A Quantification Method using Monte Carlo Sampling for the Seismic Probabilistic Safety Assessment Model of Nuclear Power Plants with Correlated Seismic Failures

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1. Introduction

In the probabilistic safety assessment (PSA) of nuclear power plants, most component failures are considered independent, excluding three dependent failures: human failure events (HFEs), common cause failures (CCFs), and correlated seismic failures. NUREG/CR-7237 [1] conducted a detailed review of numerous seismic PSAs, including seismic failure correlation analysis, and indicated that the calculated seismic core damage frequency (CDF) can considerably vary depending on the seismic failure correlation analysis method. Various seismic failure correlation analysis methodologies proposed after the seismic safety margins research program (SSMRP) [2] were also reviewed in detail therein. The NUREG report highlighted that the multivariate normal (MVN) integration method [3] can be better assess the degree of seismic correlation among the seismic correlation analysis methodologies and procedures.

2. Prior research for correlated seismic failure calculations

In 2019, Sejong University developed CORrelation EXplicit (COREX), a software that uses MVN integration to calculate the combination probability of correlated seismic failures [4]. COREX processes seismic correlation information to calculate all possible combinations of failure event probabilities for seismic failure events with correlations. It then combines these calculated combination probabilities with seismic CCF probabilities to form a set of nonlinear simultaneous equations and solves these equations to compute the probabilities of seismic CCFs. A new method based on COREX was proposed for use in the Kori Nuclear Power Plant. This method enabled the clear modeling of seismic CCFs in multi-unit seismic PSA by converting correlated seismic failures into seismic CCFs. Using this method, complex seismic correlations were reflected in Minimal Cut Sets (MCSs) or fault trees.

However, COREX initially used Monte Carlo integration of MVN integration to calculate failure combination probabilities, presenting several limitations. First, as the number of correlated failure components increases, the integration becomes impractical. Second, calculations become infeasible when noncorrelated failures exist under AND/OR logic gates. To overcome these limitations and reduce the existing uncertainties, a method based on Monte Carlo sampling was proposed to calculate failure combination probabilities.

3. Method for calculating combination probability of correlated seismic failure using MVN integration

The combination probability of a seismic failure $P_{12\ldots n}(a) = P(\cap_{i=1}^{n} A_i < a)$ can be calculated using the Monte Carlo integration of MVN distribution. Monte Carlo integration is a numerical integration method that uses random numbers and is employed only when the integration has no analytical solution [3,4].

$$P_{12\ldots n} = \int_{-\infty}^{ln\left(\frac{a}{A_{1m}}\right)} \int_{-\infty}^{ln\left(\frac{a}{A_{2m}}\right)} \cdots \int_{-\infty}^{ln\left(\frac{a}{A_{nm}}\right)} \frac{1}{\sqrt{|\Sigma|(2\pi)^n}} exp\left(-\frac{1}{2}\boldsymbol{x}^t \Sigma^{-1} \boldsymbol{x}\right) d\boldsymbol{x} \quad (1)$$

$$P_{1+2+\cdots+n} = 1 - \int_{ln\left(\frac{a}{A_{1m}}\right)}^{\infty} \int_{ln\left(\frac{a}{A_{2m}}\right)}^{\infty} \cdots \int_{ln\left(\frac{a}{A_{nm}}\right)}^{\infty} \frac{1}{\sqrt{|\Sigma|(2\pi)^n}} exp\left(-\frac{1}{2}\boldsymbol{x}^t \Sigma^{-1} \boldsymbol{x}\right) d\boldsymbol{x} \quad (2)$$

Here, $\boldsymbol{x}^t = [x_1 x_2 x_{\ldots} x_n]$. Σ is a symmetric positive definite covariance matrix, $|\Sigma|$ is a determinant of Σ, and Σ^{-1} is an inverse matrix of Σ.

$$\Sigma = \begin{vmatrix} \beta_1^2 & \beta_{12}^2 & \cdots & \beta_{1n}^2 \\ \beta_{21}^2 & \beta_2^2 & \cdots & \beta_{2n}^2 \\ \cdots & \cdots & \cdots & \cdots \\ \beta_{n1}^2 & \beta_{n2}^2 & \cdots & \beta_n^2 \end{vmatrix}, \ \beta_{ij}^2 = cov(X_i, X_j) \quad (3)$$

Each covariance β_{ij}^2 in Eq. (3) is a composite variability of uncertainty and randomness. The composite standard deviations are calculated as $\beta_i = \left(\beta_{Ri}^2 + \beta_{Ui}^2\right)^{1/2}$ and $\beta_{ij} = \beta_{ji} = \left(\beta_{Rij}^2 + \beta_{Uij}^2\right)^{1/2}$.

If x_i is replaced with $\beta_i z_i$ as $x_i = \beta_i z_i$, Eq. (1) can be converted into

$$P_{12\dots n} = \int_{-\infty}^{ln\left(\frac{a/A_{1m}}{\beta_1}\right)} \int_{-\infty}^{ln\left(\frac{a/A_{2m}}{\beta_2}\right)} \cdots \int_{-\infty}^{ln\left(\frac{a/A_{nm}}{\beta_n}\right)} \frac{1}{\sqrt{|\Sigma|(2\pi)^n}} exp\left(-\frac{1}{2}\boldsymbol{z}^t \Sigma^{-1} \boldsymbol{z}\right) d\boldsymbol{z} \tag{4}$$

$$P_{1+2+\dots+n} = 1 - \int_{\infty}^{ln\left(\frac{a/A_{1m}}{\beta_1}\right)} \int_{\infty}^{ln\left(\frac{a/A_{2m}}{\beta_2}\right)} \cdots \int_{\infty}^{ln\left(\frac{a/A_{nm}}{\beta_n}\right)} \frac{1}{\sqrt{|\Sigma|(2\pi)^n}} exp\left(-\frac{1}{2}\boldsymbol{z}^t \Sigma^{-1} \boldsymbol{z}\right) d\boldsymbol{z} \tag{5}$$

4. Method for calculating correlated seismic failure combination probability using Monte Carlo sampling

Herein, the probability of seismic failure combinations was calculated using Monte Carlo sampling because the MVN integration method for calculating failure combination probabilities had limitations. As the number of failure combinations increases, solving the integral becomes nearly impossible. The MVN integration method cannot account for non-correlated failures such as HFE or random failures, leading to increased uncertainty. Therefore, combination probability of correlated seismic failures was computed herein using Monte Carlo sampling.

$$P(a) = P(A < a) = P\left(ln(A/A_m) < ln(a/A_m)\right) \tag{6}$$

As observed in Eq. (5), it represents the probability of failure when the ground acceleration capacity, A, is less than the ground acceleration, a. This implies that a component will fail if its capacity cannot withstand the ground acceleration. The failure probabilities of each component are as follows.

$$\begin{aligned} P(A_1 < a) &= P\left(ln\left(\frac{A_1}{A_{1m}}\right) < ln\left(\frac{a}{A_{1m}}\right)\right) \\ &= \Phi\left(\frac{ln\left(\frac{a}{A_{1m}}\right)}{\beta_1}\right) \\ P(A_2 < a) &= P\left(ln\left(\frac{A_2}{A_{2m}}\right) < ln\left(\frac{a}{A_{2m}}\right)\right) \\ &= \Phi\left(\frac{ln\left(\frac{a}{A_{2m}}\right)}{\beta_2}\right) \\ &\vdots \\ P(A_n < a) &= P\left(ln\left(\frac{A_n}{A_{nm}}\right) < ln\left(\frac{a}{A_{nm}}\right)\right) \\ &= \Phi\left(\frac{ln\left(\frac{a}{A_{nm}}\right)}{\beta_n}\right) \end{aligned} \tag{7}$$

The covariance matrix is as follows:

$$\Sigma = \begin{vmatrix} \beta_1^2 & \beta_{12}^2 & \cdots & \beta_{1n}^2 \\ \beta_{21}^2 & \beta_2^2 & \cdots & \beta_{2n}^2 \\ \cdots & \cdots & \cdots & \cdots \\ \beta_{n1}^2 & \beta_{n2}^2 & \cdots & \beta_n^2 \end{vmatrix} = CC^t \tag{8}$$

Before conducting Monte Carlo sampling, the covariance matrix from Eq. (8) is decomposed using the Cholesky decomposition, followed by sampling. The ground acceleration capacity $A^t = [A_1, A_2, \dots, A_n]^t$ is determined using Eq. (9) to (12). Monte Carlo sampling using the following equations is divided into six steps as follows:

$$X = CZ + \mu \tag{9}$$

$$\begin{aligned} \mu^t &= [\mu_1, \mu_2, \dots, \mu_n]^t \\ &= [ln(A_{1m}), ln(A_{1m}), \dots, ln(A_{1m})]^t \end{aligned} \tag{10}$$

$$Z^t = [Z_1, Z_2, \dots, Z_n]^t \tag{11}$$

$$\begin{aligned} A^t &= [A_1, A_2, \dots, A_n]^t \\ &= [exp(X_1), exp(X_2), \dots, exp(X_n)]^t \end{aligned} \tag{12}$$

Step 1: Random numbers $Z^t = [Z_1, Z_2, \dots, Z_n]^t$ are generated from a standard normal distribution $N(0,1)$, where the correlation is not yet reflected.
Step 2: Eq. (9) is used to calculate $X^t = [X_1, X_2, \dots, X_n]^t$, which reflects the correlation. C represents the matrix obtained from the Cholesky decomposition of the covariance matrix.
Step 3: As the capacity values follow a log-normal distribution, the exponential function is applied to the randomly generated X values to determine capacity, as shown in Eq. (12).
Step 4: If the randomly generated capacity $A^t = [A_1, A_2, \dots, A_n]^t$ for each device is less than the ground acceleration a, then that device is considered to have failed.
Step 5: Under AND logic, if all correlated components fail, then the logic is considered to have failed. Under OR logic, if at least one of the correlated devices fails, then the logic is considered to have failed.

Step 6: Ultimately, if sampling is conducted S times and out of those, F samples resulted in failure, then F/S represents the failure probability.

Random failures can also be quantified in terms of failure probabilities using the same method. Random failures are applied to the fault tree with independent failure probabilities, separate from correlated seismic failures.

5. Comparison of the results obtained using MVN integration and Monte Carlo sampling methods

The MVN integration method did not consider various factors (such as random failure) for calculating the combination probability of failures. To reduce these uncertainties, Monte Carlo sampling implemented using COREX was used. The subsequent sections overview the results of the comparisons of failure combination probability for each condition. Table 1 summarizes the COREX input data.

Table 1. Input values for non-symmetric correlated seismic failures (n = 3) [4]

Ground acceleration	A	1.0
Median capacity	A_{1m}	0.8
	A_{2m}	1.0
	A_{3m}	1.2
Covariance $^{1/2}$	$\beta_{R1} = \beta_{U1}$	0.4
	$\beta_{R2} = \beta_{U2}$	0.5
$\beta^2 = \begin{bmatrix} 0.32 & 0.08 & 0.18 \\ 0.08 & 0.5 & 0.32 \\ 0.18 & 0.32 & 0.72 \end{bmatrix}$	$\beta_{R2} = \beta_{U2}$	0.6
	$\beta_{R12} = \beta_{U12}$	0.2
	$\beta_{R13} = \beta_{U13}$	0.3
	$\beta_{R23} = \beta_{U23}$	0.4

$\beta_{R1} = \sqrt{\beta_{Ri}^2 + \beta_{Ui}^2}$ and $\beta_{ij} = \beta_{ji} = \sqrt{\beta_{Rij}^2 + \beta_{Uij}^2}$

Table 2. Failure combination probability calculated using the MVN integration method (n = 3, non-symmetric) [4]

And based method		OR based method	
P_1	0.653381	P_1	0.653381
P_2	0.500000	P_2	0.500000
P_3	0.414953	P_3	0.414935
P_{12}	0.356307	P_{1+2}	0.797074
P_{13}	0.325187	P_{1+3}	0.743130
P_{23}	0.294712	P_{2+3}	0.620224
P_{123}	0.233555	P_{1+2+3}	0.825666

Table 3. Failure combination probability calculated using Monte Carlo sampling (n = 3, non-symmetric)

Disjoint probability		And based method		OR based method	
P_{000}	0.174227	P_1	0.652857	P_1	0.652857
P_{100}	0.203980	P_2	0.501208	P_2	0.501208
P_{010}	0.082791	P_3	0.415886	P_3	0.415886
P_{001}	0.028847	P_{12}	0.357139	P_{1+2}	0.796926
P_{110}	0.123176	P_{13}	0.325701	P_{1+3}	0.743042
P_{101}	0.091738	P_{23}	0.295301	P_{2+3}	0.621793
P_{011}	0.061338	P_{123}	0.233963	P_{1+2+3}	0.825773
P_{111}	0.233963				

Table 4. Sampling validation by comparing the mean values (n = 3, non-symmetric)

	Mean value (input)	Mean value (sampled random numbers)
X_1	-0.22314	-0.22382
X_2	0	0.00316
X_3	0.18232	0.18584

Table 5. Comparison of failure combination probabilities calculated using the MVN integration and Monte Carlo sampling methods (n = 3, non-symmetric)

Probability	MVN integration(a)	Monte Carlo sampling(b)	Relative error(c)
P_1	0.653381	0.652857	0.08
P_2	0.500000	0.501208	-0.24
P_3	0.414953	0.415886	-0.23
P_{12}	0.356307	0.357139	-0.23
P_{13}	0.325187	0.325701	-0.16
P_{23}	0.294712	0.295301	-0.20
P_{123}	0.233555	0.233963	-0.17
P_{1+2}	0.797074	0.796926	0.02
P_{1+3}	0.743130	0.743042	0.01
P_{2+3}	0.620224	0.621793	-0.25
P_{1+2+3}	0.825666	0.825773	-0.01

$(c)\ \frac{(a)-(b)}{(a)} \times 100(\%)$

6. Conclusions

If cases wherein only one group of seismically correlated components and some correlations exist between a few seismic failures, the combination failure probabilities can be calculated and reflected using the seismic cumulative distribution function. However, when multiple groups of seismically correlated components exist and seismic correlation failures complexly range across the entire fault tree, the complex combination probabilities of seismic correlation failures cannot be calculated. The MVN integration method is currently used to calculate the combination failure probabilities in seismic CCFs calculations; however, this method has several limitations. First, as the number of failure components to consider increases, the integration becomes increasingly difficult. Second, the calculation of MCSs under AND/OR conditions is not feasible. Third, when uncorrelated failures exist under the AND/OR conditions that are not seismically correlated, the combination failure probabilities cannot be calculated.

Monte Carlo sampling was therefore used herein to calculate the combination probabilities of seismically correlated failures and reduce the uncertainties posed by the MVN integration method. The advantages of using Monte Carlo sampling are that it can be implemented in COREX and can be used to calculate the combination probability regardless of the number of failure combinations. Moreover, it can be used for combination probability calculations even when the uncorrelated failures are not seismically correlated under AND/OR conditions. It also simplifies the calculation of failure combinations that would have to be solved using the

MVN integration method via sampling. Therefore, by employing Monte Carlo sampling method, correlated seismic failures combined with non-correlated seismic failures and/or random failures can be calculated and reflected into seismic CDF.

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