Model Sensitivity Analysis for the Effect of State-of-Knowledge Correlation

Gyun Seob Song, Man Cheol Kim*
School of Energy Systems Engineering, Chung-Ang University, 84 Heukseok-ro, Dongjak-gu, Seoul 06974, Korea
**Corresponding author: charleskim@cau.ac.kr*

1. Introduction

In the Probabilistic safety assessment (PSA) for a nuclear power plant, the reliability of individual components is estimated based on observed data. However, the observed data is insufficient to estimate the reliability and its uncertainty distribution because components in the nuclear power plant are expected to have high reliability. To address this problem, the data of similar components are integrated, assuming that the components have the same reliability. Thus, there is a correlation between the reliabilities which is referred to as state-of-knowledge correlation (SOKC) [1].

The SOKC may have a significant impact on the reliability of redundant systems which have similar components. If the SOKC is not considered, the reliability of these systems can be overestimated, leading to inaccurate risk assessment. Therefore, it is required to quantify the effect of the SOKC in the PSA for nuclear power plants [2]. The effect of the SOKC is affected by the number of components and the type of uncertainty distribution model assumed for the component reliability. For continuously operating components, two widely used model are lognormal and gamma distribution. Kim et al. [3] numerically analyze the effects of the SOKC for the number of components and the distribution model.

In this paper, mathematical formulations for the effects of the SOKC concerning the distribution model are proposed and compared. An example initiating event analysis for interfacing systems loss of coolant accident (ISLOCA) is used to demonstrate the effects.

2. State-of-knowledge correlation

In the PSA, the reliability of components is typically expressed by failure rate or failure probability. The failure rate or failure probability has an uncertainty distribution based on observed data and the uncertainty distribution for individual components should be estimated based on the observed data from that component, in principle. Due to the lack of data, however, the observed data from similar components are integrated, creating the SOKC between the reliabilities of the similar components. Therefore, the failure rates of failure probabilities that have SOKC are treated as the same random variable.

The failure probability of a redundant system can be calculated as the product of individual probabilities of components when the components fail independently. Then, the expected value of the system failure probability can be derived.

$$E[X_1 \cdots X_n] = E[X_1] \cdots E[X_n] \tag{1}$$

where X_i s are individual failure probabilities of components. If all the failure probabilities have the same uncertainty distribution, the system failure probability can be represented as

$$E[X_1] \cdots E[X_n] = E[X] \cdots E[X] = E[X]^n \tag{2}$$

If the failure probabilities of components have the SOKC, on the other hand, the expected value of the system failure probability has a different formula with Eq. (1) because all the random variables are treated as the same random variable.

$$E[X \cdots X] = E[X^n] \tag{3}$$

Therefore, the effect of the SOKC is related to higher moments of the uncertainty distribution for failure rate or failure probability. In this paper, the effect of the SOKC is represented as the ratio of the expected values with and without consideration of the SOKC.

$$R(X) = E[X^n]/E[X]^n \tag{4}$$

3. Model sensitivity

To analyze the effects of the SOKC concerning uncertainty distribution model, the higher moments for lognormal distribution and gamma distribution are calculated.

When the failure probability follows a lognormal distribution, the uncertainty distribution can be characterized by two model parameters - μ and σ^2, which are the mean and variance of the logarithm of the failure probability. The higher moments for the lognormal distribution can be derived as follows:

$$E[X^n] = e^{\mu + \frac{1}{2}\sigma^2} \tag{5}$$

The effect of the SOKC is then derived [4].

$$R(X) = e^{\frac{1}{2}n(n-1)\sigma^2} \tag{6}$$

Eq. (6) shows that the effect of the SOKC depends only on the number of components and the variance of the logarithm of the failure probability.

When the failure probability follows a gamma distribution, on the other hand, the uncertainty distribution can be characterized by two model parameters - α and β, which are referred as shape and rate parameters. The higher moments for the gamma distribution can be derived as follows:

$$E[X^n] = \frac{\Gamma(\alpha+n)}{\Gamma(\alpha)}\frac{1}{\beta^n} \tag{7}$$

The effect of the SOKC is then derived [4].

$$R(X) = \frac{\Gamma(\alpha+n)}{\Gamma(\alpha)\alpha^n} = \prod_{i=0}^{n-1}\left(1 + \frac{i}{\alpha}\right) \tag{8}$$

Eq. (8) shows that the effect of the SOKC depends only on the number of components and the shape parameter.

It has been mathematically proven that the effect of the SOKC is always larger when a lognormal distribution is used instead of a gamma distribution.

4. Application

ISLOCA is a loss of coolant outside the containment through the reactor coolant system pressure boundary interfacing with a low-pressure system. In the ISLOCA frequency analysis, interfacing lines as potential pathways and reliabilities of involved valves are analyzed. As the SOKC has a significant impact on the frequency of ISLOCA, the effect of the SOKC is required to be analyzed. To demonstrate the proposed mathematical formulations for the effect of the SOKC, an example reliability data is applied which is used in [3].

As mentioned, the effects of the SOKC depend only on the number of components and the model parameters in both types of distribution. There is a widely used model parameter in lognormal distribution which is a function of the standard deviation of the logarithm of the random variable.

$$EF = e^{1.645\sigma} \tag{9}$$

If the method of moments is used, the shape parameters of the gamma distribution are a function of the error function. Therefore, the model sensitivity analysis is performed concerning the number of data and the error factor.

Fig. 1 and Fig. 2 show the effects of the SOKC for both types of uncertainty distribution models when the error factors are 5 and 10, respectively. When the number of components is 2, the effects of the SOKC are the same because the effect of the SOKC depends only on mean and variance when the number of components is 2, and the means and variances are matched. Excepting for 2 components case, the effects of the SOKC with lognormal distribution are increased as the number of components increases.

Fig. 3 shows the ratio between the effects of the SOKC with lognormal distribution and gamma distribution concerning the number of components. It is shown that the ratio is increased as the number of components increases because the shape parameter is decreased as the error factor increases.

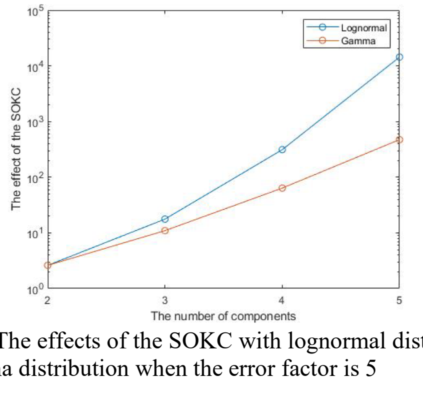

Fig. 1. The effects of the SOKC with lognormal distribution and gamma distribution when the error factor is 5

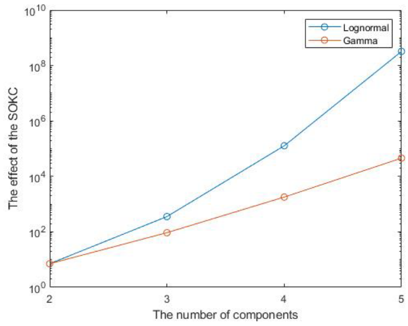

Fig. 2. The effects of the SOKC with lognormal distribution and gamma distribution when the error factor is 10

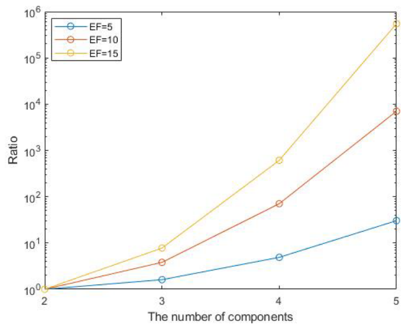

Fig. 3. The ratios between the effects of the SOKC with lognormal distribution and gamma distribution concerning the error factor

5. Conclusion

The state-of-knowledge correlation is an important factor that affects the result of risk assessment for the nuclear power plant. The effects of the SOKC depend on the type of uncertainty distribution model and therefore the impact of the model is required to be analyzed.

It is proven that the effect of the SOKC with lognormal distribution is always equal to or larger than that of the gamma distribution. In this paper, the effects of the SOKC with lognormal and gamma distribution are analyzed when the same observed data is used. The result of this paper is expected to contribute to selecting the distribution model according to the purpose of the risk assessment.

REFERENCES

[1] G. Apostolakis, S. Kaplan, Pitfalls in risk calculations, Reliability Engineering, Vol. 2, pp. 135-145, 1981.

[2] ASME/ANS RA-Sb-2013, Addenda to ASME/ANS RA-S-2008 Standard for level 1/ Large Early Release Frequency probabilistic risk assessment for nuclear power plant applications, The American Society of Mechanical Engineers, 2013.

[3] KAERI/TR-5596/2014, The effects of the state-of-knowledge correlation and component reliability data on interfacing systems LOCA frequency, Korea Atomic Energy Research Institute, 2014.

[4] G. S. Song, M. C. Kim, Comparison of state-of-knowledge correlation effect associated with lognormal and gamma uncertainty distributions, Submitted to Nuclear Engineering and Technology, 2023.