Few Group Structure Optimization based on Perturbation Theory for VHTR

Tae Young Han
Korea Atomic Energy Research Institute, 989-111, Daedeok-daero, Yuseong-gu, Daejeon, Korea
Corresponding author: tyhan@kaeri.re.kr

1. Introduction

The nuclear design code system for Very High Temperature gas-cooled Reactor (VHTR) developed by Korea Atomic Energy Research Institute (KAERI) consists of the 2D lattice code, DeCART2D [1], based on the transport theory and the 3D core simulation code, CAPP [2], based on the diffusion theory. DeCART2D uses the fine group structure by 190 group and CAPP uses the few group structure by 10 group for VHTR. The 190 group structure is originated from HELIOS [3] group structure and the 10 group structure was applied from Kim's study [4].

Kim developed the automatic optimization code for few group structure with the idea of minimizing the error of the reaction rate. The method is very simple and accurate, but it needs large calculation burden due to the brute search way which have to calculate on every group condensation cases.

As another approach, some group optimization methods [5, 6] were proposed based on the contribution theory [7]. They apply the fine group adjoint spectra for selecting group structure, which means the importance of a neutron to the multiplication factor.

In this paper, a new group structure optimization method based on the perturbation theory [8] was proposed for the few group structure in the VHTR nuclear design analysis. It applies the sensitivity coefficient which represents the product of the adjoint flux and group cross section.

2. Methodology

The proposed group structure optimization method in this study is originated from the contribution theory which applies the adjoint spectra. In this section, the simple review of the contribution theory is described and then the new group structure optimization method is presented.

2.1 Review of the Contribution Theory

The group optimization based on the contribution theory is expressed that the importance of a fine group, *g*, can be determined using the adjoint flux as follows:

$$C_g = \phi_g \cdot \phi_g^\dagger , \tag{1}$$

where ϕ_g is forward flux and $\phi_g^\dagger$ is adjoint flux.

In addition, the importance of a few group, *G*, can be calculated by:

$$C_G = \sum_{g \in G} C_g . \tag{2}$$

Thus, the few group boundaries should be chosen so that the few group importance, C_G, is equal in thermal, resonance, and fast energy regions.

2.2 Group Structure Optimization based on the Classical Perturbation Theory

In this study, the sensitivity of the classical perturbation theory [8] was applied for determining the group structure. The sensitivity is defined as the change of the multiplication factor, k_{eff}, by the change of the total cross section, *X*, as follows:

$$S_{k_{eff},X} = \frac{\dfrac{\delta k_{eff}}{k_{eff}}}{\dfrac{\delta X}{X}} = \frac{\delta k_{eff}}{\delta X}\frac{X}{k_{eff}} . \tag{3}$$

If it is applied to the multi-group transport equation, it can be expressed as follows [9]:

$$S_{k_{eff},X_g} = \frac{T_{1,X_g} + T_{2,X_g} + T_{3,X_g}}{D} , \tag{4}$$

where

$$D = \frac{1}{k_{eff}} \left(\sum_g \bar{\nu}_g \Sigma_{fg} \phi_g \right) \left(\sum_{g'} \chi_{g'} \phi_{g'}^\dagger \right),$$

$$T_{1,X_g} = -\Sigma_{tg} \phi_g \phi_g^\dagger ,$$

$$T_{2,X_g} = \frac{1}{k_{eff}} \bar{\nu}_g \Sigma_{fg} \phi_g \left(\sum_{g'} \chi_{g'} \phi_{g'}^\dagger \right),$$

$$T_{3,X_g} = \Sigma_{sg} \phi_g \phi_g^\dagger .$$

The definition represents that the sensitivity includes the reaction rate which has been used as one of the criteria for evaluating the error of the group structure. In addition, it can be seen that the sensitivity including the impact to the multiplication by the change of the cross section is more suitable than the contribution by neutrons for the group structure optimization.

After the sensitivities for the fine groups are obtained, the summation of the sensitivities for total energy range can be easily obtained as follows:

$$S_{k_{eff}} = \sum_{g}^{N} \left| S_{k_{eff},X_g} \right| . \quad (5)$$

where, N is the total number of the fine groups. Also, because the sensitivity value is positive or negative, the total contribution to the multiplication should be the summation of the absolute value for the sensitivity.

In addition, the criteria for the group boundary selection can be easily obtained by:

$$K = \frac{S_{k_{eff}}}{N_G}, \quad (6)$$

where N_G is the target number of the few groups.

If the group boundary for a few group, G, was determined, the summation of the sensitivity for the G group is defined by:

$$S_{k_{eff},G} = \sum_{g \in G} \left| S_{k_{eff},X_g} \right| \rightarrow K . \quad (7)$$

In the procedure of collapsing fine groups to a few group, the group boundary for the G group can be determined so that the summation of the sensitivity, Eq.(7), for the group is the same with the criteria, Eq.(6).

In addition, for considering various core states, the total sensitivity can be obtained by simply adding the all sensitivities for the core states as follows:

$$S_{k_{eff},G} = \sum_{i} \sum_{g \in G} \left| S^{i}_{k_{eff},X_g} \right| , \quad (8)$$

where i is a core state according to depletion, temperature variation, fuel block position, etc.

3. Numerical Results

For the verification of the proposed method, Adjoint Based Group Optimization (ABGO) code was developed. The code can calculate the sensitivity using the adjoint solutions and cross section generated by DeCART2D and determine the few group boundaries based on the above procedure.

In this study, the new group structure by ABGO code was verified with a 1/6 2D core model based on MHTGR-350 benchmark [10] as shown in Figure 1. Three cross section sets were applied for the core model and generated by the color set calculation using DeCART2D as shown in Figure 2. Actually, the fuel blocks, A, B, C, and F in Figure 1 and 2, are the same block type and but three cross section sets, A, B, and C, are different with each other due to their position in the core. The red circles in Figure 1 represent the block position for the comparison of the reaction rate.

First, DeCART2D produced the fine group adjoint solution and cross sections on the color set model, A, and ABGO determined a new few group structure with them. The target number of the few group in the optimization is 10 groups which is currently used [4]. Then, DeCART2D generated three cross section sets with the few group structure by ABGO and CAPP code calculated the core parameters for the 2D core model.

For the comparisons, the reference solutions for the 2D core, Figure 1, were directly generated by DeCART2D.

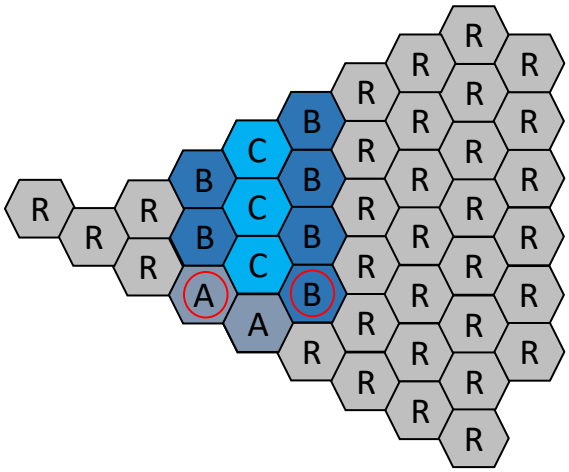

Fig. 1. 1/6 2D Core Model based on MHTGR-350 benchmark

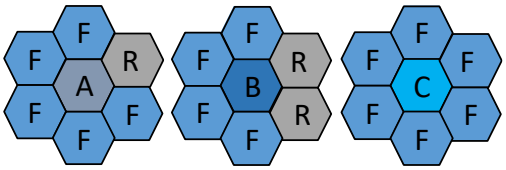

Fig. 2. Three color set models for the cross sections of the fuel block

Table 1 shows the comparison of two group structures, existing and new one. The current group structure is more groups in the thermal energy range and the new one is more groups in the resonance energy range. Table 2 is the summary of the effective multiplication factor for the core model. It is just the comparison of the global parameter and it cannot guarantee the accuracy of the group structure.

Table 3 and 4 show the absorption reaction rate errors for two group structures comparing to the reference solutions. They reveal that the reaction rate errors by the new group structure is comparable or slightly lower than that by the existing one.

This is only preliminary results on the one case and it cannot be decided that the method is more accurate. Thus, it needs more case study. However, it is noted that the proposed method is smart and efficient comparing with the existing brute search method.

Table I: Comparison of Group Upper Boundary Energy

Group	Existing (MeV)	New (MeV)
1	2.0000E+01	2.0000E+01
2	2.1445E-04	3.1203E-04
3	6.8680E-06	3.7267E-05
4	9.7100E-07	1.5536E-05
5	4.1704E-07	6.4760E-06
6	2.9074E-07	5.3000E-07
7	2.2769E-07	2.9074E-07
8	1.1157E-07	1.8443E-07
9	5.0000E-08	1.8443E-07
10	2.0492E-08	6.7000E-08

Table 2: Comparison of k_{eff} for 2D Core

	k_{eff}	Δρ (pcm)
Ref.	1.04228	-
Existing	1.04366	132
New	1.04358	125

Table 3: Comparison of Absorption Reaction Rate Errors for Block A of 2D Core

Group	Existing (%)	New (%)
1	0.37	0.34
2	0.14	0.14
3	0.12	0.00
4	0.24	0.02
5	1.17	0.37
6	1.51	0.96
7	1.84	1.58
8	2.23	1.87
9	2.67	2.17
10	3.11	2.61
RMS.	**1.70**	**1.37**

Table 4: Comparison of Absorption Reaction Rate Errors for Block B of 2D Core

Group	Existing (%)	New (%)
1	0.95	0.97
2	0.68	0.51
3	0.97	0.93
4	0.81	1.01
5	1.54	0.51
6	1.75	1.49
7	1.94	1.90
8	2.37	2.04
9	3.01	2.28
10	3.95	2.95
RMS.	**2.06**	**1.65**

4. Conclusions

In this paper, a new group structure optimization method based on the perturbation theory was proposed for the few group structure in the VHTR nuclear design analysis. It applies the sensitivity coefficient which represents the reaction rate and the impact to the multiplication by the change of the cross section. The verification results show that the reaction rate errors of new group structure by the proposed method are comparable or slightly lower to that of the existing few group structure.

If performing further case study with the proposed method, it could be applied to efficiently determine the few group structure for the two step procedure of VHTR analysis.

ACKNOWLEDGEMENT

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (No. 2020M2D4A2067322).

REFERENCES

[1] J. Y. Cho, et al., “Improvement and verification of the DeCART code for HTGR core physics analysis,” Nucl. Eng. Technol. 2019, 51, 13–30
[2] H. C. Lee, et al., “Development of HELIOS/CAPP code system for the analysis of block type VHTR cores,” PHYSOR 2012, Knoxville, Tennessee, April 15-20, (2012).
[3] R. J. Stamml’er, et al., “HELIOS Methods,” Studsvik Scandpower (1998).
[4] K. S. Kim, “Development of the Optimization Procedure for the Number of Neutron Energy Groups and Boundaries in the VHTR Physics Analysis,” Korean Nuclear Society Autumn Meeting, November 2-3, (2006).
[5] V. F. Fallah, et al., “Optimization of neutron energy-group structure in thermal lattices using ultrafine bilinear adjoint function,” Progress in Nuclear Energy, vol. 85, 648-658, (2015).
[6] F. A. Alpha, et al., “Advanced methodology for selecting group structures for multigroup cross section generation,” PHYSOR2000, Pittsburgh, PA, USA, (2000).
[7] Williams, M. L., “Generalized Contribution Response Theory,” Nuclear Science Engineering, 108: 355-383 (1991).
[8] Cacuci, D.G., “Sensitivity and Uncertainty Analysis,” vol. 1. Chapman & Hall/CRC, (2003).
[9] T. Y. Han, et al., “Development of a sensitivity and uncertainty analysis code for high temperature gas-cooled reactor physics based on the generalized perturbation theory,” Annals of Nuclear Energy, 85, 501-511, (2015).
[10] J. Ortensi, OECD benchmark for prismatic coupled neutronics/thermal fluids transient of the MHTGR-350 MW core design: benchmark definition. Tech. Rep., (2013).