Liquid metal Flow Stability of the EM Pump for the STELLA Facility

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1. Introduction

The electromagnetic (EM) pump with a nominal flowrate of 100 L/min was designed for the liquid sodium circulation in the STELLA facility [1]. The electrically conducting liquid sodium fluid in its annular channel suffers from an axial electromagnetic force. The electromagnetic force in the fluid gives a dominant contribution to the force balance due to the strong applied magnetic field. For most electromagnetic pump systems with high electrical conductivity under strong magnetic fields, viscous forces are very weak compared with electromagnetic forces as anticipated by high Hartmann number. In the electromagnetic-force-dominant MHD flow system, the radial profile of the velocity distribution turns flat so that velocity gradients become zero in the entire flow region with the exception of the narrow wall [2]. It is indicated that the flow with such a velocity distribution would be more stable than the flow with a parabolic velocity [2]. In the present study, for a simple model of axially-infinite electromagnetic pump with equivalent current sheet, the steady-state solution of unidirectional velocity distribution on the laminar flow at very narrow annular channel gap is found under a transverse magnetic field produced by three-phase magnet coils. Using the method of small perturbation for the fluid velocity, magnetic field and pressure, linear stability analysis is carried out on the flow of an incompressible liquid metal flow. The criterion for stable operation is sought from the imaginary part of perturbed angular frequency and the critical Reynolds number is thus found in terms of Hartmann number for the designed EM pump.

2. Method and Results

2.1 Steady-state solution for the equivalent model

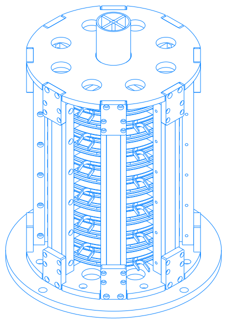

Fig. 1 The designed EM pump

Fig. 1 shows the designed three-phase annular linear induction electromagnetic pump with a flowrate of 900 kg/min. As one of the methods to find MHD solutions such as the flow velocity, magnetic field, and so on for the EM pump, an analytical model with equivalent sheet current is adopted [3].

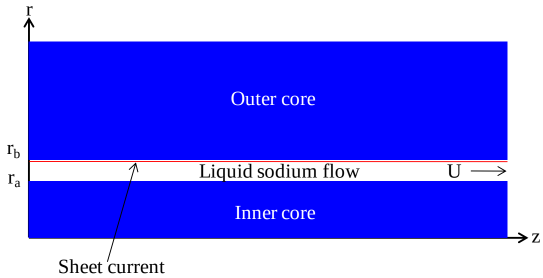

Fig. 2. Theoretical model of an ALIP

As seen in Fig. 2, the MHD induction flow system of an actual three-phase coil arrangement is idealized by the equivalent current sheet. The annular channel consists of a narrow gap between two infinite coaxial cylinders with different radii. The liquid sodium flow of the high electrical conductivity is assumed to be laminar, incompressible and axisymmetric with axial velocity components depending on the radial positions. The pumping fluid is characterized by its density, viscosity, electrical conductivity, and vacuum magnetic permeability. The magnet cores outside the annular channel have idealized silicon-iron laminations with zero conductivity and infinite permeability. The applied electrical current of the magnet coils is given by a sinusoidal equivalent sheet current that flows azimuthally on the outer wall, and generates a radial magnetic field with an axial magnetic field passing through the inner core. This current is represented by the peak line current density given by $\frac{3\sqrt{2}k_w NI}{p\tau}$. With the help of angular frequency and wave number of the travelling current, the sheet current is thus described as [4]

$$\mathbf{J}_a(r_b, z, t) = \mathrm{Re}[J_m e^{j(\omega' t - k' z)}]\hat{\boldsymbol{\theta}} \quad (1)$$

The magnetic flux density, electric field, and current are induced by the sinusoidal applied sheet current with axisymmetry. Those quantities also have the form of sinusoidal and axisymmetric fields with different phases. The mathematical descriptions for these fields are represented in Eq. (2) [3].

$$\begin{aligned} \mathbf{E}(r,z,t) &= \mathrm{Re}[(E(r)\,\hat{\boldsymbol{\theta}})e^{j(\omega' t - k' z)}] \\ \mathbf{B}(r,z,t) &= \mathrm{Re}[(B_r(r)\hat{\mathbf{r}} + B_z(r)\hat{\mathbf{z}})e^{j(\omega' t - k' z)}] \\ \mathbf{J}(r,z,t) &= \mathrm{Re}[(J(r)\,\hat{\boldsymbol{\theta}})e^{j(\omega' t - k' z)}] \end{aligned} \quad (2)$$

The conducting incompressible fluid in the time-varying magnetic field is governed by the following set of dimensionless MHD equations.

$$\nabla \bullet \mathbf{V} = 0 \tag{3}$$
$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \bullet \nabla)\mathbf{V} = -\nabla P + \frac{1}{R_e}\nabla^2 \mathbf{V} + \frac{H_a^2}{R_e}\mathbf{J} \times \mathbf{B} \tag{4}$$
$$\nabla \times \mathbf{B} = R_m \mathbf{J} \tag{5}$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{6}$$
$$\nabla \bullet \mathbf{B} = 0 \tag{7}$$
$$\mathbf{J} = \mathbf{E} + \mathbf{V} \times \mathbf{B} \tag{8}$$

where velocity (**V**), pressure (P), current density (**J**), magnetic flux density (**B**), electric field (**E**), time (t) and other geometries are dimensionless quantities normalized by their reference values with $U_0, \rho U_0^2, \sigma U_0 B_0, B_0, U_0 B_0, \frac{R_0}{U_0}, R_0,$ respectively, where B_0 is $\frac{\mu_0 J_m}{\sqrt{2} R_0 k'}$ R_0 is $r_b - r_a$ and U_0 is $\frac{\omega'}{k'}$.

2.2 Stability of liquid sodium flow in a narrow annular channel

The sodium fluid of an ALIP is developed in the axial direction under a traveling moving magnetic field with an induced current flow. Then, through some perturbations, the system may experience turbulence, and eventually, flow separation or local cavitation. In addition, an abrupt increase of induced current can lead to system damage through a growing perturbation. As is generally known, a magnetic field improves the hydrodynamic stability of the flow or suppresses the turbulence that is already present [2]. An ALIP mainly suffers from an axial developing force from the Lorentz's product of the azimuthal induced current and radial magnetic field, and experiences a radial force from the axial magnetic field that exists in the sodium fluid. In general, a perturbation can be caused in every direction, but in the present study, the stability effect of the magnetic field on the sodium metal flow is estimated from an analysis of the axisymmetric two-dimensional linear stability taking into account the perturbation on the developing direction. To analyze the magnetic field effect on an electrically conducting flow, the relation between critical Reynolds number (Re_{cr}) and Hartmann number H_a is sought by solving the 4^{th}-order differential equation reducing the Orr-Sommerfeld equation for the case of a small magnetic Reynolds number (R_m). As a result, the critical Reynolds number is found as a function of Hartmann number along with U_{av}, R_0, and k as follows.

$$R_{e_{cr}} = \frac{4\pi^2 R_0^2 k^2 + 8\pi^4 + 2\pi H_a^2 - \frac{2\pi^2}{r_{av}^2}}{2\pi^2 U_{av} R_0 k} \tag{9}$$

From Eq. (9), the critical Reynolds number ($R_{e_{cr}}$) is inversely proportional to the flow velocity (U_{av}). Actually, it is natural that the flow is more stable when the fluid does not move than when it flows. From a hydrodynamic aspect, the flow can go into the turbulent region as the fluid velocity increases and has the possibility to be unstable although it does not mean the turbulence always causes instability.

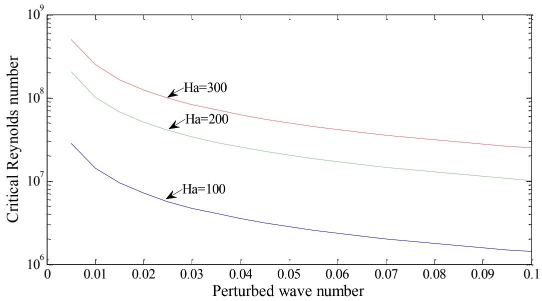

Fig. 3. Critical Reynolds number on the different Hartmann numbers according to the long wave perturbation

In Fig. 3, the critical Reynolds number ($R_{e_{cr}}$) increases as the wave number of the perturbation decreases. That is, the flow is thought to be more stable for a long wave perturbation (k << 1) in the driven magnetic field. As a result, it is shown that the flow under the high magnetic field can be kept stable even in a higher fluid velocity for low perturbation with a long wave length. The annular linear induction EM pump considered in the present study is predicted to operate stably when the Reynolds number of 32,767 of the designed pump with a maximum flowrate of 100 L/min is compared with the critical Reynolds number of an order of more than 10^6 for a long wave perturbation.

3. Conclusion

The fourth-order equation for a sinusoidal wave perturbation was obtained, and its stability was analyzed using an annular linear induction EM pump, which is used for a sodium cleaning loop. The critical Reynolds number was a function of the Hartman number, perturbation wave number, and fluid velocity from the mathematical set-up subject to a non-trivial solution for the stream function at the flow gap of the EM pump. It was shown that a magnetic field across the annular gap had a strong effect on the stability of the liquid sodium flow with a high electrical conductivity, and that it could suppress the onset of the flow instability for a long wave perturbation. It was predicted that the designed EM pump will have stable operation based on a comparison between the Reynolds number of the pump and the critical Reynolds number.

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