

Effect of Correlation Factors on the Measurement Uncertainty of Physical Inventory in Bulk Handling Facilities

Haneol Lee^{a*}, Hyun Cheol Lee^a, Jung Youn Choi^a, Hyun Ju Kim^a, Hana Seo^a, Yewon Kim^a

^aKINAC, 1534 Yuseong-daero, Yuseong-gu, Daejeon,

*Corresponding author: haneol@kinac.re.kr

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1. Introduction

The Republic of Korea (ROK) has been operating a state system of nuclear material accounting and control (SSAC) as a member state of the International Atomic Energy Agency (IAEA). To enhance the ROK's national safeguards inspection capabilities, the Korea Institute of Nuclear Nonproliferation and Control (KINAC) is planning to conduct a material balance evaluation (MBE), which will evaluate the declared amount of nuclear material in bulk handling facilities (BHF) [1]. The KINAC, as a part of the SSAC, is planning to conduct MBE to strengthen national safeguards inspection capability.

The MBE compares the material balance in a document (book inventory) and measured inventory (physical inventory) in a BHF during a specific period. It includes three main processes:

- (1) The difference found between the book and physical inventory is defined as material unaccounted for (MUF).
- (2) The uncertainty of the MUF (σ_{MUF}) is then calculated by combining individual measurement uncertainty.
- (3) The calculated MUF is then tested using z-statistics.

The expression and propagation of uncertainty can be a process using both top-down method and bottom-up method. The Guide to the Expression of Uncertainty in Measurement (GUM) was used as a bottom-up method. Previous studies have demonstrated the plausibility of applying the GUM method in national safeguards inspection [2, 3].

According to the literature, the GUM method quantifies and propagates uncertainty with reasonable assumptions or approximations, thereby neglecting uncertainty sources with little contribution. In nuclear materials accounting, these neglected uncertainties are usually correlation factors between individual uncertainty with a small fraction, resulting in the calculation of optimized combined uncertainty for a single item measurement. However, for a BHF with a large number of items, the number of correlations exponentially increases, and the accumulation of correlation factor becomes increasingly significant.

This study evaluates the effect of correlation factors in nuclear material accounting of a BHF. For our analysis,

we designed a benchmark BHF and material accounting system (weighing, U concentration analysis and ²³⁵U enrichment analysis). The combined standard uncertainty of nuclear material in the benchmark facility was then calculated with and without considering correlation factors to determine their effect on nuclear material accounting.

2. Propagation of Uncertainty in GUM

The GUM is a bottom-up uncertainty expression method, which quantifies and combines individual uncertainty components in the measurement to estimate measurement uncertainty. According to the literature, the combined standard uncertainty of X ($u_c(X)$) with a number of measurement results ($X = f(x_1, x_2, \dots, x_n)$) with standard uncertainties ($u(x_1), u(x_2), \dots, u(x_n)$), is calculated using Equation (1) [4] as,

$$u_c(X) = \sum_{i=1}^n \left(\frac{\partial X}{\partial x_i} \right)^2 u(x_i)^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^n \frac{\partial X}{\partial x_i} \frac{\partial X}{\partial x_j} u(x_i) u(x_j) r(x_i, x_j) \quad (1)$$

where the correlation factor ($r(x_i, x_j)$) denotes the degree of correlation between two measurement results, which is defined as the covariance divided into individual uncertainty ($r(x_i, x_j) = u(x_i, x_j) / u(x_i) u(x_j)$). If all measurement uncertainties are independent of each other, the correlation factor becomes zero.

The GUM classifies measurement uncertainty into "type A" and "type B". The uncertainty from repeated measurements is classified into type A. The standard uncertainty of x_i (type A), with sample standard deviation (s_{x_i}) and n measurements, is calculated using Equation (2) as,

$$u(x_i) = \frac{s_{x_i}}{\sqrt{n}} \quad (2)$$

Measurement uncertainty other than type A is classified into type B. The quantification of type B uncertainty depends on its characteristics. Typical type B uncertainty includes readability in the measurement system or measurement results in a certificate. The standard uncertainty (type B) is calculated depending on its type (uniform, triangular, certified uncertainty, ...) [4].

The GUM also introduces “expanded uncertainty ($U(X) = ku_c(X)$)” for demonstrating the confidence level (or interval) of the measurement results. The coverage factor (k) is calculated as two-tailed distribution statistics ($t_{\alpha/2, \nu_{eff}}, z_{\alpha/2}$) for the given confidence level ($1 - \alpha$) of the measurement results. The degrees of freedom of combined uncertainty (ν_{eff}) is calculated using Equation (3) as,

$$\nu_{eff}(X) = \frac{(u_c(X))^4}{\sum_{i=1}^n \left(\frac{\partial X}{\partial x_i} u(x_i) \right)^4} \nu_{x_i} \quad (3)$$

Usually, a t-value is used as a coverage factor for $\nu_{eff} < 10$ and a z-value is used for $\nu_{eff} \geq 10$.

3. Measurement Systems for Material Accounting

The mass of nuclear material (M) in a bulk handling facility is calculated using Equation (4):

$$M = \sum_{i=1}^I \sum_{j=1}^J (m_{ij} f_{U,ij} w_{235,ij}), \quad (4)$$

where the mass (m), U concentration (f_U) and ^{235}U enrichment (w_{235}) are the measurement results. The I and J in equation (4) indicate the stratum information and number of items in a stratum respectively.

Since over several thousand items exist in a bulk handling facility and the same measurement systems are applied for items in the same stratum, relative standard uncertainty (δ) is usually used for nuclear material accounting. The relative standard uncertainty is defined as the standard uncertainty divided by the measurement result.

The uncertainty components of individual measurement include both independent uncertainty components and uncertainty components with covariance. According to Equation (1), the independent components and components with covariance must be propagated separately. Therefore, we define independent uncertainty components as “relative random uncertainty components (δ_r)” and uncertainty components with covariance as “relative systematic uncertainty components (δ_s)”. We assumed the correlation factor as 1, since the exactly same equipment and process are applied, for items with covariance, in nuclear material accounting.

3.1 Weighing process using EBAL

The mass of an item (m) was measured using an electric balance (EBAL) which is calibrated using a standard mass. The key uncertainty components for weighing are the EBAL measurement (X), buoyancy effect (f_b) and calibration (f_c). The mass of an item and its relative standard uncertainty ($\delta(m)$) are calculated using Equations (5) and (6). The uncertainty components (X, f_b, f_c) in Equation (6) and their standard

uncertainties ($u(X), u(f_b), u(f_c)$) are calculated in Equations (7) ~ (12) respectively:

$$m = X f_b f_c, \quad (5)$$

$$\delta(m) = \left(\frac{1}{m} \right) \times \sqrt{\left(\frac{\partial m}{\partial X} \right)^2 u(X)^2 + \left(\frac{\partial m}{\partial f_b} \right)^2 u(f_b)^2 + \left(\frac{\partial m}{\partial f_c} \right)^2 u(f_c)^2}, \quad (6)$$

$$X = X_m + \varepsilon, \quad (7)$$

$$f_b = (1 - d_a/d_s)/(1 - d_a/d), \quad (8)$$

$$f_c = m_s/(X_{sm} + \varepsilon), \quad (9)$$

$$u(X) = \sqrt{u(X_m)^2 + u(\varepsilon)^2}, \quad (10)$$

$$u(f_b) = \sqrt{\left(\frac{\partial f_b}{\partial d_a} \right)^2 u(d_a)^2 + \left(\frac{\partial f_b}{\partial d_s} \right)^2 u(d_s)^2 + \left(\frac{\partial f_b}{\partial d} \right)^2 u(d)^2}, \quad (11)$$

$$u(f_c) = \sqrt{\left(\frac{\partial f_c}{\partial m_s} \right)^2 u(m_s)^2 + \left(\frac{\partial f_c}{\partial X_{sm}} \right)^2 u(X_{sm})^2 + \left(\frac{\partial f_c}{\partial \varepsilon} \right)^2 u(\varepsilon)^2}. \quad (12)$$

X_m and X_{ms} indicate the measurement result of EBAL for the sample and standard mass. m_s indicates the standard mass, ε indicates the combined uncertainty of readability related factors of the EBAL, and d_{air} , d_{std} and d indicate the density of air, standard mass and sample respectively.

Calibration uncertainty was considered as a systematic uncertainty component, since the mass of all items in a facility are measured between EBAL calibration.

Figure 1 depicts the effect of uncertainty components in the weighing process using a fish-bone diagram. The red colored components in Figure 1 are classified into systematic components. Table 1 summarizes the results of relative standard uncertainty calculation for weighing using EBAL, including random and systematic component calculation. The results indicate that the effect of buoyancy is negligible compared to the other random components. The contribution of random and systematic components was 83.78 % and 16.22 % respectively.

Since all uncertainty components in Figure 1 are independent of each other, relative standard random and systematic uncertainty are propagated using Equation (1) without correlation factors.

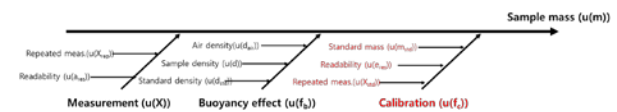


Fig. 1. Uncertainty components in weighing.

Table I. Relative standard uncertainty of weighing(m)

$\delta(m)$	$u(m)$	m	X (rand)	f_b (rand)	f_c (sys)
1.014E-04	1.014E-04	1.0000	1.0001	1.0000	1.0000
$\delta_r(m)$	$u_r(m)$	$nu(m)$	$u(X)$	$u(f_b)$	$u(f_c)$
9.279E-05	9.279E-05	3.4909	9.280E-05	8.476E-08	4.082E-05
$\delta_s(m)$	$u_s(m)$		$c(X)$	$c(f_b)$	$c(f_c)$
4.082E-05	4.083E-05		1.0000	1.000	1.000
			cont(X)	cont(f_b)	cont(f_c)
			8.378E-01	6.992E-07	1.622E-01

3.2 U concentration analysis using GRAV

The U concentration (f_U) in uranium oxide samples were analyzed by gravimetric method(GRAV). The GRAV analyzes the oxygen to uranium ratio (O/U) by measuring the mass difference of a sample before and after oxidation. We then convert it into the U concentration of the sample. The key uncertainty components for U concentration are the mass ratio before and after oxidation (m_i/m_f), impurity concentration before (w_I) and after oxidation (w_F) and ^{235}U enrichment (w_{235}).

The U concentration (f_U) and its relative standard uncertainty ($\delta(f_U)$) are calculated using Equations (13) and (14) as,

$$f_U = \frac{A_U}{A_U + A_O(O/U)}, \quad (13)$$

$$\delta(f_U) = \left(\frac{1}{f_U}\right) \sqrt{\left(\frac{\partial f_U}{\partial \left(\frac{O}{U}\right)}\right)^2 \left(u_r\left(\frac{O}{U}\right)\right)^2 + u_s\left(\frac{O}{U}\right)^2}. \quad (14)$$

We neglected the effect of atomic mass uncertainties, since their effect can be considered to be negligible compared to the uncertainty of measurement results.

The O/U and its relative uncertainty ($u(O/U)$) are calculated using Equations (15)~(17):

$$O/U = \frac{[(1-w_I)(m_i/m_f) - F_S(1-w_F)]A_U}{F_S(1-w_F)A_O}, \quad (15)$$

$$u_r\left(\frac{O}{U}\right) = \sqrt{\sum_i \left(\frac{\partial f_U}{\partial x_i}\right)^2 u(x_i)^2}, \quad (16)$$

$$u_s\left(\frac{O}{U}\right) = \sqrt{\sum_j \left(\frac{\partial f_U}{\partial x_j}\right)^2 u(x_j)^2}. \quad (17)$$

x_i in Equations (16) and (17) indicate the measurement results in Equation (15). F_S is the U concentration of U_3O_8 , which is a constant. The calibration uncertainty of measuring sample mass before oxidation and after oxidation was considered as a systematic uncertainty component. Random components (repeatability, readability) are classified as x_i in Equation (16) and the systematic component (calibration) is classified as x_j in Equation (17).

Figure 2 depicts the overall process to calculate $\delta(f_U)$. The standard uncertainties of impurity concentrations ($u(w_I)$, $u(w_F)$) and enrichment ($u(w_{235})$) were considered to be independent and uniformly distributed. We identified the systematic uncertainty components

which are indicated in red in the figure. Uncertainty components which cancelled out are also identified as red arrows. Table II summarizes the results of relative standard uncertainty calculation for U concentration analysis using GRAV, including random and systematic component calculation. The results also indicate that the effect of enrichment is negligible compared to other random components. The contribution of random and systematic components was >99.99% and <0.01% respectively.

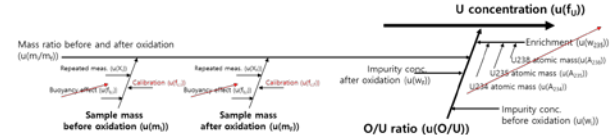


Fig. 2. Uncertainty components in U conc. analysis.

Table II. Relative standard uncertainty of f_U with correlation.

$\delta(f_U)$	$u(f_U)$	f_U	$u_r(mi/mf)$	$u(w_I)$	$u(w_F)$	$u(w_{235})$
6.939E-04	6.116E-02	8.814E+01	6.672E-04	2.125E-05	5.952E-06	6.300E-05
$\delta_r(O/U)$	$u_r(f_U)$	$nu(f_U)$	$c(mi/mf)$	$c(w_I)$	$c(w_F)$	$c(w_{235})$
6.939E-04	6.116E-02	1.5280E+01	17.5308	-16.8669	16.8664	0.0072
			cont_r(mi/mf)	cont(w_I)	cont(w_F)	cont(w_{235})
			9.990E-01	9.382E-04	7.359E-05	1.511E-09
$\delta_s(O/U)$	$u_s(f_U)$		$u_s(mi/mf)$			
1.101E-06	9.707E-05		1.060E-06			
			$c(mi/mf)$			
			17.5308			
			cont_s(mi/mf)			
			2.519E-06			

However, the correlation factors within a single measurement are often considered to be negligible. We calculated the relative standard uncertainty for the same analysis process without correlation factors to demonstrate the effect of correlation factors within a measurement. Therefore, systematic uncertainty components in weighing (calibration) were propagated using Equation (16) instead of Equation (17). The results of the relative standard uncertainty of U concentration using GRAV for “no correlation” are summarized in Table III.

Table III. Relative standard uncertainty of f_U (no correlation).

$\delta(f_U)$	$u(f_U)$	f_U	$u(mi/mf)$	$u(w_I)$	$u(w_F)$	$u(w_{235})$
1.003E-03	8.839E-02	8.814E+01	9.645E-04	2.125E-05	5.952E-06	6.300E-05
		$nu(f_U)$	$c(mi/mf)$	$c(w_I)$	$c(w_F)$	$c(w_{235})$
			76.3953	-16.8669	16.8664	0.0065
			cont(mi/mf)	cont(w_I)	cont(w_F)	cont(w_{235})
			9.995E-01	4.492E-04	3.524E-05	5.798E-10

We then compared the calculated relative standard uncertainties using two propagation methods. The results shown in Table II and III indicate that the relative standard uncertainty significantly increased. The systematic components for sample mass before and after oxidation were almost cancelled out when the correlation factor was considered, due to Equation (17). Therefore, the correlation factor must be identified within a measurement if its contribution is not negligible.

3.3 ^{235}U enrichment analysis using TIMS

The enrichment of ^{235}U (w_{235}) in samples were analyzed by thermal ionization mass spectrometry

(TIMS). TIMS initially ionizes U in a sample, separates U isotopes using a curved magnetic field and measures each isotope using a number of detectors. Major isotopes (^{235}U , ^{238}U) and minor isotopes (^{234}U , ^{236}U) are measured using a Faraday Cup (FC) and Secondary Electron Multiplier (SEM) respectively. We then analyze the ^{235}U enrichment using the measured isotopic ratios ($R_{23x/238}$, $x = 4,5,6$). The measured isotopic ratios were also corrected using the CRM measurement results.

The key uncertainty components for ^{235}U enrichment analysis are the isotopic ratio for the sample and the CRM measurement ($RS_{23x/238}(m)$, $RC_{23x/238}(m)$), CRM certificate ($RC_{23x/238}(c)$), background measurement ($RB_{23x/238}$) and detector efficiency factors (d_Y , d_{LDT} , d_{Flat}) [5].

^{235}U enrichment (w_{235}) is calculated using Equation (18) as,

$$w_{235} = \frac{A_{235}R_{235/238}}{A_{234}R_{234/238} + A_{235}R_{235/238} + A_{236}R_{236/238} + A_{238}}. \quad (18)$$

Its relative standard uncertainty and random and systematic components ($\delta(w_{235})$, $\delta_r(w_{235})$, $\delta_{s(FC)}(w_{235})$, $\delta_{s(SEM)}(w_{235})$) are calculated using Equations (19) ~ (22):

$$\delta(w_{235}) = \left(\frac{1}{w_{235}}\right) \sqrt{u_r(w_{235})^2 + u_{s(FC)}(w_{235})^2 + u_{s(SEM)}(w_{235})^2}, \quad (19)$$

$$u_r(w_{235}) = \sqrt{\sum_x \left(\frac{\partial w_{235}}{\partial R_{23x}}\right)^2 u_r(R_{23x})^2}, \quad (20)$$

$$u_{s(FC)}(w_{235}) = \sqrt{\left(\frac{\partial w_{235}}{\partial R_{235}}\right)^2 u_s(R_{235})^2}, \quad (21)$$

$$u_{s(SEM)}(w_{235}) = \sqrt{\sum_x \left(\left(\frac{\partial w_{235}}{\partial R_{23x}}\right) u_s(R_{23x})\right)^2}. \quad (22)$$

We divided the systematic components based on detector types (FC, SEM) since they are independent of each other depending on the detector types. We neglected the effect of atomic mass uncertainties since their effect can be considered negligible compared to the uncertainty of measurement results.

The isotopic ratios between major isotopes ($R_{235/238}$) and minor isotopes ($R_{234/238}$, $R_{236/238}$) are calculated using Equations (23) and (24):

$$R_{235/238} = \left(\frac{RS_{235}(m)RC_{235}(c)}{RC_{235}(m)}\right) - RB_{235/238}, \quad (23)$$

$$R_{23x/238} = \left(\frac{RS_{23x}(m)RC_{23x}(c)\delta(Y)\delta(L)\delta(F)}{RC_{23x}(m)}\right) - RB_{23x/238}. \quad (24)$$

We assumed the same detector efficiency was applied in the minor isotope ratio calculation and all samples were analyzed within a short time and shared background isotopic ratios; therefore, detector efficiency factors and background measurements were considered as systematic uncertainty components. The relative standard uncertainty for isotopic ratios are then calculated using Equations (25) ~ (27) as,

$$u_r\left(R_{\frac{235x}{238}}\right) = \sqrt{\sum_i \left(\frac{\partial R_{235}}{\partial x_i}\right)^2 u_r(x_i)^2}, \quad (25)$$

$$u_s\left(R_{\frac{235}{238}}\right) = \sqrt{\left(\frac{\partial R_{235}}{\partial RB_{\frac{235}{238}}}\right)^2 u_s\left(RB_{\frac{235}{238}}\right)^2}, \quad (26)$$

$$u_s\left(R_{\frac{23x}{238}}\right) = \sqrt{\sum_j \left(\frac{\partial R_{23x}}{\partial x_j}\right)^2 u_s(x_j)^2}. \quad (27)$$

Random components (the sample measurement, CRM measurement and CRM certificate) are classified as x_i in Equation (25). Systematic components for the major isotopic ratio (background isotopic ratio) and minor isotopic ratio (background isotopic ratio, detector efficiency factors) are propagated using Equations (26) and (27) respectively.

Figure 3 depicts the overall process to calculate $\delta(w_{235})$. The purple and red colored components are systematic component for the major isotopic ratio measurement and minor isotopic ratio measurement respectively. Table IV summarizes the results of the relative standard uncertainty calculation for ^{235}U enrichment analysis using TIMS, including random and systematic component calculation. The results indicate that the uncertainty of measuring $R_{235/238}$ is dominant. The contribution of random and systematic component was 98.25 % and 1.75 % respectively.

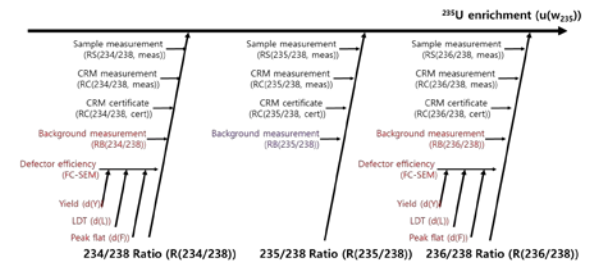


Fig. 3. Uncertainty components in ^{235}U enrichment analysis.

Table IV. Relative standard uncertainty of w_{235} with the correlation factor.

$\delta(w_{235})$	$u(w_{235})$	w_{235}	$u_r(R234/238)$	$u_r(R235/238)$	$u_r(R236/238)$
5.154E-04	1.747E-05	3.389E-02	6.866E-06	1.879E-05	1.849E-08
$\delta_r(w_{235})$	$u_r(w_{235})$	$nu(w_{235})$	$c(R234/238)$	$c(R235/238)$	$c(R236/238)$
5.109E-04	1.731E-05	5.458E+01	-0.03218	-0.92128	-0.03246
			$cont(R234/238)$	$cont(R235/238)$	$cont(R236/238)$
			1.600E-04	9.823E-01	1.180E-09
$\delta_s(w_{235})$	$u_s(w_{235})$		$u_s(R234/238)$	$u_s(R235/238)$	$u_s(R236/238)$
6.826E-05	2.314E-06		2.959E-07	2.500E-06	2.508E-08
			$c(R234/238)$	$c(R235/238)$	$c(R236/238)$
			-0.03218	-0.92128	-0.03246
			$cont(R236/238)$		
					1.754E-02

We also calculated the relative standard uncertainty of the same process without considering correlation factors. Therefore, the systematic uncertainty components in minor isotopic ratios (RB(23x/238), d(Y), d(L), d(F)) were propagated using Equation (20) instead of Equations (21) and (22). The results of the relative standard uncertainty of measuring ^{235}U enrichment using TIMS for “no correlation” are summarized in Table V.

Table V. Relative standard uncertainty of w_{235} (no correlation).

$\delta(w_{235})$	$u(w_{235})$	w_{235}	$u_r(R234/238)$	$u_r(R235/238)$	$u_r(R236/238)$
5.154E-04	1.747E-05	3.389E-02	6.873E-06	1.896E-05	3.116E-08
		$nu(w_{235})$	$c(R234/238)$	$c(R235/238)$	$c(R236/238)$
		52.6832	-0.03218	-0.92128	-0.03246
			$cont(R234/238)$	$cont(R235/238)$	$cont(R236/238)$
			1.603E-04	9.998E-01	3.352E-09

We then compared the calculated relative standard uncertainties using two propagation methods. The results of Table IV and V indicate that the relative standard uncertainty is almost consistent.

The effect of the correlation factor in minor isotope ratios are neglected in Table V. However, since the uncertainty contribution of minor isotope ratios were almost negligible ($10^{-9} \sim 10^{-4}$), the correlation factor can be neglected within a single ^{235}U enrichment analysis using TIMS.

3.4 Relative standard uncertainty for measurements

In Sections 3.1 ~ 3.3, we calculated the relative standard uncertainties for EBAL, GRAV and TIMS. Table VI summarizes the total relative standard uncertainty (δ), random component (δ_r) and systematic component (δ_s) of the measurement systems in Table I, II and IV.

Table VI. Relative standard uncertainty for all methods.

Method	δ	δ_r	δ_s
EBAL(m)	1.014×10^{-4}	9.279×10^{-5}	4.082×10^{-5}
GRAV(f_U)	6.939×10^{-4}	6.939×10^{-4}	1.101×10^{-6}
TIMS(w_{235})	5.154×10^{-4}	5.109×10^{-4}	6.826×10^{-6}

4. Benchmark Facility based Uncertainty Analysis

We evaluated the effect of the systematic uncertainty component among the total measurement uncertainty by varying the characteristics of a benchmark BHF and

methods of propagation of uncertainty. The benchmark facility is a simplified fuel fabrication plant, which contains homogeneous UO_2 powder and pellets. The characteristics of the benchmark facility are described in Table VII. We assumed the benchmark facility has two measurement systems for powder and pellet strata (EBAL_{PD}, EBAL_{PL}, GRAV_{PD}, GRAV_{PL}, TIMS_{PD} and TIMS_{PL}) with the relative standard uncertainty shown in Table IV.

Table VII. Characteristics of the benchmark BHF.

Stratum	N	\bar{m} (kg)	\bar{f}_U (%)	\bar{w}_{235} (%)
Powder	100	500	88.145	4.5
Pellet	1,000	50	88.145	4.5
M (kg-^{235}U)				3,966

The net mass of nuclear material in the facility is then calculated using Equation (4). We then calculated the measurement uncertainty of the total ^{235}U mass in the benchmark facility ($u(M)$) with (Equation (28)) and without (Equation (29)) the correlation factor between each measurement.

$$u(M) = \sqrt{\begin{aligned} &(\sum_{PD}^{NPD} M_{PD}^2)(\delta_{rm(PD)}^2 + \delta_{rfU(PD)}^2 + \delta_{rw_{235}(PD)}^2) \\ &+ (\sum_{PL}^{NPL} M_{PL}^2)(\delta_{rm(PL)}^2 + \delta_{rfU(PL)}^2 + \delta_{rw_{235}(PL)}^2) \\ &+ (\sum_{PD=1}^{NPD} M_{PD})^2 \delta_{sm(PD)}^2 + (\sum_{PD=1}^{NPD} M_{PD})^2 \delta_{sfU(PD)}^2 \\ &+ (\sum_{PD=1}^{NPD} M_{PD})^2 \delta_{sw_{235}(PD)}^2 + (\sum_{PL=1}^{NPL} M_{PL})^2 \delta_{sm(PL)}^2 \\ &+ (\sum_{PL=1}^{NPL} M_{PL})^2 \delta_{sfU(PL)}^2 + (\sum_{PL=1}^{NPL} M_{PL})^2 \delta_{sw_{235}(PL)}^2 \end{aligned}} \quad (28)$$

$$u(M) = \sqrt{\begin{aligned} &(\sum_{PD=1}^{NPD} M_{PD}^2 + \sum_{PL=1}^{NPL} M_{PL}^2) \\ &\times (\delta_m^2 + \delta_{fU}^2 + \delta_{w_{235}}^2) \end{aligned}} \quad (29)$$

Figures 4 and 5 describe the results of the measurement uncertainty analysis for the benchmark facility using the two propagation methods.

u(M)				
0.214				
	u_r(M)	Fraction		
	0.180	0.707		
			u_r(PD)	Fraction
			0.172	0.909
			u_r(PD(m))	Contribution
			0.018	0.011
			u_r(PD(f,u))	Contribution
			0.138	0.641
			u_r(PD(w_235))	Contribution
			0.101	0.348
			u_r(PL)	Fraction
			0.054	0.091
			u_r(PL(m))	Contribution
			0.006	0.011
			u_r(PL(f,u))	Contribution
			0.044	0.641
			u_r(PL(w_235))	Contribution
			0.032	0.348
	u_s(M)	Fraction		
	0.116	0.293		
			u_s(EBAL_PD)	Contribution
			0.081	0.202
			u_s(GRAV_PD)	Contribution
			0.002	0.000
			u_s(TIMES_PD)	Contribution
			0.014	0.006
			u_s(EBAL_PL)	Contribution
			0.081	0.202
			u_s(GRAV_PL)	Contribution
			0.002	0.000
			u_s(TIMES_PL)	Contribution
			0.014	0.006

Fig. 4. Measurement uncertainty of the benchmark facility and its contribution considering correlation factors.

u(M)				
0.181				
	u(PD)	Fraction		
	0.173	0.909		
			u(PD(m))	Contribution
			0.020	0.014
			u(PD(f,u))	Contribution
			0.138	0.636
			u(PD(w_235))	Contribution
			0.102	0.351
			u(PL)	Fraction
			0.055	0.091
			u(PD(m))	Contribution
			0.006	0.014
			u(PD(f,u))	Contribution
			0.044	0.636
			u(PD(w_235))	Contribution
			0.032	0.351

Fig. 5. Measurement uncertainty of the benchmark facility and its contribution without considering correlation factors.

We also compared the contribution of random and systematic components in measurement for single item and the entire facility (Table VIII).

Table VIII. Uncertainty contribution for random and systematic components.

	Uncertainty contribution (%)			
	Single Item		Entire BHF	
	Rand.	Sys.	Rand.	Sys.
EBAL	83.78	16.22	4.91	95.09
GRAV	99.99	0.01	99.97	0.03
TIMS	98.25	1.75	98.25	1.75
Total	99.77	0.23	70.68	29.32

These results indicate that the uncertainty difference between two methods is significant (~ 15%), while

individual relative standard uncertainty remained the same (Table IV). The uncertainty difference will become increasingly significant as the amount of nuclear material in a BHF increases.

The systematic uncertainty becomes dominant among EBAL and TIMS. Therefore, management of the systematic uncertainty component of EBAL and TIMS is important for nuclear material accounting in large BHF's.

5. Conclusions

GUM based uncertainty expression and propagation has been considered for domestic MBE in the ROK. Since the GUM method adopts reasonable assumptions, the correlation factors with a small contribution are typically neglected in individual measurement. However, a BHF for the MBE contains many items which shares the measurement system. Therefore, the accumulation of correlation factors should not be neglected. We defined uncertainty factors with correlation as systematic uncertainty components and uncertainty factors without correlation as random uncertainty components.

A benchmark BHF was designed with a number of UO₂ powders and pellets. The amount of ²³⁵U in the benchmark facility was measured using EBAL, GRAV and TIMS. We examined the effect of correlation factors in the uncertainty of individual measurement and entire physical inventory.

Our results indicate the effect of correlation is significant for GRAV and negligible for the other methods. Therefore, the effect of correlation must be tested even in individual measurements. The fraction of systematic uncertainty components in individual measurements for EBAL, GRAV and TIMS were 16.22 %, <0.01 % and 1.75 % respectively. However, the fraction of systematic uncertainty components in the BHF inventory became 29.32 %. Therefore, correlation factors between systematic uncertainty components should be carefully propagated for BHF's.

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