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Comparison for the calculation methods of a weighted average temperature

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1. Introduction

Arrhenius equation is introduced to calculate the qualified life of equipment [1]. In Arrhenius equation, the activation energy is considered to reflect the relationship between temperature and aging. Therefore, the weighted average temperature should be utilized when the service life of an equipment is calculated using Arrhenius equation. In this study, two methods to calculate a weighted average temperature are introduced and compared to confirm which method is the most optimal and basic equation.

2. Calculation for a weighted average temperature

In this section, the deduction of Arrhenius equation for the calculation of a weighted average temperature is demonstrated. Arrhenius equation indicates that the reactivity rises exponentially with increasing temperature [2].

$$\frac{dR}{dt} = \exp\left(-\frac{E_a}{kT}\right) \tag{1}$$

where

- R is the reaction rate $[s^{-1}]$,
- E_a is the activation energy [eV],
- t is the time to reaction $[s]$ or the end of the qualified life,
- k is Boltzmann coefficient [8.617E-05 eV/K]
- T are the temperature at the certain time $[K]$.

The solution of Equation (1) can be derived as the following where A denotes the pre-exponential factor.

$$t = \frac{1}{R} = \frac{1}{A}\exp\left(\frac{E_a}{kT}\right) \tag{2}$$

When t is replaced with the entire measure time ($t_{measure}$) in Equation (2), T should be replaced with a weighted average temperature ($T_{weighted}$) as the following.

$$t_{measure} = \frac{1}{A}\exp\left(\frac{E_a}{kT_{weighted}}\right) \tag{3}$$

Since the measured temperature is discriminative, a method to calculate a weighted average temperature should be introduced. Therefore, two methods to calculate a weighted average temperature are compared to confirm which method is the most optimal and basic in this study.

2.1 Method 1: Using a pre-equivalent temperature

In this section, the method of using pre-equivalent temperature to calculate a weighted average temperature is introduced. The calculation steps of this method are:

1) Assume a pre-equivalent temperature ($T_{pre-equi}$),
2) Calculate a pre-equivalent time ($t_{pre-equi}$) with a measured temperature ($T_{measure}$), then
3) Calculate a weighted average temperature ($T_{weighted}$) with the pre-equivalent time and the measured temperature.

The pre-equivalent temperature ($T_{pre-equi}$) is a virtual parameter. And, the entire pre-equivalent time ($\Sigma_{i=1}^{n} t_{pre-equi}$) can be calculated as the following according to Equation (2).

$$\Sigma_{i=1}^{n} t_{pre-equi}^{i} = \frac{1}{A}\exp\left(\frac{E_a}{kT_{pre-equi}}\right) \tag{4}$$

$$\Sigma_{i=1}^{n} t_{pre-equi}^{i} = \Sigma_{i=1}^{n} \frac{1}{A}\exp\left(\frac{E_a}{kT_{measure}^{i}}\right) \tag{4`}$$

When Equations (3) and (4) are organized for $1/A$, the following equation to calculate the weighted average temperature is derived.

$$\begin{aligned}\Sigma_{i=1}^{n} t_{pre-equi}^{i} &= t_{measure}\exp\left(\frac{E_a}{kT_{weighted}}\right)^{-1}\cdot\exp\left(\frac{E_a}{kT_{pre-equi}}\right) \\ &= t_{measure}\exp\left[\frac{E_a}{k}\left(\frac{1}{T_{pre-equi}} - \frac{1}{T_{weighted}}\right)\right]\end{aligned} \tag{5}$$

When organizing Equation (5) for $T_{weighted}$, the following equations is deduced.

$$\frac{1}{T_{weighted}} = \frac{1}{T_{pre-equi}} + \frac{k}{E_a}\cdot\ln\left(\frac{t_{measure}}{\Sigma_{i=1}^{n} t_{pre-equi}^{i}}\right) \tag{6}$$

So far, a method to calculate a weighted average temperature utilizing the pre-equivalent temperature is introduced.

2.2 Method 2: the formula in IEEE-1205

In this section, the formula in IEEE-1205 [2] to calculate a weighted average temperature is introduced. The entire measured time ($t_{measure}$) can be calculate with the measured temperature according to the following equation.

$$t_{measure} = \frac{1}{A}\left[\exp\left(\frac{E_a}{kT^1_{measure}}\right) + \exp\left(\frac{E_a}{kT^2_{measure}}\right) + \cdots + \exp\left(\frac{E_a}{kT^n_{measure}}\right)\right] = \frac{1}{A}\Sigma^n_{i=1}\exp\left(\frac{E_a}{kT^i_{measure}}\right) \quad (7)$$

When organizing Equations (3) and (7), the following equation is deduced.

$$T_{weighted} = \frac{\frac{E_a}{k}}{\ln\left(\frac{t_{measure}}{\Sigma^n_{i=1}\exp\left(\frac{E_a}{kT^i_{measure}}\right)}\right)} \quad (8)$$

Comparing Equations (6) and (8), the formula in IEEE-1205 is remarkably simple.

3. Comparison of two methods

3.1 The measured temperature data

In this section, the virtual data to compare Methods 1 and 2 for the calculation of a weighted average temperature is introduced.

Table 1: The virtual data of measured temperature

$T^i_{measure}$[°C]	$t^i_{measure}$[min]
10	25
15	15
25	25
35	5
45	10
35	15
25	10
15	20
10	5

According to Table I, the total measured time ($t_{measure}$) is 130 minutes. In addition, the activation energy is assumed to be 0.8eV as the virtual data.

3.2 The comparison of Methods 1 and 2

In this section, the comparison between Methods 1 and 2 is demonstrated. $\Sigma^n_{i=1}t^i_{pre-equi}$ and $\exp\left(\frac{E_a}{kT^i_{measure}}\right)$ can be calculated using the virtual data in Table 1. And, those results are tabulated in Table 2.

Table 2: The calculation process measured temperature

Data No.	Method 1		Method 2
	$t^i_{pre-equi}$ [min]	$t^i_{pre-equi}$ [min]	$\exp\left(\frac{E_a}{kT^i_{measure}}\right)$
	$T_{pre-equi}$ =50°C	$T_{pre-equi}$ =100°C	
1	0.4319	0.0092	1.440E-13
2	0.4577	0.0097	1.526E-13
3	2.2477	0.0479	7.495E-13
4	1.2349	0.0263	4.118E-13
5	6.3667	0.1355	2.123E-12
6	3.7046	0.0789	1.235E-12
7	0.8991	0.0191	2.998E-13
8	0.6103	0.0130	2.035E-13
9	0.0864	0.0018	2.880E-14
Sum	16.0392	0.3415	5.3483E-12

1) Method 1 at $T_{pre-equi}$ =50°C

Using Equation (6) and the result in Table II, a weighted average temperature can be calculated as the following.

$$\frac{1}{T_{weighted}} = \frac{1}{50+273.15} + \frac{8.617\text{E}-05\ \text{eV/K}}{0.8eV}\cdot\ln\left(\frac{130}{16.0392}\right) = 3.320\text{E}-03[\text{K}^{-1}] \quad (9)$$
$$\therefore T_{weighted} = 28.0612℃$$

2) Method 1 at $T_{pre-equi}$ =100°C

When calculating the weighted temperature in the same way of Method 1 at $T_{pre-equi}$ =100°C, the weighted temperature is the same as the following.

$$\frac{1}{T_{weighted}} = \frac{1}{100+273.15} + \frac{8.617\text{E}-05\ \text{eV/K}}{0.8eV}\cdot\ln\left(\frac{130}{0.3415}\right) = 3.320\text{E}-03[\text{K}^{-1}] \quad (10)$$
$$\therefore T_{weighted} = 28.0612℃$$

In short, the pre-equivalent does not affect the calculation result of a weighted average temperature.

3) Method 2

The weighted average temperature can also be calculated using Equation (8) and the calculation result in Table II as the following.

$$T_{weighted} = \frac{\frac{0.8eV}{8.617\text{E}-05\ \text{eV/K}}}{\ln\left(\frac{130}{5.3483\text{E}-12}\right)} = 28.0612℃ \quad (11)$$

As shown above, the calculation results of a weighted average temperature using Methods 1 and 2 are the same.

3. Conclusions

In this study, the calculation methods and results of a weighted average temperature are compared. The two pre-equivalent temperatures for Method 1 are introduced to prove that the pre-equivalent temperature does not affect the calculation of a weighted average temperature.

As the result, Method 2 is the best calculation formula to calculate a weighted average temperature because:

1) The formula of Method 2 is comparatively simple and intuitive, and
2) The pre-equivalent temperature does not affect the calculation result of a weighted average temperature. In other words, the pre-equivalent temperature is not necessary to calculate a weighted average temperature.

Therefore, using Equation (8) in Method 2 is recommended to calculate a weighted average temperature.

REFERENCES

[1] A. Mantey, A Review of Equipment Aging Theory and Technology Revision 1, EPRI NP 1558, p-74, 2020.
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