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Discussion on the Development of Numerical Analysis Based on Staggered Grid for Arbitrary 3-dimensional Mesh Shape

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1. Introduction

In numerical analysis in fluid dynamic, finite volume method (FVM) is very popular scheme because it can easily conserve the fluid properties. FVM is based on two grid systems; staggered grid and collocated grid. In staggered grid system scalar variables such as pressure, temperature, and so on are located in the cell center, and the vector variables such as velocity are located in cell face. Thus for the implementation of FVM the staggered control volume (or mesh) for momentum equation should be additionally defined, as shown in Fig.1.

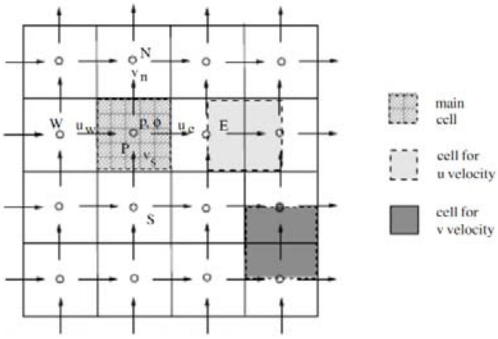


Fig. 1 Numerical Control Volume for Momentum Equation in Staggered Grid System

In collocated grid system all the variables, regardless that they are scalar variables or vector variables, are located in cell center. The merit of staggered grid system is that it can avoid the unrealistic check board type pressure distribution and it can easily conserve the convective properties through the faces because the velocities are defined in the faces. However in spite of such merits the staggered grid system is not generally adopted in 3-dimensional complex geometry because of the difficulties in the generation of staggered control volume in arbitrary mesh shapes such as tetrahedral mesh. So commercial computational fluid dynamics codes usually adopt collocated grid system for the analysis of complicated domain.

In conventional staggered grid system the staggered cell takes the halves of the neighboring two scalar cells, as shown in Fig.1. However Wenneker et al. proposed new type staggered cell such as the shaded part in Fig. 2 [1]. And they applied it successfully to compressible flow.

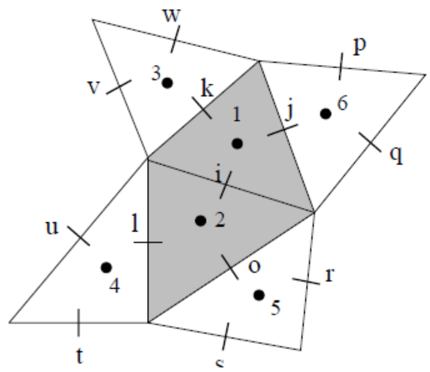


Fig. 2 Numerical Control Volume for Momentum Equation in Staggered Grid System Proposed by Wenneker et al.(2000)

Momentum cell in Fig. 2 make it easy to integrate convective terms, because the surfaces of momentum cell are surfaces of scalar cell, at which the velocities are already defined. Compressible flow in numerical analysis is far different from two-phase flow which is based on two-fluid model. This paper discusses on the applicability of such momentum cell to two-phase flow of two-fluid model.

2. Governing Equations and Grid System

2.1 Governing equations

Governing equations for the numerical implementation are those of CAP which are based on 3-fluid, 2-phase flow model [2]. For the sake of simplicity the equations for gas phase are presented here.

Continuity equation for gas phase:

$$\frac{\partial}{\partial t}\left(\alpha_g \rho_g\right)+\nabla\cdot\left(\alpha_g \rho_g \mathbf{v}_g\right) = -\frac{\frac{p_v}{p} H_{gli\to g}\left(T^s\left(p_v\right)-T_g\right)+H_{gli\to l}\left(T^s\left(p_v\right)-T_l\right)}{\left(h_g^*-h_l^*\right)} - \frac{\frac{p_v}{p} H_{dgi\to g}\left(T^s\left(p_v\right)-T_g\right)+H_{dgi\to d}\left(T^s\left(p_v\right)-T_d\right)}{\left(h_g^*-h_d^*\right)} + \Gamma_g \quad (1)$$

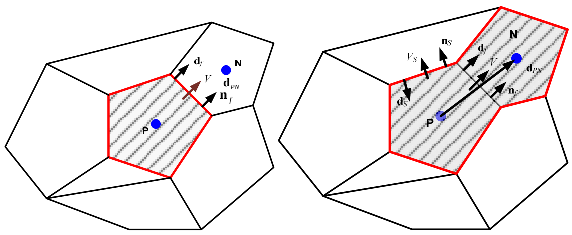
Momentum equation for gas phase:

$$\alpha_g\rho_g\frac{\partial \mathbf{v}_g}{\partial t}+\alpha_g\rho_g\mathbf{v}_g\cdot\nabla\otimes\mathbf{v}_g$$
$$=-\alpha_g\nabla p+\alpha_g\rho_g\mathbf{g}+\nabla\cdot\left(\alpha_g\left(\mu_g+\mu_g^t\right)\nabla\otimes\mathbf{v}_g\right)$$
$$+\Gamma_{lg}^{EV}\left(\mathbf{v}_l-\mathbf{v}_g\right)+\Gamma_{dg}^{EV}\left(\mathbf{v}_d-\mathbf{v}_g\right)-\mathbf{v}_g\Gamma_g$$
$$+\left[F_{gl}\left(\mathbf{v}_l-\mathbf{v}_g\right)+F_{gd}\left(\mathbf{v}_d-\mathbf{v}_g\right)\right]-F_g^w\mathbf{v}_g$$
$$+\left[C_{gl}^{VM}\alpha_g\alpha_l\rho_{gl}\frac{\partial}{\partial t}\left(\mathbf{v}_l-\mathbf{v}_g\right)\right.$$
$$\left.+C_{gd}^{VM}\alpha_g\alpha_d\rho_{dg}\frac{\partial}{\partial t}\left(\mathbf{v}_d-\mathbf{v}_g\right)\right]$$
$$+\mathbf{M}_g \tag{2}$$

Energy equation for gas phase: (abbreviated)

2.2 Grid System

Grid system in this implementation is given in Fig. 3.



(a) Scalar Cell (b) Momentum Cell
Fig.3 Grid System for Numerical Implementation

Velocity vector in the face of scalar cell is defined as

$$\mathbf{v}_{g,f}=V_g\mathbf{d}_f \tag{3}$$

V_g is normal velocity at face *f*.

3. Implementation of FVM

Some noticeable integration results are presented here.

Convective term in continuity equation:

$$\frac{1}{\delta t}\int_{t^{(n)}}^{t^{(n+1)}}\int_{CV}H(\mathbf{x})\nabla\cdot\left(\alpha_g\rho_g\mathbf{v}_g\right)dvdt$$
$$=\sum_S\left(\tilde{\alpha}_g^{(n)}\tilde{\rho}_g^{(n)}F_g^{(n+1)}\right)_S \tag{4}$$

, where

$$F_{g,S}^{(n+1)}\equiv\gamma_{A,S}A_S\mathbf{v}_{g,S}^{(n+1)}\cdot\mathbf{n}_S=\gamma_{A,S}A_SV_{g,S}^{(n+1)}\left(\mathbf{n}_S\cdot\mathbf{d}_S\right)$$

Convective term in momentum equation:

$$\frac{1}{\delta t}\int_{t^{(n)}}^{t^{(n+1)}}\int_{CV}H(\mathbf{x})\alpha_g\rho_g\mathbf{v}_g\cdot\nabla\otimes\mathbf{v}_g\,dvdt$$
$$=\sum_S\left\{\left(\tilde{\alpha}_g^{(n)}\tilde{\rho}_g^{(n)}V_g^{(n)}\right)_S F_{g,S}^{(n)}\mathbf{d}_S\right\}$$
$$-V_{g,f}^{(n)}\mathbf{d}_f\sum_S\left(\tilde{\alpha}_g^{(n)}\tilde{\rho}_g^{(n)}\right)_S F_{g,S}^{(n)} \tag{5}$$

Pressure term in momentum equation:

$$\frac{1}{\delta t}\int_{CV}\int_{t^{(n)}}^{t^{(n+1)}}-H(\mathbf{x})\alpha_g\nabla p dtdv$$
$$=-\gamma_{v,f}V_f\hat{\alpha}_{g,f}^{(n)}\nabla p_f \tag{6}$$

, where ∇p_f is not known still, but multiplying $\mathbf{d}_{f,LR}$ make the calculation easy

$$-\gamma_{v,f}V_f\hat{\alpha}_{g,f}^{(n)}\nabla p_f\cdot\mathbf{d}_{f,LR}$$
$$=-\gamma_{v,f}V_f\hat{\alpha}_{g,f}^{(n)}\left(p_R-p_L\right) \tag{7}$$

Viscosity term in momentum equation:

$$\frac{1}{\delta t}\int_{CV}\int_{t^{(n)}}^{t^{(n+1)}}H(\mathbf{x})\nabla\cdot\left[\alpha_g\left(\mu_g+\mu_g^t\right)\nabla\otimes\mathbf{v}_g\right]dtdv \tag{8}$$
$$=\sum_S\alpha_{g,S}^{(n)}\left(\mu_{g,S}^{(n)}+\mu_{g,S}^{t,(n)}\right)\left\{\left(\mathbf{n}_S\cdot\mathbf{d}_S\right)\left(\sum_{S'}\left(\frac{A_{S'}V_g^{(n)}\mathbf{d}_{S'}}{V_{S'}}\right)\right)\right\}\left(\gamma_{A,S}A_S\right)$$

The other terms are easily integrated without difficulties. For the final arrangement face directional vector $\mathbf{d}_f$ is multiplied to difference momentum equation.

4. Conclusions

All the results of integration for each term make it ease to compose the system pressure equation. Actual calculation results will be presented in conference.

ACKNOWLEDGMENTS

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