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Efficient Technique based on Model Order Reduction for Dynamic Response Analysis of Structures

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1. Introduction

Nowadays, the numerical analysis of complex structures with huge degrees of freedom utilizing numerical method such as finite element analysis became practical thanks to the great advances in the computing power and resources. However, the analysis models are also getting larger and larger to evaluate the detail responses of the structures containing the complex geometry without any simplifications.

Therefore, there remains a demand for an efficient method for solving large and complex problems. In this study, the approach using model order reduction (MOR) technique [1] is introduced to analyze the dynamic problem such as modal analysis and response spectrum analysis in an efficient way.

The full model size can be significantly reduced to a model with small size through this technique without unacceptable loss of accuracy. In addition, the process to link the MOR method to the ABAQUS solution routine is also suggested in order to use the necessary finite element information for model reduction.

2. Model Reduction for Eigenvalue Problems

Model order reduction enables us to obtain an accurate low-dimensional representation of high-dimensional finite element models.

The general eigenvalue problem having large degrees of freedom (N) can be described as follows;

$$\{K-\lambda M\}x(t)=0 \quad (1)$$

If we apply a suitable transformation matrix T such as,

$$q(t)=Tx(t) \quad (2)$$

where $x(t)\in R^N$, $T\in R^{N\times n}$, $q(t)\in R^n$

Then the full model can be transformed to the reduced system with small number of degrees of freedom (n) as follows;

$$\{K_r-\lambda M_r\}q(t)=0 \quad (3)$$

where $K_r=T^TKT$, $M_r=T^TMT$

After all, the key of the reduction of the full model is how effective the transformation matrix is constructed. That is, very efficient eigenvalue analysis is possible by obtaining a transformation matrix consisting of base vectors in the Krylov subspace through the moment matching method [2].

The general dynamic system neglecting the damping term can be described as follows;

$$M\ddot{x}(t)+Kx(t)=fu(t) \quad (4)$$

After the Laplace transformation of Eqn. (4), the transformation function can be written as,

$$s^2MX(s)+KX(s)=FU(s),$$
$$(s^2M+K)X(s)=FU(s),$$
$$H(s)=\frac{X(s)}{Y(s)}=\frac{F}{(s^2M+K)}=(s^2K^{-1}M+I)^{-1}K^{-1}F \quad (5)$$

If we apply the Taylor series expansion to the transformation function (H(s)) with respect to $s=0$, then the following equation can be obtained.

$$H(s)|_{s=0}=[I-s^2K^{-1}M+s^4(K^{-1}M)^2-s^6(K^{-1}M)^3+\cdots]K^{-1}F$$
$$=\sum_{i=0}^{\infty}(-1)^i(K^{-1}M)^iK^{-1}Fs^{2i} \quad (6)$$

The coefficient $(-1)^i(K^{-1}M)^iK^{-1}F$ is defined as the moment of the transformation function $H(s)$.

$$m_i=(-1)^i(K^{-1}M)^iK^{-1}F \quad (7)$$

This can be thought of an index that represents the similarity between the full system and reduced system.

$$H(s)=m_0+m_1(s-s_0)+\cdots+m_n(s-s_0)^n$$
$$\text{: full system} \quad (8\text{-}1)$$
$$\hat{H}(s)=\hat{m}_0+\hat{m}_1(s-s_0)+\cdots+\hat{m}_n(s-s_0)^n$$
$$\text{: reduced system} \quad (8\text{-}2)$$

If we can construct the system that satisfies the following relation up to the desired n^{th} order, then the dynamic characteristics of the reduced system become to be matched with that of the full system [3].

$$m_i=\hat{m}_i,\ i=1,2,\cdots,n \quad (9)$$

The moments of the full system and the reduced system can be matched if the transformation function is constructed with the basis vectors of Krylov subspace.

The n^{th} order Krylov subspace can be defined as follows;

$$Kr_n(A,b)=span(b,Ab,A^2b,\cdots,A^{n-1}b) \quad (10)$$
$$A\in R^{n\times n},\ b\in R^n\text{:starting vector}$$

Here, if $A=K^{-1}M$, $b=K^{-1}F$, then the n^{th} order Krylov subspace can be written as,

$$Kr_n(K^{-1}M,K^{-1}F)=span(K^{-1}F,K^{-1}MK^{-1}F,\cdots,(K^{-1}M)^{n-1}K^{-1}F) \quad (11)$$

When all linear combinations of each column vector of the transformation matrix construct the n^{th} Krylov subspace in Eqn. (11), the moments of the full system and the reduced system become identical up to n^{th} order.

However, the basis vectors shown in the Krylov subspace in Eqn. (11) can cause some numerical errors in practical applications because they may not meet the requirement that all the basis vectors should be linearly independent. Therefore, the new basis vectors need to be generated through the Anoldi process [4] in order to satisfy the linearly independent requirement while maintaining the same Krylov subspace as before.

3. Model Reduction for Response Spectrum Analysis

The response spectrum analysis (RSA) is used to obtain the maximum response of the dynamic behavior of the structural system. The seismic design against the earthquake often requires the maximum displacements or forces due to earthquake loading, so the response spectrum analysis is frequently used dynamic analysis method in the seismic design.

The basic procedure of response spectrum analysis is as follows.

First, the multi-degree of freedom system shown in Eqn. (12) is divided into the independent equation of motion of single degree of freedom by utilizing the orthonormal properties of each mode.

$$M\ddot{x}(t)+C\dot{x}(t)+Kx(t)=-Mr\ddot{x}_g(t) \qquad (12)$$

where M = mass matrix; C=damping matrix, K=stiffness matrix, and r = influence coefficient vector

Secondly, the natural frequencies (ω_i) and natural modes ($\emptyset_i$) is determined by the modal analysis. And then the peak response in the n^{th} mode is estimated by the following steps;

A. The deformation (D_i) and pseudo-acceleration (A_i) corresponding to a specific natural frequency (ω_i) and damping ratio (ζ_i) are obtained from the earthquake response spectrum.
B. The displacements of i^{th} mode are computed as follows;

$$u_i=\Gamma_i\Phi_i D_i \qquad (13)$$

where

$$\Gamma_i=\frac{\Phi_i^T Mr}{\Phi_i^T M\Phi_i} \text{ : mode participation factor} \qquad (14)$$

C. The maximum response can be usually determined through SRSS (Square Root of Sum of Squares) rule because the maximum response of each mode rarely occurs simultaneously.

In order to apply the MOR method, the transformation matrix is first derived from the Krylov subspace as described in previous section. Then we can get natural frequencies (ω_i) and mode shape vectors (Φ_i) from the reduced system in an efficient way. The displacement of each mode can be obtained from Eqn. (13) based on the eigen mode calculated by the reduced system.

4. Integration of ABAQUS and MOR method and Numerical Examples

The integration of commercial finite element codes and MOR method is required for the application to the practical complex problems. The commercial software such as ABAQUS has already equipped various type of elements so we can construct complex model by utilizing the proper elements that the software provides.

Basically, the mass and stiffness matrix information need to be transferred to the MOR module from ABAQUS solution routine. This can be implemented some keywords that the ABAQUS supports and dedicated code developed by MATLAB.

A simple building model constructed with beam element is considered to verify the accuracy and efficiency of the proposed methodology as depicted in Fig. 1. The relevant finite element model has been prepared to have 1116 nodes and 1430 elements with 2-node linear beam element.

The steel material properties (E = 200000MPa, Poisson's ratio = 0.3, density=7.8 e^{-9} ton/mm^3) are applied for the beam elements of the building and the height of the building is 30 m and its width is 15 m.

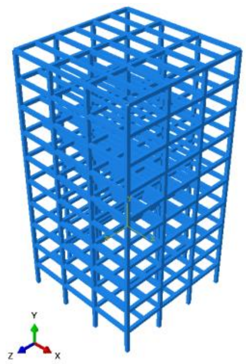

Figure 1 Steel Beam Structure

The modal analysis results are summarized in the Table 1 where the N means the order of the reduced system and M1~M10 stand for each mode number. The results of the reduced systems having different order were compared with those of the full system computed by ABAQUS. The mode shapes up to mode number 4 are shown in Figure 2.

Table 1 Modal Analysis Results (unit: Hz)

	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10
Abaqus	1.787	1.852	2.124	5.469	5.649	6.454	9.519	9.776	11.011	11.900
N=10	1.787	1.852	2.124	5.469	5.649	9.591	10.269	15.612	18.435	38.230
N=20	1.787	1.852	2.124	5.469	5.649	6.454	9.519	9.776	11.015	12.889
N=30	1.787	1.852	2.124	5.469	5.649	6.454	9.519	9.776	11.010	11.899

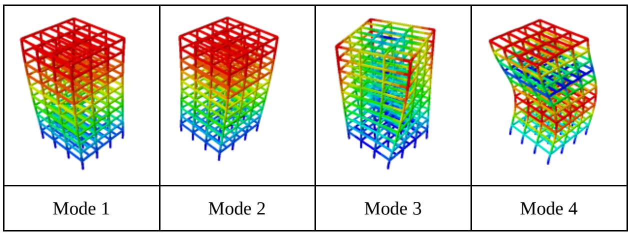

Figure 2 Mode Shapes

The total degree of freedom (TDOF) of this building model is 1116 but the natural frequencies calculated with the reduced system having only 10 order (degrees of freedom) is identical up to fifth mode (M5) and the

accuracy is getting improved as the order of reduced system increases as shown in Table 1.
For the response spectrum analysis, the response spectrums shown in Figure 3 were used as an example.

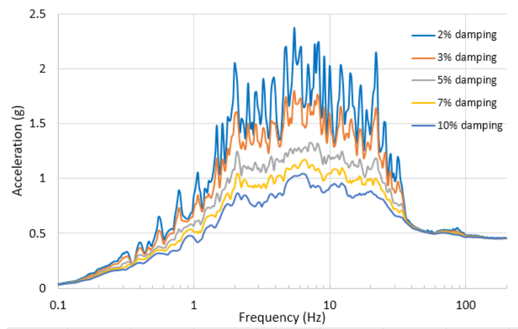

Figure 3 Response Spectrum with Different Damping Ratio

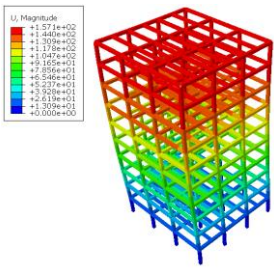

Figure 4 RSA Result (disp.)

Table 2 Comparison of RSA Results

		Full Model	Reduced Model	Relative Error
	DOF	1116	30	
Disp. (mm)	2% damping	157.1	161.1	2.48%
	3% damping	127.5	129.3	1.39%
	5% damping	100.9	101.3	0.39%
	7% damping	86.99	86.61	0.44%
	10% damping	77.55	76.32	1.61%

The maximum displacement of the top story of the building was compared between the reduced model and full model from response spectrum analysis based on the response spectrum in Figure 3. There is only about 2.5% maximum discrepancy between two of them within the considered typical damping range from 2% to 10% regardless of the damping ratio even though the reduced model has much smaller degrees of freedom.

5. Conclusions

The numerical accuracy and efficiency of the reduced model utilizing Krylov subspace based-model order reduction method was validated through its application to the eigenvalue problem and response spectrum analysis of a simple building structure.
It was confirmed that the proposed method could provide accurate results compared to the those of the full system in spite of a significantly reduced degree of freedom.
The advantages of this approach would be greater as the size of the model increases.

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