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Study on Wolter mirror design for a neutron microscope development

Jin Man Kim [a*], Cheonkyu Lee [a], Jung-Gil Lee [a], Tae Joo Kim[b], Jongyul Kim[b]
[a]*Carbon Neutral Technology R&D Department, Korea Institute of Industrial Technology, Cheonansi, 31056, Republic of Korea*
[b]*Neutron Science Division, Korea Atomic Energy Research Institute, Daejeon, 34057, Republic of Korea*

[*]*Corresponding author: jinmankim@kitech.re.kr*

1. Introduction

Neutron imaging, unlike visible light, is a technology that can nondestructively inspect the inside of a material by penetrating it using neutron radiation. In specific, since the neutron can penetrate materials such as metals, it enables observation of unknown regions that cannot be seen by other methods. Therefore, it is an important technology in that it can be applied to various industries, research, and medical fields.

In addition, it is possible to analyze hidden objects with higher resolution using a neutron microscope. The neutron microscope is different from a general optical microscope because neutrons cannot be refracted using a lens or reflected with an optical reflector. Neutrons are reflected only when they are incident almost parallel to a smooth metal surface. As a result, the neutron microscopy can be implemented using Wolter mirror designed to increase the neutron flux by reflecting the neutron beam incident below a certain angle (critical angle) on the surface.

In this study, basic research on the design of the Wolter mirror was conducted. In order to implement the Wolter mirror, geometrical analysis and simulation were performed, and the feasibility of implementation was examined.

2. Method and results

For the design of the Wolter mirror, the geometrical design and simulation to verify it were performed using Matlab and McStas, respectively.

2.1 Design of Wolter mirror

The conceptual design of the Wolter mirror referred to Jackson's literature [1], and Table I shows the factors related to the Wolter mirror. Five factors were set to determine the shape of the Wolter mirror.

Table I: Factors of Wolter mirror

Factors	Meaning
M	Magnification
L	Instrument length from the object to the image plane
θ_{cf}	The maximum collection angle
$\Delta\theta_c$	Acceptance angle on a hyperbola mirror
b_c	Minor axis on hyperbola

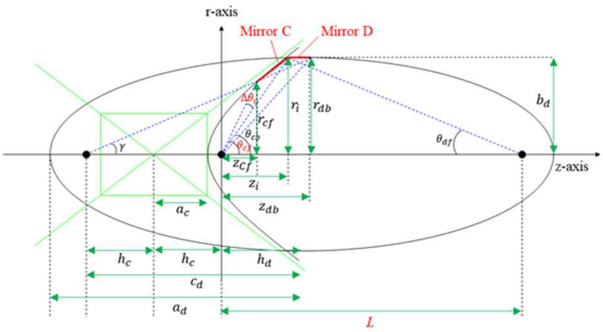

Fig. 1. Basic geometry for the design of Wolter mirror

As shown in Fig. 1, θ_{cb} can be obtained by the following relation.

$$\theta_{cb} = \theta_{cf} - \Delta\theta_c \quad (1)$$

In Fig. 2, r_i means the height of the point where the elliptical mirror and the hyperbolic mirror intersect.

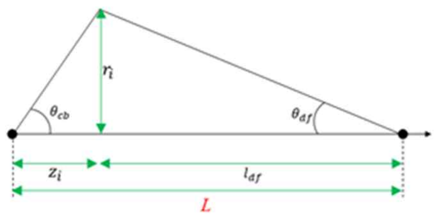

Fig. 2. Geometry for decision of r_i and z_i

Since L and M are already defined, θ_{df} can be obtained through the following formula.

$$\theta_{df} = \sin^{-1}\left(\frac{\sin\theta_{cb}}{M}\right) \quad (2)$$

Also, z_i and r_i can be obtained using the following formulas.

$$\frac{\tan\,\theta_{df}}{\tan\,\theta_{db}+\tan\,\theta_{df}}=\frac{r_i/l_{df}}{r_i/z_i+r_i/l_{df}}=\frac{z_i}{L} \qquad (3)$$

$$z_i=L\frac{\tan\,\theta_{df}}{\tan\,\theta_{db}+\tan\,\theta_{df}} \qquad (4)$$

$$r_i=z_i\times\tan\,\theta_{db} \qquad (5)$$

The hyperbola is defined when the major and minor axes and a focal length are determined. The major axis, the minor axis, and the focal length are marked a_c, b_c, and h_c, respectively. Since the coordinates of the center of the hyperbola are $(-c_c,0)$, the hyperbolic equation can be written as

$$b_c^2(z_i+c_c)^2-a_c^2r_i^2=a_c^2b_c^2 \qquad (6)$$

$$a_c^2+b_c^2=c_c^2 \qquad (7)$$

Since b_c is already defined, a_c and c_c can be obtained by combining the two expressions.

Likewise, an ellipse can be defined only when the major axis (a_d) and minor axis (b_d) are determined.

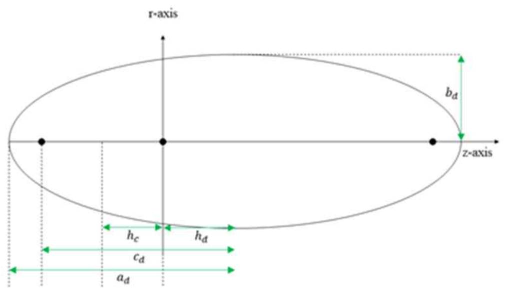

Fig. 3. Parameters for an ellipse

From Fig. 3, the following relationship is established.

$$h_c=c_c \qquad (8)$$

$$c_c=\frac{L+2h_c}{2} \qquad (9)$$

$$h_d=\frac{L-2h_c}{2} \qquad (10)$$

Also, from the definition of the ellipse, the following formula is established.

$$b_d^2(z_i-h_d)^2+a_d^2r_i^2=a_d^2b_d^2 \qquad (11)$$

$$a_d^2=b_d^2+c_d^2 \qquad (1)$$

Since h_c, h_d and the focal length (c_d) have already been determined, a_d and b_d can be obtained.

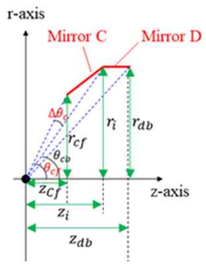

Fig. 4. Parameters for a hyperbola

In Fig. 4, since θ_{cf} is a known value, r_{cf} and z_{cf} can be obtained using the following relationship.

$$\tan\,\theta_{cf}=\frac{r_{cf}}{z_{cf}} \qquad (12)$$

Also, the following relation is made by combining previous equations.

$$b_c^2\left(z_{cf}+h_c\right)^2-a_c^2z_{cf}^2\tan\,\theta_{cf}^2=a_c^2b_c^2 \qquad (13)$$

Therefore, r_{cf} and z_{cf} are obtained.

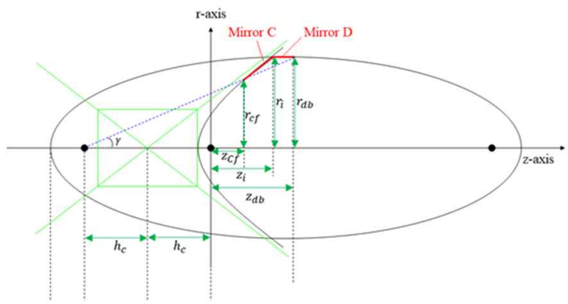

Fig. 5. Parameters of an elliptical mirror

In an ellipse, the following formula is established by the relationship between each factor.

$$\tan\gamma=\frac{r_{cf}}{z_{cf}+2\_h_c}=\frac{r_{db}}{z_{db}+2h_c} \qquad (14)$$

Since z_{cf} and r_{cf} were obtained from the hyperbolic mirror earlier, the $\tan\gamma$ can also be obtained.

In addition, since (r_{db} , z_{db}) is a point on an ellipse, z_{db} and r_{db} can be obtained by using the equation of the ellipse together.

$$b_d^2(z_{db}-h_d)^2+a_d^2(z_{db}+2h_c)^2\tan\,\gamma^2=a_d^2b_d^2 \qquad (15)$$

Based on the pervious relations, a ray-tracing for a neutron with the Wolter mirror was simulated. Fig. 6 is an example of the calculation result. A neutron passing through the sample was successfully reflected off the mirror and reached the image plane, creating a magnified image of the original sample.

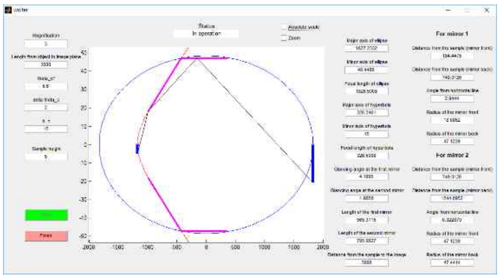

Fig. 6. Result of ray tracing on the designed Wolter mirror using Matlab (y-axis magnified image)

2.2 Simulation of neutron imaging with the designed Wolter mirror

After designing a single cone-shaped mirror, we implemented a Wolter mirror by connecting two cone mirrors by adjusting the factors in McStas.

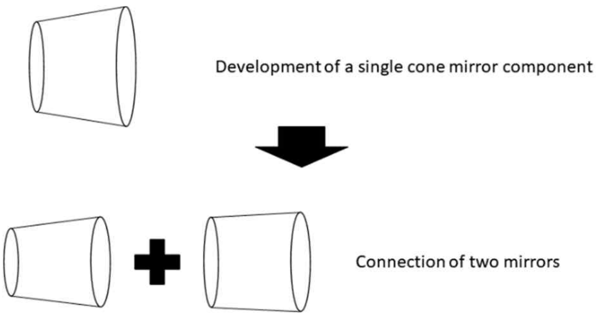

Fig. 7. Strategy of the development of the Wolter mirror in McStas

In order to simulate a cone-shaped mirror in McStas, it was expressed as an equation for a body of rotation using hyperbolic or elliptic equations as shown in Fig. 8.

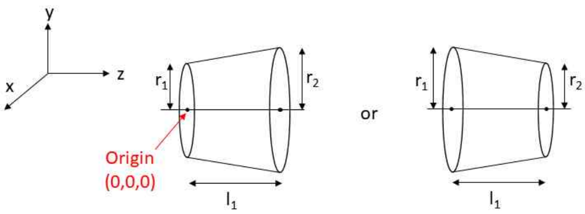

Fig. 8. Rotated cones in order to construct Wolter mirror

The neutrons that reach the entrance of the cone mirror continue to move and meet the inner surface of the cone mirror or reach the exit of the cone mirror. Since we know the neutron velocity when it reaches the entrance, we can calculate the times dt and t1 to meet the mirror or reach the exit. And reflection occurs in the mirror only when dt is smaller than t1.

To confirm that the implemented single cone mirror functions properly, a simple simulation was performed by configuring a source and detector under arbitrary conditions within McStas. As a result, it was found that the cone-shaped mirror appeared intact, and it was also possible to confirm that reflection occurred when neutrons hit the inner wall as shown in Fig. 9.

The change of neutron flux with and without a single cone mirror was also confirmed. In order to check the performance of the single cone mirror, the mirror was arranged in a direction in which the inner diameter gradually decreased so that the neutrons were collected.

Uniform neutron flux was detected when the neutron source was directly detected without the single cone mirror. When the single cone mirror was placed, neutrons were collected with higher neutron density within a radius of about 1 cm. This meant that the neutron flux was increased through the single cone as described in Fig. 10.

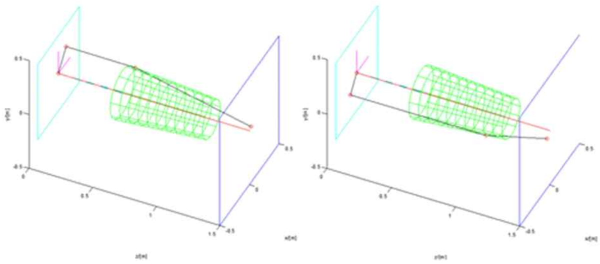

Fig. 9. The path of a neutron hitting a single cone mirror

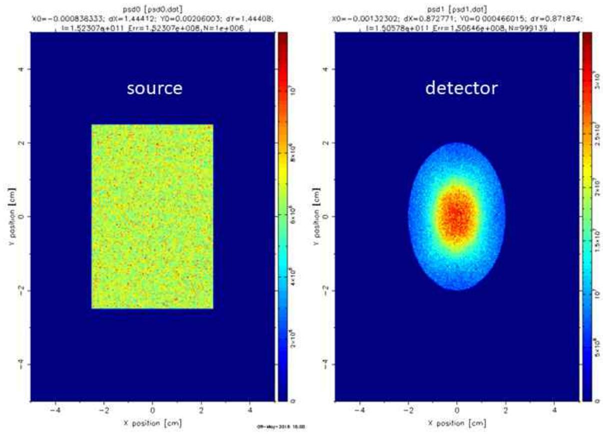

Fig. 10. The comparison of the simulated neutron images without the single cone mirror(left) and with the single cone mirror(right)

3. Conclusion

In the present study, we conducted study on the basic design of Wolter mirror, which is essential for realizing a neutron microscope.

The mirror was designed using ellipses and hyperbolas, and it was confirmed that neutrons passing through the sample normally reach the image plane as a result of ray tracing simulation.

In addition, from the simulation using McStas, it was observed that the neutron flux can be increased using the Wolter mirror.

Therefore, it is expected that a neutron microscope can be implemented by increasing the neutron flux using the Wolter mirror.

ACKNOWLEDGEMENT

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[1] J. A. Jackson, "Wolter X-Ray Microscope Computed Tomography Ray-Trace Model with Preliminary Simulation Results," Lawrence Livermore National Laboratory (LLNL), Livermore, CA, 2006.