Uncertainty analysis for frequency of heat pipe cascading failure event in micro modular reactor

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1. Introduction

As the attention for micro modular reactor increased, various types of reactor design have been developed in recent years. Especially, heat pipe is widely considered as core cooling system in micro modular reactors. Even though heat pipe is expected to have high reliability, there is a potential for heat pipe cooling system to lose their ability due to cascading failure of heat pipes. Multiple failure of heat pipes due to cascading effect can threaten the integrity of reactor core, and hence it should be analyzed as initiating event in probabilistic safety assessment (PSA) for heat pipe cooled reactor.

Heat pipe is passive heat transfer device, and the performance depends on assigned thermal load. In the heat pipe cooling system, all the heat pipes share total thermal load and therefore, the reliability of the system is analyzed by load sharing model. Because of load redistribution property in load sharing model, time to failure distribution of the system depends on failure scenario. Therefore, the frequency of heat pipe cascading failure event is represented by combination of occurrence probabilities and failure times of all the failure scenarios. However, it is practically impossible to calculate occurrence probability of each scenario and hence it is estimated by the results of simulation. Thus, the estimated occurrence probabilities have uncertainties, and the uncertainties are propagated to estimated frequency of cascading failure event.

In this paper, the uncertainties of estimated occurrence probability are introduced. And then, the result of estimated frequency of cascading failure is shown with an example heat pipe cooling system in micro modular reactor.

2. Methods and Results

In this section, time to failure distribution for load sharing model is introduced to estimate frequency of heat pipe cascading failure event. And then, the uncertainty of estimated frequency is analyzed in accordance with uncertainties of parameters in the load sharing model.

2.1 Load sharing system

Load sharing system is a redundant system that components of the system share total load and the reliability of each component depends on assigned load. If some components are failed in load sharing system, the loads of failed components are distributed to remaining components. Therefore, the reliabilities of components in the load sharing system have interdependence. In the load sharing model, it is generally assumed that the components in the system are independently failed one by one. In each failure step, the load of failed component is distributed, and the failure rates of remaining components are changed as a result of load distribution. Then, the time to cascading failure is the sum of independent exponential random variables which have total failure rate of the system in each step as a parameter. Consequently, the time to cascading failure of each failure scenario follows hypoexponential distribution.

If the Aalen's additive hazard model [1] is applied, the failure scenarios can be categorized with respect to the number of failed components until cascading failure is occurred. Then, the time to cascading failure distribution which reflects all the failure scenarios can be obtained by marginalizing over all the failure scenarios.

$$\mathrm{P}(T \leq t) = \sum_{i=1}^{N} \mathrm{P}(X = i)\,\mathrm{P}(T \leq t | X = i) \qquad (1)$$

Therefore, the time to cascading failure follows finite mixture of hypoexponential distributions.

2.2 Uncertainty of the number of failure scenarios

If the occurrence probabilities of the failure scenarios are estimated by the result of simulation, the likelihood function given an observation is same as multinomial distribution given true probabilities. When the system is too complex to identify all the failure scenarios, the number of failure scenarios is obtained as the number of observed failure scenarios. However, it is possible that there are some unobserved failure scenarios which have low occurrence probabilities. Therefore, the uncertainty of then umber of failure scenarios should be reflected to estimated frequency of cascading failure event.

As the observed data follows multinomial distributions, the likelihood function for the number of hypothetical failure scenarios given observed data can be defined with multinomial distribution for all case of failure scenario selection from the number of hypothetical failure scenarios. If we assume that the prior probabilities for all case are all equal and use bayes theorem, the maximum of likelihood function depends on the number of simulations and the number of observed failure scenarios [2].

$$K_{MLE} = \begin{cases} \infty & (N = M) \\ N & (N > M^2) \\ NM/(M-N) & (M < N \leq M^2) \end{cases} \qquad (2)$$

where K_{MLE} is maximum likelihood estimator for the number of hypothetical failure scenarios, N is the number of observed failure scenarios, and M is the number of simulations.

2.3 Uncertainty of the occurrence probabilities

As the number of failure scenarios is estimated based on multinomial distribution, a common method to estimate occurrence probability of failure scenario is maximum likelihood estimation. However, the estimated probabilities for unobserved failure scenarios are assigned zero in maximum likelihood estimation. Therefore, we assign probability distribution for the occurrence probabilities and perform uncertainty analysis in this paper.

The widely used probability distribution in uncertainty analysis is conjugate distribution for likelihood function. The conjugate distribution for multinomial likelihood function is Dirichlet distribution.

$$(p_1, \cdots, p_k) \sim Dirichlet(\alpha_1, \cdots, \alpha_k) \qquad (3)$$

Then, the probability distribution in Eq. (1) is linear combination of Dirichlet components and the frequency which is hazard function for time to failure distribution can be represented as ratio of linear combination of Dirichlet components. Because the Dirichlet components can be represented as normalized chi-square random vector which have $2\boldsymbol{\alpha}$ as degree of freedom vector [3]. Therefore, the linear combination of Dirichlet components and the ratio can be represented as the linear combination of independent chi-square random variables.

$$\mathrm{P}\left(\sum_{i=1}^{k} w_i p_i \le z\right) = \mathrm{P}\left(\frac{\sum_{i=1}^{k} w_i c_i}{\sum_{i=1}^{k} c_i} \le z\right) = \mathrm{P}\left(\sum_{i=1}^{k} (w_i - z) p_i \le 0\right) \qquad (4)$$

where c_i is chi-square random variable having $2\alpha_i$ degree of freedom.

In this paper, the non-informative prior is used, and the posterior distribution is used as uncertainty distribution for the occurrence probabilities.

3. Application to an example system

Hybrid Micro Modular Reactor (H-MMR) is design phase heat pipe cooled micro modular reactor [4]. H-MMR uses hexagonal arrayed heat pipes to remove heat from the reactor core. Fig.1 shows the geometry of heat pipes in H-MMR. In H-MMR, there is no other heat removal system for the reactor core. Therefore, heat pipe cascading failure event should be considered as initiating event in PSA for H-MMR and the frequency should be estimated to quantify risk.

The heat pipe encounters functional limit when the assigned thermal load is larger than designed capacity. In general, heat pipes are designed to have enough safety margin. When a heat pipe is overloaded, therefore, there have already been several failed adjacent heat pipes and it can be assumed that large thermal loads are assigned to remaining heat pipes. For this reason, the criterion for heat pipe cascading failure is that more than one heat pipe is overloaded during the failure process. The assumed frequency of each heat pipe failure is used in this paper and the Aalen's additive hazard model is also used to reflect interdependence. Above mentioned, the failure scenarios can be categorized by the number of failed heat pipes until cascading failure event is occurred and all the scenarios in the same category have same time to cascading failure distribution. The failure frequency of each heat pipe in normal state is assumed as 1E-5.

There are 55 heat pipes in a single fuel assembly and the failure effect of each heat pipe is not identical because of geometry. Because of the failure criterion, the minimal number of failed heat pipes until failure criterion is achieved is identified. However, the maximum number of failed heat pipes cannot be identified practically. Therefore, the occurrence probabilities of all scenarios should be estimated based on the result of simulation.

Fig. 2 shows the results of frequency of heat pipe cascading failure event and the 90% confidence interval when the number of failure scenarios is estimated as the number of observed failure scenario. The 90% confidence interval is estimated with Eq. (4). Contrary to exponential time to failure distribution, the frequency is increased as time goes on because of the characteristic of hypoexponential distribution.

Fig. 3 shows the results considering the uncertainty of the number of failure scenarios. The estimated frequency based on maximum likelihood estimation is same whether considering the uncertainty of the number of scenarios or not because the probabilities for unobserved scenarios are zero in maximum likelihood estimation. However, the upper and lower bound of confidence interval is lower than Fig. 2 because the additional failure scenarios in Fig. 3 have large number of failed heat pipes until the scenario is ended.

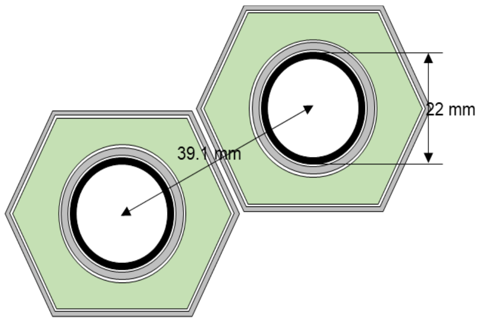

Fig. 1. Geometry of heat pipes in H-MMR

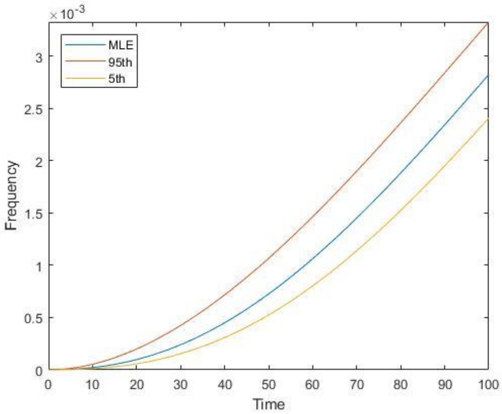

Fig. 2. Estimated frequency of heat pipe cascading failure and the 90% confidence interval without the uncertainty of the number of failure scenarios

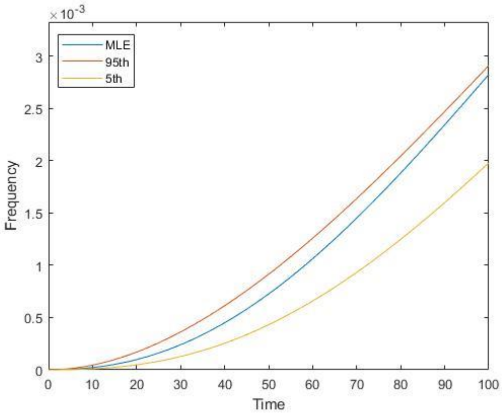

Fig. 3. Estimated frequency of heat pipe cascading failure and the 90% confidence interval with the uncertainty of the number of failure scenarios

4. Conclusions

There have been few attentions about the uncertainty of the results of simulation. Because of the use of simulation results, however, the number of observed scenarios and their probabilities have uncertainty based on the multinomial distribution. In this paper, the uncertainty analysis for estimated frequency of heat pipe cascading failure event is performed, with respect to two types of uncertainties came from observed data. It is shown that both uncertainties affect the result of estimation. However, the uncertainty analysis is performed separately in this paper. If the uncertainty distributions for two uncertainties are integrated, the insight for the total uncertainty of the simulation can be obtained and it is expected that the integrated confidence interval can be calculated.

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