Transactions of the Korean Nuclear Society Autumn Meeting
Changwon, Korea, October 20-21, 2022

Intensive Review on the Bird Suction Factor for the Condensation of Air-steam Mixture

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1. Introduction

Condensation of steam on the cold surface in the presence of air, a kind of non-condensable gas, has been a long interest of thermal hydraulic area. It is, in particular, important phenomena in the nuclear containment, and become more important in case of passive containment cooling system of recent prevalence. So many studies have been done, and so many theories have been introduced. Among them, so-called 'Bird suction factor' or simply 'suction factor' is one of the most sophistic term which explains the effect of mass generation or depletion on the evaporative/ condensive heat transfer (Bird et al., 2002). So the modification factor considering such effect has been frequently used in the nuclear thermal hydraulic codes such as CONTEMPT-LT and GOTHIC (Hargroves et al., 1979, Rahn, 2004). And this theory is also introduced in the discussion of condensation phenomena in Collier et al. (1994).

In spite of that, the derivation process of the suction factor looks not so simple and not so sound, although Bird et al.(2002) provides the outline of the derivation process. Recently, Lee, J.H. et al. (2015) pointed out the ambiguity of the density expression in the condensing mass flux in Collier et al.(1994).

This paper discusses the derivation process of the suction factor and relevant issues. The symbols in each equation follow the Bird et al. (2002)'s

2. Mass transfer in nonstationary media

Mass diffusion of species A is given by Fick's law.

$$\mathbf{j}_A = -\rho D_{AB} \nabla \omega_A \tag{1}$$

Here, $\mathbf{j}_A$ is mass flux relative to the mixture velocity.

$$\mathbf{j}_A = \rho(\mathbf{v}_A - \mathbf{v}) \tag{2}$$

So, the absolute mass flux or combined mass flux, which consider the convective motion is defined as

$$\mathbf{n}_A = \mathbf{j}_A + \rho_A \mathbf{v} \tag{3}$$

By the definition of mixture velocity and Fick's law, above equation leads to

$$\mathbf{n}_A = -\rho D_{AB} \nabla \omega_A + \omega_A(\mathbf{n}_A + \mathbf{n}_B) \tag{4}$$

Corresponding molar expression is easily obtained in the similar way.

$$\mathbf{N}_A = -c D_{AB} \nabla x_A + x_A(\mathbf{N}_A + \mathbf{N}_B) \tag{5}$$

3. Expressions for Conservation

Continuity equation can be expressed as followings for mass and molar form, respectively.

$$\frac{\partial}{\partial t}(\rho_\alpha) = -(\nabla \cdot \mathbf{n}_\alpha) + r_\alpha \tag{6}$$

$$\frac{\partial}{\partial t}(c_\alpha) = -(\nabla \cdot \mathbf{N}_\alpha) + R_\alpha \tag{7}$$

, where r_α and R_α are generation rate of α in the form of mass and mole per unit time and unit volume, respectively.

For energy,

$$\frac{\partial}{\partial t}\rho\left(\hat{U} + \frac{1}{2}v^2\right) = -(\nabla \cdot \mathbf{e}) + (\rho \mathbf{v} \cdot \mathbf{g}) \tag{8}$$

4. Profile near the condensing boundary layer

Let's consider the situation that a condensable gas species A in the presence of the non-condensable gas species B is moving toward cold surface with being condensed as shown in Fig. 1

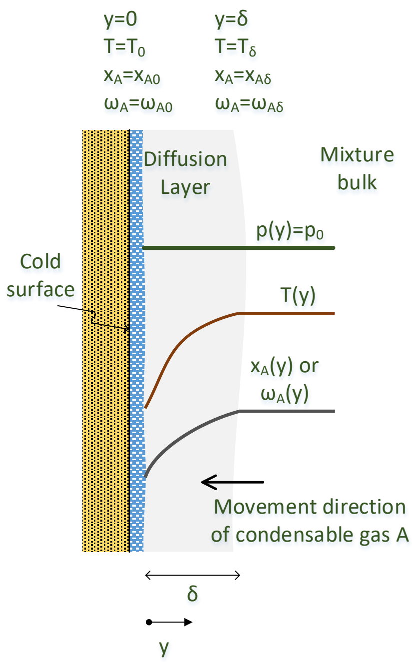

Fig. 1. Condensation of gas A on the cold surface in the presence of non-condensable gas B

Conservation equations in section 3 can be applied to Fig. 1 in steady-state and one-dimensional form,

$$\frac{dn_A}{dy} = 0 \tag{9}$$

$$\frac{dN_A}{dy} = 0 \tag{10}$$

$$\frac{de}{dy} = 0 \tag{11}$$

4.1. Absolute mass flux and mole flux

Eqs. (4) and (5) can be applied in one-dimensional form to obtain the flux. Note that the gas B doesn't move, and the flux of gas A is constant because of continuity.

$$n_A = -\rho D_{AB}\frac{d\omega_A}{dy} + \omega_A n_A \tag{12}$$

$$N_A = -cD_{AB}\frac{dx_A}{dy} + x_A N_A \tag{13}$$

With the boundary condition of ω_{A0}, x_{A0} (at $y = 0$) and $\omega_{A\delta}, x_{A\delta}$ (at $y = \delta$) and with the fact of uniform gas A flux, above equations can be solved by the aid of Eqs. (9) and (10), assuming transport property and densities are constant.

$$\frac{1-\omega_A}{1-\omega_{A0}} = \left(\frac{1-\omega_{A\delta}}{1-\omega_{A0}}\right)^{y/\delta} \tag{14}$$

$$\frac{1-x_A}{1-x_{A0}} = \left(\frac{1-x_{A\delta}}{1-x_{A0}}\right)^{y/\delta} \tag{15}$$

These two results can be more developed by inserting into Eqs. (12) and (13) to get the mass flux and mole flux.

$$n_{Ay} = \frac{\rho D_{AB}}{\delta}\ln\left(\frac{1-\omega_{A\delta}}{1-\omega_{A0}}\right) \tag{16}$$

$$N_{Ay} = \frac{cD_{AB}}{\delta}\ln\left(\frac{1-x_{A\delta}}{1-x_{A0}}\right) \tag{17}$$

Here the two fluxes must be negative because the gas A fraction at y=0 is smaller than at y=δ, and this is identical to the observation in Fig. 1.

Using Eqs. (14), (15), (16), and (17) the profile of mass fraction or mole fraction can be expressed as followings.

$$\frac{\omega_A-\omega_{A0}}{\omega_{A\delta}-\omega_{A0}} = \frac{1-\exp\left(\frac{n_{Ay}}{\rho D_{AB}}y\right)}{1-\exp\left(\frac{n_{Ay}}{\rho D_{AB}}\delta\right)} \tag{18}$$

$$\frac{x_A-x_{A0}}{x_{A\delta}-x_{A0}} = \frac{1-\exp\left(\frac{N_{Ay}}{cD_{AB}}y\right)}{1-\exp\left(\frac{N_{Ay}}{cD_{AB}}\delta\right)} \tag{19}$$

4.2. Temperature profile in diffusion layer

The energy flux is given by following relation that it is composed of the conductive flux and convective flux (Gas B is stagnant.).

$$\begin{aligned} e_y &= -k\frac{dT}{dy} + \left(\tilde{H}_A N_{Ay} + \tilde{H}_B N_{By}\right) \\ &= -k\frac{dT}{dy} + N_{Ay}\tilde{C}_{pA}(T-T_0) \end{aligned} \tag{20}$$

$$e_y = -k\frac{dT}{dy} + n_{Ay}C_{pA}(T-T_0) \tag{21}$$

Insertion of each equation into Eq. (11) and integration between the limits $T = T_0$ at y = 0, and $T = T_\delta$ at y = δ gives

$$\frac{T(y)-T_0}{T_\delta-T_0} = \frac{1-\exp\left(\frac{n_{Ay}C_{pA}}{k}y\right)}{1-\exp\left(\frac{n_{Ay}C_{pA}}{k}\delta\right)} \tag{22}$$

$$\frac{T(y)-T_0}{T_\delta-T_0} = \frac{1-\exp\left(\frac{N_{Ay}\tilde{C}_{pA}}{k}y\right)}{1-\exp\left(\frac{N_{Ay}\tilde{C}_{pA}}{k}\delta\right)} \tag{23}$$

These equations have analogous form with Eqs. (18) and (19).

4.3. Comments on the derivation

4.3.1. Mass and mole

As is seen in the above derivation the flux and profiles can be obtained from mass based equation and mole based equation, and the results show very similar or analogous form (See Eqs. (14) & (15), Eqs. (16) & (17), Eqs. (18) & (19), and Eqs. (22) and (23)).

Eq. (17) can be transformed into mass flux from the fact that $n_{Ay} = M_A N_{Ay}$ and $\rho = \rho_A + \rho_B = M_A c_A + M_B c_B = c(M_A x_A + M_B x_B)$

$$n_{Ay} = \frac{D_{AB}}{\delta}\rho\frac{M_A}{(M_A x_A + M_B x_B)}\ln\left(\frac{1-x_{A\delta}}{1-x_{A0}}\right) \tag{24}$$

Or the other form can be obtained using $\rho_A = M_A c_A = M_A(cx_A)$, thus, $c = \rho_A/(M_A x_A)$.

$$n_{Ay} = \frac{D_{AB}}{\delta}\frac{\rho_A}{x_A}\ln\left(\frac{1-x_{A\delta}}{1-x_{A0}}\right) \tag{25}$$

These two equations look different, but both are identical after following manipulation

$$\frac{\rho_A}{\rho} = \left(\frac{M_A c_A}{M_A c_A + M_B c_B}\right) = \left(\frac{M_A x_A}{M_A x_A + M_B x_B}\right) \tag{26}$$

Here, will Eq. (16) and Eq. (25) (or Eq. (24) be really same each other?

4.3.2. Temperature profile and mass/mole profile

Eqs. (22) and (23) are believed the temperature profile for the same diffusion layer, but just the starting governing equations are different: one is based on mole parameter, the other on mass parameter. Will both equations generate the same temperature profile?

It is believed that both equations will usually generate different temperature profile. And mass or mole distribution of Eqs (16) & (17) and Eqs. (18) & (19) are also believed to generate different profile.

A past study on the similar subject was carried out by Lee, J.H. et al. (2015). They studies on the condensation flux in several thermal hydraulic codes such as MARS-KS, RELAP5, MELCOR, CFX, and TRACE. Here MARS-KS and RELAP5 codes had some bug in the codding, which is now corrected and will be discussed in detail in following subsection, so that the flux related functions were far different from the other codes as shown in Fig. 2. Among these codes, MELCOR and CFX are based on the mole properties and TRACE on the mass properties, respectively. For dilute steam, there were just a little difference, but for dense steam relatively large difference was revealed. This result is identical to the insistence of this study that the profiles are different according to mass base or mole base.

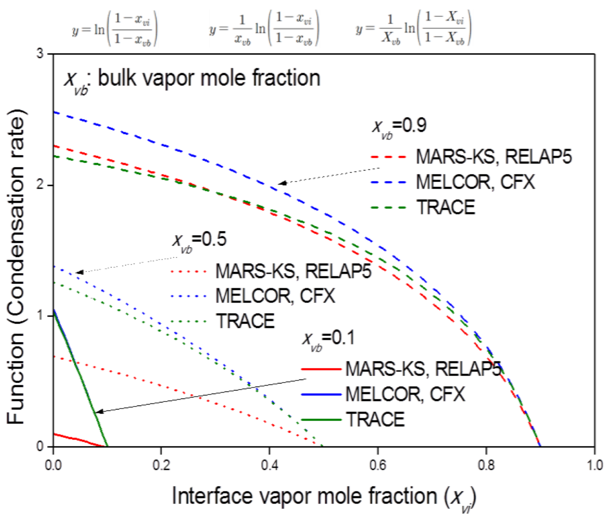

Fig. 2. Condensation flux related function for several codes (Lee, J.H. et al., 2015)

Then why do these derivations result in different profile? It may partially be caused by the fact that the gas mixture mass density (ρ) and mole density (c) over the diffusion layer were assumed constant during the integration. These two parameters must not be constant because the temperature in this diffusion layer is never constant as shown in Fig. 1 and Eqs. (22) & (23). The different temperature would generate different mass density and mole density according to the equation of state.

By the way, Bird et al. (2002) and Incropera et al. (2011) discussed anther case of mas transfer: column evaporation. The evaporative flux of mole form is similarly derived like this study. But there is no mention on the gas temperature profile. And uniform temperature seems assumed and resultantly a constant density assumption is valid. However, looking at the derivation process, the mass flux relation can be similarly obtained, but the two solutions look different. This means that the constant assumption in the derivation in this study or Bird et al. (2004) is not all of the contribution to the difference in mass form and mole form.

4.3.3 Comments on the Collier et al.(1994)'s derivation

<u>Flux direction</u>

As discussed above subsection, the direction of absolute flux of mas or mole is negative, i.e., left-ward. But Eq. (10.37) or Eq. (10.38) has positive sign. It goes against overall discussions of the section in Collier et al.(1994). It may be caused by the positive sign for the diffusion term of Fick's law in governing equation, Eq. (10.33). Eqs. (4) & (5) in this study specify negative sign for diffusion term of Fick's law.

<u>Density in mass flux equation</u>

Collier's derivation result for mole flux is identical to this study except the sign. However, the mass flux of Eq. (10.38) has ambiguous notation for density, which is shown as ρ_g. It is believed that 'diving by mole fraction of gas g' may be omitted if ρ_g is mass density of gas g. This fact was previous pointed out by Lee, J.H. et al. (2015).

<u>Energy flux</u>

The wall is cold and the bulk is hot so that the heat flux is left-ward, i.e. negative sign, and this is well matched with Eqs. (20) & (21). Reviewing Collier et al. (1994), the first term in right hand side of Eq. (10.41) must be positive and the second term is also positive because the mass flux is positive as discussed above. So, the heat flux of left hand side is positive. It is contradictory to this situation but the consistency of the direction of mass flux and heat flux is maintained.

5. Suction factor

5.1.Derivation of suction factor

Suction factor means the heat transfer promotion factor by means of the mass transfer compared to pure heat transfer case. The heat flux with mass transfer at y=0 can be obtained from Eq. (22) and (23).

$$-k\left(\frac{dT}{dy}\right)_{y=0} = \frac{k}{\delta}(T_\delta - T_0)\frac{\tilde{a}}{1-e^{\tilde{a}}}$$

(27)

$$-k\left(\frac{dT}{dy}\right)_{y=0} = \frac{k}{\delta}(T_\delta - T_0)\frac{a}{1-e^{a}}$$

(28)

The newly introduced parameters are defined as followings, and these parameters must be negative.

$$\tilde{a} = \frac{N_{Ay}\tilde{C}_{pA}}{k}\delta$$

(29)

$$a = \frac{n_{Ay}C_{pA}}{k}\delta$$

(30)

The heat flux when there is no mass transfer can be derived from Eqs. (21) & (22) by letting $N_{Ay} = 0$ and $n_{Ay} = 0$.

$$\frac{T - T_0}{T_\delta - T_0} = \frac{y}{\delta}$$

(31)

This means the temperature profile is linear when there is no mass transfer, and it is identical to the conduction problem.

The heat flux at y=0 is given as following with the distinguishing notation superscript 0.

$$-k\left(\frac{dT}{dy}\right)_{y=0}^{0} = -k\frac{T_\delta - T_0}{\delta}$$

(32)

The heat flux ratio between two is give as

$$\frac{-k\left(\frac{dT}{dy}\right)\Big|_{y=0}}{-k\left(\frac{dT}{dy}\right)^{0}\Big|_{y=0}} = \frac{\tilde{a}}{e^{\tilde{a}} - 1}$$

(33)

$$\frac{-k\left(\frac{dT}{dy}\right)\Big|_{y=0}}{-k\left(\frac{dT}{dy}\right)^{0}\Big|_{y=0}} = \frac{a}{e^{a} - 1}$$

(34)

The denominators are negative because a and $\tilde{a}$ are negative, and these suction factors are positive.

Another derivation method is as follows. From Eq. (22)

$$\frac{dT}{dy} = \left(\frac{T_\delta - T_0}{1 - e^{\tilde{a}}}\right)\left(-\frac{\tilde{a}}{\delta}\right)\exp\left(\tilde{a}\frac{y}{\delta}\right)$$

(35)

Putting it into Eq. (20) results in

$$e_y = \tilde{a}\left(\frac{T_\delta - T_0}{1 - e^{\tilde{a}}}\right)h\exp\left(\tilde{a}\frac{y}{\delta}\right) + \tilde{a}h(T - T_0)$$

(36)

Integration between $y = 0$, and $y = \delta$ and collaborate arrangement with $h = D_{AB}/\delta$ gives

$$e_y = \left(\frac{\tilde{a}}{e^{\tilde{a}} - 1}\right)[-h(T_\delta - T_0)]$$

(37)

In the similar way, following equation will be derived, for the mass based equation.

$$e_y = \left(\frac{a}{e^{a} - 1}\right)[-h(T_\delta - T_0)]$$

(38)

The expressions in Eqs. (37) and (38) are mutually similar to Eqs. (33) & (34), or more extended up to convective heat transfer.

The graphical interpretation of suction factor is shown in Fig. 3 (red line). For Eqs. (29) and (31) the graph shows monotonous increasing trend, and there is a singular point at x=0 (a=0 or $\tilde{a} = 0$). And for positive value of x, the suction factor is larger than 1, and it means that the heat transfer is enhanced by the mass transfer.

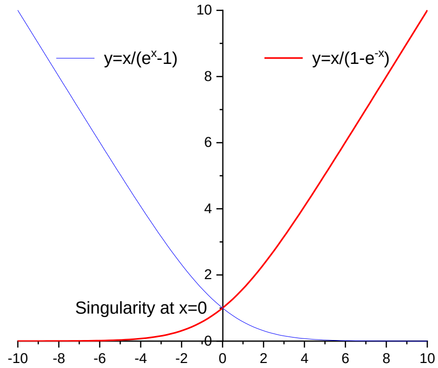

Fig. 3. Graphical interpretation of suction factor

5.2. Comments on the derivation

In Collier et al.(1994), the suction factor shows slightly different form.

$$\frac{a}{1 - e^{-a}}$$

(39)

Here ‘a’ is defined as

$$a = \frac{j_g c_{pg}}{h}$$

(40)

This difference is caused by the governing equation as discussed in subsection 4.3.3. The mass flux, j_g, is actually negative, and the suction factor is larger than 1 as shown in blue line of Fig. 3.

But the sign of mass flux, j_g, looks conflicting with the previous derivation process in Collier et al..

6. Conclusion

This study reviews the detailed derivation process of suction factor in Bird et al. (2002) and Collier et al. (1994). Detailed processes were checked and mathematical stringency was also checked. Findings can be summarized as follows:

1) Mathematical robustness was found out weaken in the integration over the diffusion layer since

the mass density and mole density are not constant, but they were assumed constant.

2) The obtained mass profile and temperature derived from mass equation and mole equation seem different each other. It may be partially caused by the assumption of constant density over the diffusion layer. But it is not all because the same problem arises for the column evaporation where the constant temperature and resultantly constant density are maintained.
3) The derivation in Collier et al. (1994) has some confusing aspect because the sign of the flux is not matched with the coordinate or Bird et al. (2002).
4) The density in the condensing mass flux equation in Collier et al. (1994) has some ambiguity or typo.
5) The suction factor form in Collier et al. (1994) has slightly different form. And it may be caused by the confusing sign of the flux.
6) Suction factor has a singular point at zero, and the value is smaller or larger than one according to the sign of independent variable ‘a’. Smaller value may be corresponding to evaporation and larger value to condensation.

This study is expected to clarify the ambiguity in condensation formulation in the presence of non-condensable gas.

6. Acknowledgment

This work was supported by Korea Institute of Energy Technology Evaluation and Planning (KETEP) grant funded by the Korea government (MOTIE) (20217810100020, Development of Unique Thermal Hydraulic Analysis System of Containment Building Based on SPACE for APR Nuclear Power Plant).

NOMENCLATURES

c = mole density, $c_A + c_B$ $[\mathrm{kmol/m^3}]$
C_p = specific heat, $[\mathrm{kJ/kg \cdot K}]$
$\tilde{C}_p$ = mole specific heat, $= C_p/M$, $[\mathrm{kJ/kmol \cdot K}]$
D_{AB} = Binary diffusion coefficient, $[\mathrm{m^2/s}]$
$\mathbf{e}$ = energy mass flux, $[\mathrm{kg/m^2 s}]$
$\mathbf{g}$ = gravity, $[\mathrm{m/s^2}]$
$\tilde{H}$ = mole specific enthalpy, $= Mh$, $[\mathrm{kJ/kmol}]$
h = specific enthalpy, $[\mathrm{kJ/kg}]$
$\mathbf{j}$ = mass flux relative to mixture velocity, $[\mathrm{kg/m^2 s}]$
M = mole weight, $[\mathrm{kg/kmol}]$
$\mathbf{N}$ = absolute mole flux, $[\mathrm{kmol/m^2 s}]$
$\mathbf{n}$ = absolute mass flux, $[\mathrm{kg/m^2 s}]$
p = pressure, [Pa]
T = temperature, [K]

$\hat{U}$ = specific internal energy, $[\mathrm{kJ/kg}]$
$\mathbf{v}$ = mixture velocity, $\mathbf{v} = \omega_A \mathbf{v}_A + \omega_B \mathbf{v}_B$, $[\mathrm{m/s}]$
x_α = mole fraction, $x_\alpha = c_\alpha / c, [-]$
y = coordinate, $[\mathrm{m}]$
ρ = mass density, $= \rho_A + \rho_B$, $[\mathrm{kg/m^3}]$
ω_α = mass fraction, $\omega_\alpha = \rho_\alpha / \rho$, $[-]$

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