# Geometric Beta Optimization of the Superconducting Linac for PEFP<sup>\*</sup>

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# 1. Introduction

The normal conducting linac of the proton engineering frontier project (PEFP) has been developed to accelerate proton beams up to 100 MeV. One of the possible extensions is a spallation neutron source which needs 1-GeV proton beams. We are now studying the superconducting linac which accelerates proton beams from 100 MeV to 1 GeV. This work summarized the results on the transit time factor (TTF) and energy gain of the cavity in order to determine the optimized values of geometric beta.

### 2. Methods and Results

We studied the TTF based on the multi-cell cavity theory, the energy gain per cavity under some assumptions as explained below. Based on these results, we optimized the geometric beta values. We studied in 2 and 3 regions of the geometric beta values.

#### 2.1 TTF: Multi-Cell Cavity Theory

The TTF in a multi-cell cavity theory[1] is given by

$$T = T_g S(N, \beta_s$$

where  $S(N,\beta_s/\beta) = \begin{cases} \frac{1}{N} \left[ 1 + \sum_{m=1}^{(N-1)/2} (-1)^m 2 \cos(m\pi\beta_s/\beta) \right], & \text{N odd} \\ \frac{2}{N} \left[ + \sum_{m=0}^{N/2-1} (-1)^m \sin([m+1/2]\pi\beta_s/\beta) \right], & \text{N even} \end{cases}$ 

 $T_g = \frac{\pi}{4} \left[ \frac{\sin[(\beta_s/\beta - 1)\pi/2]}{(\beta_s/\beta - 1)\pi/2} + \frac{\sin[(\beta_s/\beta + 1)\pi/2]}{(\beta_s/\beta + 1)\pi/2} \right]$ 

The parameter N represents the cell number. The subscript s means the synchronous particle. We note that S(N,1)=1 at  $\beta = \beta_s$ . The gap factor T<sub>g</sub> is valid for the  $\pi$ -mode in the superconducting cavity.

## 2.2 TTF with 3 regions for $\beta_g$

We note that the TTF depends on the number of cells in a cavity and the  $\beta_g$  value. In this subsection we studied the case with 3 different regions of  $\beta_{g}$ . We used cell numbers of 4, 5, and 6. In each combination of the cell numbers, we obtained the  $\beta_g$  values under the condition that the TTF values are similar at 100 MeV, 1 GeV, and 2 overlapping points. Fig. 1 shows the transit time factor for several cases. The numbers above each plots represents the numbers of the cells in each  $\beta_{g}$ 

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region. For example, 556 means 5, 5, and 6 cells in the low, medium, and high beta cavities, respectively.



Fig. 1. Transit time factor for 3 regions for  $\beta_g$ 

## 2.3 Energy Gain

The energy gain per cavity is given by

$$\Delta W = q E_0 T L \cos \phi$$

where L and  $\phi$  represent the cavity length and the synchronous phase. In order to calculate the energy gain we need to know the accelerating field,  $E_0$ . First of all we assumed that the peak surface electric field is 30 MV/m. After defining  $E_{acc} = E_0 \times (\pi/4)$  and using the SNS simulation result [2] of the ratio,  $E_{peak}/E_{acc}$ , we can obtain the ratio as a function of  $\beta$  by a fitting. The fitting result is given in Fig. 2 and the fitting function is given by

$$f(\beta) = -5.60606 + \frac{3.17305}{\beta} + 4.12188\beta$$

For a given  $\beta_g$ , we can obtain  $E_{acc}$  and  $E_0$  from this formula. This method was applied to both 3 and 2 region cases for  $\beta_{g}$ .



Fig. 2.  $E_{peak}/E_{acc}$  as a function of  $\beta$  (fitting result).

# 2.4 Energy Gain with 3 regions for $\beta_g$

The result of the energy gain per cavity is given in Fig. 3 for 3-region case for  $\beta_g$ . We note that the energy gain is not efficient in the low beta region. We also found that cell number of 6 in high beta region is much efficient than the cases with fewer cell numbers like 4 and 5. From this study, we determined the 2 region case of should be better than 3 regions.



Fig. 3. Energy gain per cavity for 3 regions for  $\beta_{g...}$ 

# 2.5 TTF with 2 regions for $\beta_g$

We studied the 2-region case by using similar method as the 3-region case. Fig. 4 shows the transit time factor for several cases. The numbers in each plot represent the number of cells in each region.



Fig. 4. Transit time factor for 2 regions for  $\beta_{g}$ .

### 2.6 Energy Gain with 2 regions for $\beta_g$

The result of the energy gain per cavity is given in Fig. 5 for 2-region case for  $\beta_g$ . Because the case with the larger number of cells is efficient for acceleration, we choose the cell number of 6 both for low and high beta regions where  $\beta_g = 0.50$  and 0.74. Fig. 6 and Fig. 7 show the TTF and the energy gain per cavity in this case. The transition energy is 224.8 MeV from low to high beta cavity.









Fig. 7. Energy gain per cavity for 6-cell cavit. ..

### 3. Conclusions

We studied the TTF and the energy gain per cavity for the PEFP superconducting linac in order to optimize the geometric beta values. We obtained that 2-beta region is better than 3-region case. The geometric beta values are 0.50 and 0.74 and the transition energy is 224.8 MeV.

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