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Preliminary Study on Location Selection for Emergency Response Using P-Median Algorithm

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1. Introduction

In the event of a nuclear accident, appropriate protection measures play an important role in minimizing damage to residents around nuclear facilities. There are four main ways to protect residents in the early stages of the nuclear accident: sheltering, evacuation, food distribution and intake control, and thyroid protection [1]. One of the technical concerns related with these ways may be how and where to decide the locations for shelters, supply storages, control centers, and so on. From the mathematical viewpoint, this belongs to graph theory, clustering, and optimization. It should be noted that these places should be a spot where everyone knows and public transportation is easy to access, which can be regarded as special constraints in the optimization problem. We found the P-Median algorithm would effectively make the location of the assembly places easier for residents to access at the shortest distance. In this study, the P-Median algorithm was demonstrated in two cases under certain conditions.

2. Method

The places for emergency response should be positioned where people can access and effectively include capacity(i.e., emergency supplies). One way to measure the efficiency and effectiveness of places for emergency response is by evaluating the average distance between the customers and the places. When the average distance decreases, the accessibility of the places increase and average response times decrease. This is known as the P-Median problem, which was introduces by Hakimi [2]. The P-Median algorithm has been used to find locations to minimize average distance, average travel time, and average travel cost of consumers using facilities [3]. This study attempted to optimize the places for emergency response based on the geographical data containing information on population number, latitude, and longitude. Considering the constraints such as awareness or capacity, finding the optimal location of the places corresponds to the category of linear programming and the P-Median algorithm have been facilitating these technical concerns [3].

2.1. P-Median Algorithm

The P-Median algorithm which is a method of finding the optimal location of the supply location according to the demand of consumers helps solve various location problems such as electric vehicle charging stations, bicycle parking lots, and temporary COVID-19 screening stations. Assuming that P-Median has a candidate location for installation of public facilities, each candidate location represents a consumer demand-producing area, and that each consumer is given a transportation cost and transportation distance per unit, it is a problem of determining the location of p facilities that can meet the needs of all consumers at a minimum transportation cost [3]. The basic model of the P-Median algorithm in this study is as follows:

Inputs:

$$d_{ij} = \text{shortest distance from node } i \text{ to node } j \quad (1)$$
$$h_i = \text{population demand at node } i \quad (2)$$
$$p = \text{the number of facilities that are to be located.} \quad (3)$$

Objective Function:

$$Y^* = arg \min_Y f(h_i,\ d_{ij},\ y_{ij}) \quad (4)$$
$$f(h_i,\ d_{ij},\ y_{ij}) = \sum_i \sum_j h_i d_{ij} y_{ij} \quad (5)$$
$$Y = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1j} \\ y_{21} & y_{22} & \cdots & y_{2j} \\ \vdots & \vdots & \ddots & \vdots \\ y_{i1} & y_{i2} & \cdots & y_{ij} \end{bmatrix} \quad (6)$$

Constraints:

$$\sum_j y_{ij} = 1 \ \text{(for all } i\text{)} \quad (7)$$
$$\sum_j x_j = p \quad (8)$$
$$y_{ij} \le x_j \ \text{(for all } i,\ j\text{)} \quad (9)$$
$$y_{ij} \in 0,\ 1 \ \text{(for all } i,\ j\text{)} \quad (10)$$
$$x_j \in 0,\ 1 \ \text{(for all } j\text{)} \quad (11)$$

Decision variables:

$x_j = 1, if\ facility\ located\ at\ j\ 0, otherwise$
$y_{ij} = 1, if\ demand\ node\ i\ assigned\ to\ facility\ located\ at\ j\ 0, otherwise$

The objective function shown in Equation (4) means that the P-Median algorithm finds Y with a minimum value when the function f is located at the location point j of the facility. Equation (5) represents the total distance between the facility and the consumer. d_{ij} means the shortest distance between the facility and the consumer. Considering the total distance, the width of the road and the number of the people can be formulated in h_i. In this study, h_i was only considered as a residential population. In Equation (6), Y is a matrix consisting of y_{ij}. Y, whose value of y_{ij} is 0 or 1, finds the minimum value of the objective function. Constraint (7) indicates that the point of demand must be serviced by one facility, and that there is no overlapping service or service absence area. Constraint (8) indicates that the number of areas serviced by facilities in one's area is equal to the number of facilities. Constraint (9) indicates that if $x_j = 0$, there are no facilities in area j and users in area i are not assigned to facilities in area j, so $y_{ij} = 0$ and if $x_j = 1$, it means that there are facilities in area j, so y_{ij} has a value of 0 or 1. Constraint (10), (11) indicate that the decision variables x_j and y_{ij} have a value of 0 or 1 [3].

2.2. Example problem

This is an example problem in which an arbitrary distance consisting of five nodes is set.

Table I: Distance between node i and node j

Node	**1**	**2**	**3**	**4**	**5**
1	*0*	*1*	*1*	*2*	*2*
2	*1*	*0*	*1*	*2*	*2*
3	*1*	*1*	*0*	*1*	*1*
4	*2*	*2*	*1*	*0*	*1*
5	*2*	*2*	*1*	*1*	*0*

Table I represents the distance of each of the five nodes. Distance information is stored in a two-dimensional list called length. Fig.1 shows the result at $p = 1$. The first factor of each term is the $h_i d_{ij}$, and the second is y_{ij}. The last line means optimal solution. When the facility has a location point $j = 3$ ($y_{i3} = 1$), the sum of all terms becomes 4 and has a minimum value. Fig. 2 shows the case of weighting node 4. Unlike Fig. 1, the optimal solution has a minimum value when $j = 4$ ($y_{i4} = 1$). Therefore, the facility should be located at node 4.

```
print("min f = ")
for i in range(len(length)):
    print("i = " + str(i+1))
    for j in range(len(length)):
        print("j = " + str(j+1) + ": [" + str(D[i][j]) + " * " + str(int(value(X[i][j]))), end="]  ")
    print()
    print()
print("sum = " + str(value(prob.objective)))
✓ 0.3s
min f =
i = 1
j = 1: [0 * 0]  j = 2: [1 * 0]  j = 3: [1 * 1]  j = 4: [2 * 0]  j = 5: [2 * 0]

i = 2
j = 1: [1 * 0]  j = 2: [0 * 0]  j = 3: [1 * 1]  j = 4: [2 * 0]  j = 5: [2 * 0]

i = 3
j = 1: [1 * 0]  j = 2: [1 * 0]  j = 3: [0 * 1]  j = 4: [1 * 0]  j = 5: [1 * 0]

i = 4
j = 1: [2 * 0]  j = 2: [2 * 0]  j = 3: [1 * 1]  j = 4: [0 * 0]  j = 5: [1 * 0]

i = 5
j = 1: [2 * 0]  j = 2: [2 * 0]  j = 3: [1 * 1]  j = 4: [1 * 0]  j = 5: [0 * 0]

sum = 4.0
```

Fig. 1. Results of example at $p = 1$. (The first factor of each term: $h_i d_{ij}$, The second factor of each term: y_{ij}).

```
print("min f = ")
for i in range(len(length)):
    print("i = " + str(i+1))
    for j in range(len(length)):
        print("j = " + str(j+1) + ": [" + str(D[i][j]) + " * " + str(int(value(X[i][j]))), end="]  ")
    print()
    print()
print("sum = " + str(value(prob.objective)))
✓ 0.3s
min f =
i = 1
j = 1: [0 * 0]  j = 2: [1 * 0]  j = 3: [1 * 0]  j = 4: [2 * 1]  j = 5: [2 * 0]

i = 2
j = 1: [1 * 0]  j = 2: [0 * 0]  j = 3: [1 * 0]  j = 4: [2 * 1]  j = 5: [2 * 0]

i = 3
j = 1: [1 * 0]  j = 2: [1 * 0]  j = 3: [0 * 0]  j = 4: [1 * 1]  j = 5: [1 * 0]

i = 4
j = 1: [8 * 0]  j = 2: [8 * 0]  j = 3: [4 * 0]  j = 4: [0 * 1]  j = 5: [4 * 0]

i = 5
j = 1: [2 * 0]  j = 2: [2 * 0]  j = 3: [1 * 0]  j = 4: [1 * 1]  j = 5: [0 * 0]

sum = 6.0
```

Fig. 2. Results of example when node 4 was weighted.

2.3. Data Setting

In order to obtain the d_{ij} of the P-Median, we obtain the direct distance of the j node from the i node. Nodes are based on resident population datasets. Among the EPZ, nodes within a 5 km radius of the Kori nuclear site corresponding to the Precautionary Action Zone(PAZ) are used. Fig. 3 shows Ulju-gun nodes within 5 km of the Kori nuclear site. The haversine module was imported to calculate d_{ij} by using DataFrame(Fig. 4) including latitude and longitude information. d_{ij} becomes a 177x177 matrix. In order to multiply d_{ij} by h_i, the number of residents was selected as a weight in this study. In the case of the 'current population number,' it is appropriate to stop by during the purposeful passage and visit. The place in the event of a nuclear accident corresponds to the destination to gather. Therefore, the 'number of residents' is appropriate. By weight, p optimal positions are distributed to places with more people.

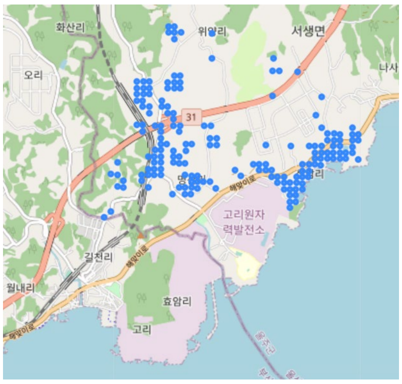

Fig. 3. Configuring d_{ij} nodes within a 5 km radius of the Kori nuclear site.

	MEGA_NM	MEGA_CD	CTY_NM	CTY_CD	X_AXIS	Y_AXIS	HOUS	POP	POP_10	POP_20	POP_30	POP_40	POP_50	POP_60_O	배포처	latitude	longitude
0	울산광역시	31	울주군	31710	519450	306150	3.28	9.12	1.32	1.16	0.66	1.68	2.12	2.18	biz-gis.com	35.347398	129.311987
1	울산광역시	31	울주군	31710	518750	305850	1.64	4.56	0.66	0.58	0.33	0.84	1.06	1.09	biz-gis.com	35.344777	129.304244
2	울산광역시	31	울주군	31710	518050	307450	1.5	4.13	0.65	0.39	0.34	0.62	0.99	1.14	biz-gis.com	35.359279	129.296776
3	울산광역시	31	울주군	31710	517650	307050	10.5	28.91	4.55	2.73	2.38	4.34	6.93	7.98	biz-gis.com	35.355722	129.292318
4	울산광역시	31	울주군	31710	517550	307350	1.5	4.13	0.65	0.39	0.34	0.62	0.99	1.14	biz-gis.com	35.358437	129.291261
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
172	울산광역시	31	울주군	31710	519850	307650	1.5	4.06	0.53	0.24	0.25	0.74	0.8	1.5	biz-gis.com	35.360867	129.316607
173	울산광역시	31	울주군	31710	520850	306550	5.25	14.88	1.11	2.31	1.2	2.55	3.93	3.78	biz-gis.com	35.350834	129.327445
174	울산광역시	31	울주군	31710	518150	306850	3.0	8.26	1.3	0.78	0.68	1.24	1.98	2.28	biz-gis.com	35.35386	129.297789
175	울산광역시	31	울주군	31710	518150	308250	4.5	12.39	1.95	1.17	1.02	1.86	2.97	3.42	biz-gis.com	35.366477	129.297991
176	울산광역시	31	울주군	31710	518750	306750	3.0	8.26	1.3	0.78	0.68	1.24	1.98	2.28	biz-gis.com	35.352888	129.304375

Fig. 4. DataFrame of d_{ij} nodes within a 5 km radius of the Kori nuclear site.

3. Results

Python was used for the program, and it was selected to use the PuLP module to obtain the optimal solution of the objective function. The number of places was chosen by 9 as a preliminary study. Visual studio code is used as IDE, and the computer's performance is CPU: i7-12700KF, Memory: 32 GB. In all cases PuLP.solve() took 7 seconds on average. The P-Median algorithm was used in two cases: (1) optimal location considering the number of people, and (2) optimal location considering senior people over 60s. The key factor in P-Median algorithm is to setup the weighting function, *h*, such that meaningful constraints can be applied.

Case 1. Optimal locations considering the number of people.

In Fig. 5, hav_len corresponds to Equation 1, and df_node does to Equation 2. Since *n* is 177, which is the number of all nodes, weight_length is calculated by multiplying the 177x177 array by the number of people corresponding to each node. Fig. 6 shows the population-weighted optimal location. Because the resident population is weighted, it can be seen that the results are concentrated in places where many people stay.

```
weight_length = [[0] * (n) for _ in range(n)]
for i in range(n):
    for j in range(n):
        weight_length[i][j] = hav_len[i][j] * df_node['POP'][i]
```

Fig. 5. Weighting Algorithm for Residential Population.

Fig. 6. Optimal location with a weighted number of people.

Case 2. Optimal locations a place considering senior people.

Fig. 7 shows that the P-Median algorithm ran by weighing 100 times where there are more than 10 senior people over 60s, which mean higher transportation cost for seniors. Obviously, the weighting factor 100 can be adjusted depending on a specific problem. According to the P-Median algorithm, the optimal location occurs on the node where *h* is weighted. In Fig. 8, it can be seen that optimal locations are moved to the places where the population in their 60s is concentrated.

```
weight_length = [[0] * (n) for _ in range(n)]
for i in range(n):
    for j in range(n):
        if df_node['POP_60_O'][i] > 10:
            weight_length[i][j] = hav_len[i][j] * df_node['POP'][i] * 100
        else:
            weight_length[i][j] = hav_len[i][j] * df_node['POP'][i]
```

Fig. 7. Weighting Algorithm when there are more than 10 senior people.

Fig. 8. Optimal location with more than 10 people in their 60s and older.

5. Conclusion

In this study, the places for emergency response were searched by running the P-Median algorithm based on the residential population data. The performance of the P-Median algorithm was demonstrated with some case studies. In case of emergency response, it could be said that the material that ensures people's safety is provided by optimizing the locations of shells, supply stores, and control centers. It is expected that the algorithm can be improved if the weight can be formulated such as road conditions, age distribution, or supply policies. These are on going and will be further developed in the next phase.

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