Reviews on the Nonlinear Stability of Single Pressure Two-fluid Model

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1. Introduction

Multi-fluid models [1,2] have complex characteristics [4,5,6]. This means that it is not properly posed for the initial value problem[3]. Since this fact was found at early seventies, many researchers have studied to overcome the difficulties. Brief reviews on the method taken in two-fluid code RELAP5[8] were made and the impacts on SPACE[9] were investigated[10].

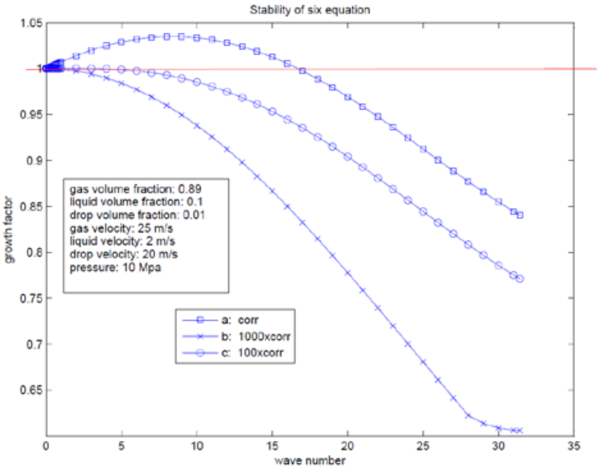

Fig. 1. Growth factor for six equation model

As can be seen in Fig. 1, the growth factor of von Neumann stability analysis for a typical annular mist flow regime shows unstable behavior(a), where a typical interfacial drag force correlation is used. Increasing drag force by factor of 1000 guarantees the stability(c). But these factors cannot be justified with any physical basis. Even though the von Neumann (linear) stability analysis shows unstable region, most of the real problem calculations show no distinct instability. Many people believe that the nonlinear effects are the main reason for this apparent contradictory situation. In this paper, reviews on references for the nonlinear stability are executed for better understanding of the problem.

2. Reviews on works that deals with non-linear stability of the multi-fluid Model

2.1 Thyagaraja's study [11]

He solves the single pressure two-fluid model using the *explicit upwind* finite difference scheme for a dead end pipe. It has been checked that the multi-phase equations are nonhyperbolic everywhere. Initial conditions and necessary parameters are as follows;

$u_1 = u_2 = 0\ m/s,\ \ T_1 = T_2 = 413\ K$
$\alpha_1 = 0.5 + 0.1\sin(\pi x/L),$
$p = 12(1+\sin(\pi x/L))\ MPa$
length of pipe $(L)\ = 50\ m,$
number of grid $= 100,$
time step $= 5\ \mu s$

The results show that the motion is regular (i.e. no evidence of high-k instability) and that symmetry about the mid-point is preserved. The constants of the motion are conserved to a high accuracy.

He argues that, when properly interpreted, multi-fluid equations may be solved without explicit regularizing terms by the use of "faithful numerical schemes", which preserves the positivity and conservation properties of the physical system.

2.2 Krishnamurthy's thesis[12]

Krishnamurthy performs extensive parametric runs to cover a comprehensive range of system parameters with the torus problem (shown in Fig. 2) using RELAP5.

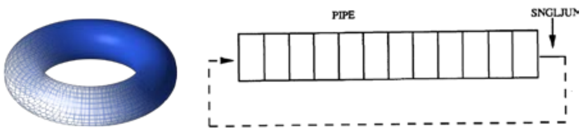

Fig. 2. Torus and nodalization

His problem consists of a horizontal toroidal tube with following data;

length = 2.0 m, diameter = 0.1 m, cell number = 40
void fraction = constant + variation
velocities → constant gas and liquid volumetric flux
pressure → 0.1 MPa and 10.0 MPa.

Case #	α_g	v_f (m/s)	v_g-v_f (m/s)	P (MPa)	$\Delta\alpha$	Wavenumber	Max. Growth Ratio $\frac{\Delta\alpha_{max}}{\Delta\alpha_0}$
1	0.5	2	12.5	0.1	0.005	2	12.32
2	0.1	2	1.9	0.1	0.005	2	4.97
3	0.9	2	52.5	0.1	0.005	2	2.92
4	0.5	4	12.5	0.1	0.005	2	8.98
5	0.5	2	8.0	0.1	0.005	2	1.04
6	0.5	2	17.0	0.1	0.005	2	1.26
7	0.5	2	12.5	0.1	0.05	2	3.30
8	0.5	2	4.1	1.0	0.005	2	5.98
9	0.1	2	0.6	1.0	0.005	2	3.08
10	0.5	2	12.5	0.1	0.005	8	5.14
11	0.5	2	12.5	0.1	0.005	2,5	9.00,6.8
12	0.5	2	12.5	0.1	0.005	2,10	9.00,3.76

Table-1. List of cases, conditions and growth ratio

There is no manifestation of the ill-posed differential behavior with the numerical results obtained. Initially some growth (linear) is identified but it is limited. Maximum growth factor is shown in Table-1 for individual cases. As shown in Fig. 3, steady state is approached.

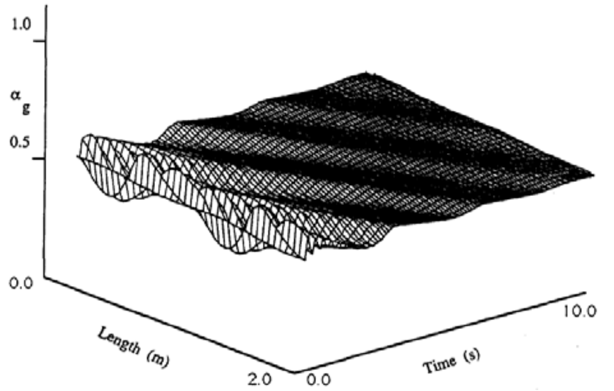

Fig. 3. Void fraction variation in for nominal case

He tries to explain the reason for the limited growth with the cascading process which transfers the energy from a growing wave to shorter wavelengths.

2.3 Keyfitz's singular shock approach[13]

She uses two-equation incompressible two-phase flow model to illustrate how one can construct a predictive mathematical theory for equations which are not hyperbolic. Her equations are as follows;

$$\beta_t + \left(\upsilon B_1(\beta)\right)_x = 0 \qquad \begin{cases} \beta \equiv \rho_2\alpha_1 + \rho_1\alpha_2 \\ \upsilon \equiv \rho_1 u_1 - \rho_2 u_2 \end{cases} \qquad (1)$$
$$\upsilon_t + \left(\upsilon^2 B_2(\beta)\right)_x = 0$$

Using the explicit solution to the Riemann problem by the singular shock theory, she illustrates the three points: the role of localization of perturbations of a nonhyperbolic state; the finite amplitude of oscillations; and the fixed frequency of oscillations. This approach, however, is not successful to overcome the ill-posedness problem because her solution fails to meet uniqueness.

2.4 Energy norm estimation method of Kreiss[14]

The parabolic nonlinear conservation laws are written as;

$$u_t + A(u)u_x = \nu u_{xx} \qquad \nu \ge 0$$
$$u(x,0) = f(x), \quad f \in C^\infty, \quad f(x) = f(x+1) \qquad (2)$$

Kreiss argues that energy norm for eq.(2) can be expressed as follows;

$$\frac{d\|u\|^2}{dt} = \frac{C}{\nu}\left|u_{\max}\right|^2 - \frac{\left|u_{\max}\right|^3}{\nu} \qquad (3)$$

where $u_{\max}$ is peak value, ν is viscosity and C is constant. When $u_{\max}$ is small, i.e. $u_{\max} \le C$, then, the norm can grow. But once $u_{\max}$ becomes greater than C, then, the right hand side of eq.(3) becomes negative and the norm stops glowing.

3. Discussions and Conclusions

Usually steady state solutions are obtained by running so called "null transient" with RELAP5 or SPACE. The result of Fig. 3 can be understood as a typical behavior of a null transient. Even though the initial conditions are in the unstable region (Taitel-Dukler criterion), it approaches to the steady state without any conspicuous unstable behavior.

Such a nonlinear stability may be explained by the estimation of the energy norm for eq.(2) as;

$$\frac{d\|u\|^2}{dt} = \overbrace{2\int_0^1 uA(u)u_x dx}^{I} - \overbrace{2\nu\int_0^1 u_x^{\,2} dx}^{II} \qquad (4)$$

Usually $A(u)$ is roughly equal to u. For a sine wave perturbation, $I \approx 0$ and $II > 0$. It means energy norm cannot grow. The assumption $A(u) \approx u$ is too rough to be applicable to real problem. But it can be used as a basis to investigate the nonlinear stability by numerical calculation approaches [11,12].

Further elaboration of energy norm method is definitely necessary in the future.

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