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New Methodology for Proportional Damping under Three-directional Excitations

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1. Introduction

Recent technical standards [1,2] specifies that seismic load shall be simultaneously excited in three directions, when performing nonlinear seismic response analysis of nuclear power plant structures with seismic isolation. This is because the linear combination may be useless in the non-linear analysis. Since the nonlinearity of seismic isolation system must be considered, the seismic response analysis is generally performed by the direct integration method in time domain. However, it may be difficult to apply the proportional damping matrix under simultaneous three-directional excitations.

A determination of accurate damping properties of the structural system has long been a challenging problem to many researchers. The damping generally includes a phenomenon that vibration is absorbed or reduced over time while energy is dissipated to an external system. The conclusions of most researchers in the past regarding application of damping to the dynamic analysis were limited to simple formulas with a damping value determined by empirical judgement or experimental result. The damping ratio which is used in dynamic analyses varies with the type of structural material, the type of structure, and the level of load. Guidelines for the damping ratio are presented in various technical standards according to the stress level as well as material and structure type. The reason why it is difficult to accurately evaluate the damping of a structural system is due to the limitations of classical mechanics, which does not deal with the micro regions but the macro properties (mass and stiffness) of the structural system.

Rayleigh proposed that the damping could be constructed as a linear summation of stiffness and mass matrices multiplied by proportional coefficients. The coefficients(a_k) correspond to a weight of the natural vibration mode of the structure. If *n*-modes are applied in *N*-degrees of freedom(DOF) system, *i*-th modal damping ratio(ξ_i) can be expressed with a power expansion series as like $\xi_i = \frac{1}{2}\sum_{k=0}^{n-1} a_k \omega_i^{2k-1}$. Therefore, the modal damping ratio converges to an accurate value as the number of modes to be applied increase[3].

The proportional damping matrix usually considers two modes (i, j) with high mass participation in the direct integration method. The pair of proportional coefficients(α, β) can be obtained from two dominant modes (ω_i, ω_j) and two modal damping ratio (ξ_i, ξ_j) shown in Eq.-1.

$$\frac{1}{2}\begin{bmatrix} 1/\omega_i & \omega_i \\ 1/\omega_j & \omega_j \end{bmatrix} \begin{Bmatrix} \alpha \\ \beta \end{Bmatrix} = \begin{Bmatrix} \xi_i \\ \xi_j \end{Bmatrix} \qquad \text{(Eq.-1)}$$

Therefore the proportional damping matrix of the global structural system is simply obtained by multiplying the mass($[M]$) and stiffness matrix($[K]$) by two coefficients, as like $[C] = \alpha[M] + \beta[K]$. As mentioned above, it may be difficult to determine the proportional coefficients, due to simultaneous excitation in three directions and two-way system of the structure. The two-way system of a structure means that it is flexible in the horizontal direction while it is stiff in the gravity direction. Hence, vibration modes corresponding to the three directions must be considered. Eq.-1 is insufficient to satisfy the three directional condition with two proportional coefficients.

There are still no research works or specified guidelines of the technical standards for the proportional damping, when simultaneously excited in three directions. New methodology for calculating the multi-directional proportional damping matrix considering simultaneous multi-excitation is proposed in this paper.

2. Methodology

The proposed methodology for the calculation of proportional damping is similar to the formulation of the finite element method, and is derived for a general use. In addition, the derivation process considers the orthogonal characteristic of the constitutive equations of material. The following initial assumptions are necessary for this formulation.

- The governing equation is partially modified through the derivation. The proposed governing equation is inversely inferred from the damping force($\{F_d(t)\}$) of the discrete system.
- The proportional coefficients for each direction consider only two modes and modal damping ratio.
- The multi-directional proportional coefficients depend on the DOF number in a structure, and number of the loading directions.
- The proposed proportional damping is expressed as an individual finite element.

Firstly, the equation of motion of the discrete system is inversely estimated to derive the governing equation, then a new proportional damping matrix is formulated. The proposed equation of motion and force equilibrium equation in time domain are as follows.

$$\begin{aligned} &[M]\{\ddot{u}(t)\} + \left[C^{(R)}\right]\{\dot{u}(t)\} + [K]\{u(t)\} = \{F(t)\} \\ &\{F_a(t)\} + \{F_d(t)\} + \{F_s(t)\} = \{F_e(t)\} \end{aligned} \qquad \text{(Eq.-2)}$$

Where, $\left[C^{(R)}\right]$ is the expression of the (Rayleigh) proportional damping matrix by dividing the mass damping term ($\left[C_\alpha\right]$) and the stiffness damping term ($\left[C_\beta\right]$), which also consists of a linear combination ($\left[C^{(R)}\right]=\left[C_\alpha\right]+\left[C_\beta\right]$). $\{F_a(t)\}$ is inertia force, $\{F_d(t)\}$ is the damping force, $\{F_s(t)\}$ is the stiffness force, and $\{F_e(t)\}$ is the external force. Especially the damping force is defined separately as $\{F_d(t)\}=\{F_\alpha(t)\}+\{F_\beta(t)\}$. If the above equation of motion (Eq.-2) is the result of variation on the governing equation, the governing equation in the solid state can be reconstructed in vector form as follows.

$$\left[L_\sigma\right]\left\{\sigma^{(R)}\right\}+\{b\}=\rho\{\ddot{u}+\alpha\dot{u}\},\left\{\sigma^{(R)}\right\}=\{\sigma\}+\beta\{\dot{\sigma}\} \quad \text{(Eq.-3)}$$

Where, $\{\dot{\sigma}\}$ is the flow stress vector. ρ is unit mass and $\{\ddot{u}\}$, $\{\dot{u}\}$ are the acceleration and velocity vectors respectively. $\left[L_\sigma\right]$ is a differential operator. The stress vector ($\left\{\sigma^{(R)}\right\}$) is re-expressed as follows.

$$\left\{\sigma^{(R)}\right\}=[D][B]\{u\}+\left[D_\beta\right]\left[\dot{B}\right]\{u\} \quad \text{(Eq.-4)}$$

where $\left[D_\beta\right]$ is an isotropic damped constant matrix, and $\left[\dot{B}\right]$ is a flow strain matrix. Strain rate refers to the time-dependent gradient of strain. $\left[D_\beta\right]$ is expressed as the product of the coefficient(β) of stiffness and the elastic material matrix($[D]$). And assuming that $\left[D_\beta\right]$ is a symmetric and isotropic material, it can be expressed as

$$[D_\beta]=\begin{bmatrix} \beta_1(\lambda+2G) & \beta_{12}\lambda & \beta_{13}\lambda & & & \\ \beta_{12}\lambda & \beta_2(\lambda+2G) & \beta_{23}\lambda & & & \\ \beta_{13}\lambda & \beta_{23}\lambda & \beta_3(\lambda+2G) & & & \\ & & & \beta_{12}G & & \\ & & & & \beta_{23}G & \\ & & & & & \beta_{13}G \end{bmatrix} \quad \text{(Eq.-5)}$$

$$\beta_{ij}=\frac{1}{2}(\beta_i+\beta_j)$$

The indices(*i,j*) indicate the direction in the proportional coefficient β_{ij} of stiffness. Using the finite element formulation procedure, the proportional damping matrix ($\left[C^{(R)}\right]^e$) of the unit element(e) can be obtained as follow.

$$\left[C_\beta\right]^e=\int_\Omega\left[\bar{B}\right]^T\left[D_\beta\right]\left[\bar{B}\right]dV,\left[\bar{B}\right]=[B]\{u\} \quad \text{(Eq.-6)}$$

$\left[\bar{B}\right]$ is the strain matrix for the element displacement vector, and dV denotes the unit volume ($dx_1dx_2dx_3$).The proportional damping matrix ($\left[C_\alpha\right]^e$) of the mass is also derived in the unit element. In the right term of Eq.-3, $\rho\alpha\{\dot{u}\}$ is the internal force generated by the damping force ($\{F_\alpha(t)\}$) by the mass. The proportional coefficient matrix $[\alpha]$ of mass is also derived as below.

$$\alpha_{ij}\delta_{ij}=[\alpha]=\begin{bmatrix}\alpha_1 & & \\ & \alpha_2 & \\ & & \alpha_3\end{bmatrix}.\ if\ i\neq j,\ \delta_{ij}=0 \quad \text{(Eq.-7)}$$

The damping matrix ($\left[C_\alpha\right]^e$) of mass is easily formulated as follows.

$$\left[C_\alpha\right]=\rho\int_V[N]^T[\alpha][N]dV \quad \text{(Eq.-8)}$$

Where, $[N]$ is the matrix of shape function. Finally the damping force obtained by the proposed proportional damping matrix is expressed as follows.

$$\{F_d(t)\}^e=\{F_\alpha(t)\}^e+\{F_\beta(t)\}^e=\left[\left[C_\alpha\right]+\left[C_\beta\right]\right]^e\{\dot{u}(t)\}^e \quad \text{(Eq.-9)}$$

The verification of the proposed methodology can be seen through numerical analyses.

3. Conclusions

In this paper, a multi-directional proportional damping matrix calculation methodology was presented. In addition, to verify this methodology, a linear or non-linear seismic response analyses was performed with parametric studies. The summary and conclusions are as follows.

- The current proportional damping cannot reflect the characteristics of two-way structural system, when simultaneously excitation in three directions.
- A new multi-proportional damping method is presented. The governing equations were corrected through several assumptions. The derivation process is similar to the formalization process of the finite element method.
- The proposed proportional damping was verified through various numerical analyses. The disadvantages of the current proportional damping matrix can be overcome.

The authors will show the results using the proposed methodology in the upcoming presentation.

REFERENCES

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