydraulic resistance be $p_a', \rho_a', v_a', \rho_{a3}', v_{a3}'$, then the hydraulic resistance to get the desired pressure, p_a, can be approximated,

$$K_{a3} = (p_a - p_a')/\frac{1}{2}\rho_{a3}' v_{a3}'^2 \quad (12)$$

Once the desired condition at the point '*a*' obtained, then the upstream hydraulic resistances can be adjusted by increasing a small amount until the desired values are obtained. At this time, the following relationship is applied between the hydraulic resistance of the steam flow path and the hydraulic resistance of the water flow path.

$$K_{1a}' = \frac{\rho_{2a}' v_{2a}'^2}{\rho_{1a}' v_{1a}'^2} K_{2a}' \quad (13)$$

3.4 Two-phase flow merging with heat transfer

In the case of heat transfer from one volume to the outside, it starts with the case where all hydraulic resistances are zero. It is assumed that heat is removed through a heat structure attached to the hydrodynamic volume, in the modeling of the problem. Heat removal is simulated by imposing heat flux as a boundary condition. For heat exchangers such as feedwater heaters, this heat flux (q_a) is defined as the difference between the total enthalpy of incoming water ($m_2 h_{f2}$) and steam ($m_1 h_{g1}$) and the total enthalpy of the drained water (h_{f3}).

$$q_a A_H = m_2 h_{f2} + m_1 h_{g1} - (m_1 + m_2) h_{f3} \quad (14)$$

where A_H means heat transfer area of the heat structure.

In code calculation, the pressure of point '*a*' is determined from the momentum equation considering hydraulic resistance, but since the thermodynamic state of point '*a*' is changed by forced heat transfer, a value that converged through repeated calculation will be found. Once the pressure and flow states of point '*a*' are determined, the hydraulic resistances for the two upstream flow paths of point '*a*' are determined in an approximate manner, similar to the method in the Section 3.3.

4. Results and Discussion

The previously determined hydraulic resistance values are applied to the following simple problems having dividing paths and merging paths, respectively (Fig. 4). The lengths of all the volumes were set to 1m for simplicity and the areas were specifically set depending on the problem concerned. In the merging problem, a heat structure to consider the case of heat transfer is attached to the volume. In this case, vertical upstream volumes were applied to realize the effect of water pressure on the merged volume. In the MARS code input, the wall friction model was turned off to exclude the effect of the length of the pipe arbitrarily assumed in the input.

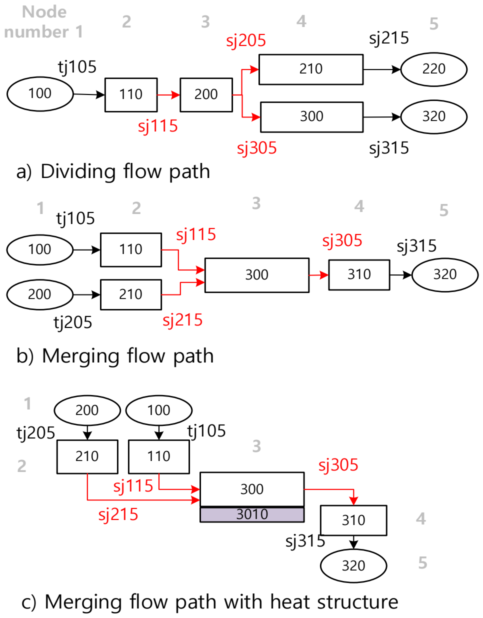


Fig. 4. MARS-KS nodalization of flow paths with division merging, and heat transfer

4.1 Single-phase flow

Fig. 5 shows a calculation result for the dividing case of different pressure and area at an inlet and two outlet boundaries (Fig.4a).

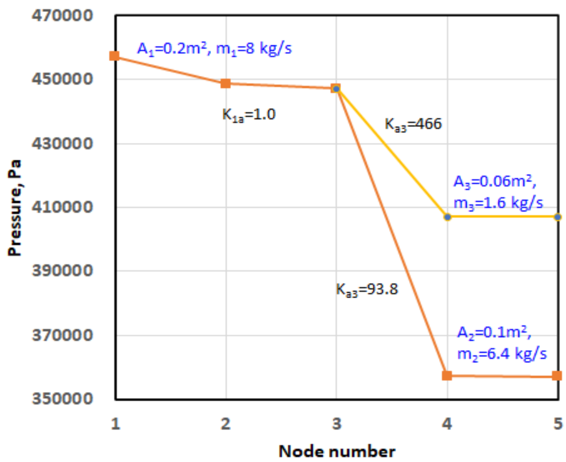


Fig. 5. Calculated pressure distribution for dividing case

Fig. 6 shows a calculation result for the merging case of different pressure and area at two inlet and one outlet boundaries (Fig.4b). The hydraulic resistance determined by the model in the Section 3 were also described in the figures. Any non-monotonous pressure distribution was not found in the both calculation results, which indicated those hydraulic resistances can be one of the solutions of the inverse problem.

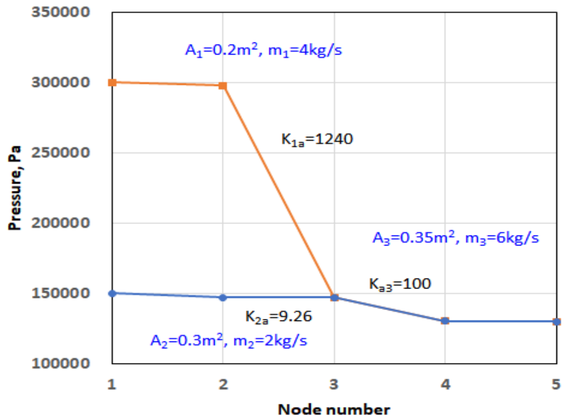


Fig. 6. Calculated pressure distribution for merging case

4.2 Two-phase merging flow

Fig. 7 shows a calculated pressure distribution in the problem of for two-phase flow merging (Fig.4b). In this problem, steam and water are introduced in 2 kg/sec and 5 kg/sec at a pressure 0.2MPa and 0.22 MPa, through pipes having areas of 0.2m^2 and 0.1m^2, respectively. In this figure, the results for the case where the hydraulic resistances are all 0, the case where only the hydraulic resistance at the merged path is increased, and the case where both the hydraulic resistance is increased are compared together. From this comparison, it can be shown that the hydraulic resistances that satisfies all constraints for the two-phase flow merging can be obtained using the method of the present study.

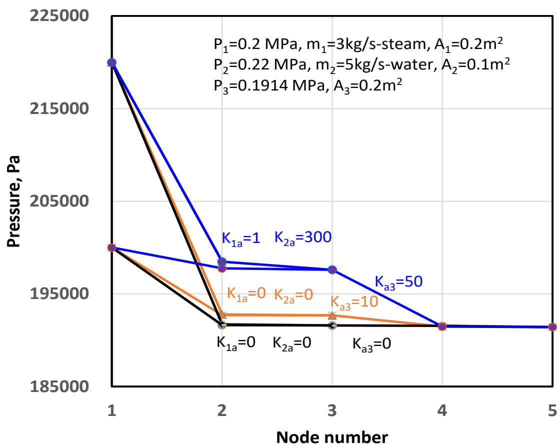
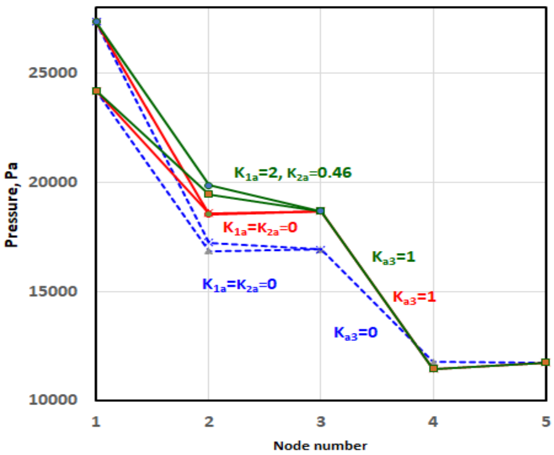


Fig. 7. Calculated pressure distribution for two-phase flow merging case

4.3 Two-phase flow with heat transfer

Fig. 8 shows a calculated pressure distribution in the problem of two-phase flow merging and heat transfer at the merged flow path. In this problem, steam and water are introduced in 3.628 kg/sec and 24 kg/sec at a pressure

0.0242 MPa and 0.027 MPa with pipe flow areas are 0.34m^2 and 0.018m^2, respectively. Similar to the previous problem, three cases of the combination of hydraulic resistances are compared. One can find that the hydraulic resistances that satisfies all constraints for the two-phase flow merging and heat transfer can be obtained using the method of the present study. Although there is a point having a slightly lower pressure than the pressure at the outlet boundary, the difference is 0.0002



MPa at most, and it is expected to be improved by applying the heat flux boundary condition in detail.

Fig. 8. Calculated hydraulic resistances for case of two-phase flow merging and heat transfer.

5. Summary and Conclusions

The present paper discussed a method to determine the hydraulic resistances in flow networks with division and merger, in both single-phase flow and two-phase flow.

For single-phase flow, equations for the hydraulic resistances were theoretically derived from the Bernoulli equations and their dimensionless form can be obtained. For two-phase flow, however, they should be determined by adjusting the code calculation result due to difficulty in theoretical approach.

The present model calculates the possible combination of hydraulic resistances of dividing or merging flow paths from the constraints such as flow properties, flow dividing or merging ratio. In the present method, among the combinations of hydraulic resistances with a positive value, those that cause a non-monotonous pressure distribution through the code calculation are excluded.

From the MARS-KS code calculations using the hydraulic resistances determined by the present method, the flow dividing ratio and merging ratio can be achieved without non-monotonous pressure distribution under the various conditions of upstream and downstream.

The method is expected to contribute to solve an inverse problem related to the installation of nuclear renewable hybrid energy system.

ACKNOWLEDGEMENT

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NOMENCLATURE

A	Area
D	Diameter of pipe
Eu	Euler number in equation (11)
f	Friction factor
g	gravitational acceleration
h	enthalpy
L	Length of pipe
K	Hydraulic resistance
k	Form loss factor
M_{ij}	Coefficient of equations (5), (7)
Nz	Elevation non-dimensional number in equation (11)
P	Pressure with elevation ($p_i + \rho_i g z_i$)
p	Static pressure
p_{di}	Dynamic pressure at i ($\frac{1}{2}\rho_i {v_i}^2$)
q	heat flux
v	velocity
z	elevation
δP_{ij}	Differential pressure between i and j
φ_1	flow dividing/merging ratio at path 1
ρ	density

Superscripts

*	Nondimensional parameters
‘	properties assumed zero hydraulic resistance

Subscripts

a	Point for dividing/merging
f	liquid
g	gas
H	related to heat transfer
1, 2, 3	Points for upstream and downstream
1a, 2a	flow path from 1 to a, 2 to a
a2, a3	flow path from a to 2, a to 3