

A Fast Variance Reduction Technique for Efficient Radiation Shielding Calculations in Nuclear Reactors

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1. Introduction

The growing demand for cleaner and more sustainable energy sources has spurred significant interest in the development of Small Modular Reactors (SMRs). Unlike traditional large-scale nuclear reactors, SMRs are designed to be compact, modular, and capable of being deployed in a wide range of environments, from remote locations to densely populated areas. These characteristics, while offering considerable advantages in terms of flexibility and scalability, also present unique challenges, particularly in the area of radiation shielding. SMRs, with their compact size and modular construction, necessitate a more refined and comprehensive approach to radiation shielding. Efficient, light weight, compact radiation shielding Analysis is needed [1].

Reference Reactor is Lead-Bismuth Eutectic (LBE) cooled fast-spectrum SMR. Rated power of Reference Reactor is 40 MW_{th} and provided for at least 15 years [2].

For Reactor Radiation shielding Analysis, we will utilize the Monte Carlo radiation transport code, MCNP6, to perform k-code criticality calculations for a lead-bismuth eutectic (LBE) cooled reactor. K-code calculation provides not only the effective multiplication factor but also data on neutron flux, reaction rates, particle energy spectra, and more. The focus of this research is on calculating neutron flux and dose rates by simulating individual particle histories and recording key aspects to describe the average behavior of the reactor. To achieve this, we will use a three-dimensional heterogeneous reactor model to accurately analyze the reactor's behavior under different conditions.

Using MCNP k-code criticality calculations for primary radiation shielding design can be time-consuming due to the need for very low relative errors, making it impractical for detailed radiation mapping. To efficiently generate extensive radiation maps, variance reduction techniques (VRT) are essential. Traditional methods like SSW/SSR (Surface source write/Surface source read) record sources on a designated surface and use these surface sources instead of the original ones. However, these methods have limitations, such as the inability to utilize multi-threading during recording and the potential for insufficient particle capture. For example, when considering the analysis of radiation leakage from a shielded structure around a reactor, if the shielding is highly effective and neutrons rarely reach the surface where the source is recorded, a bias could occur between the surface source and the actual source.

Similarly, the WWG (Weight Window Generation) technique may fail to record weights accurately. To overcome these challenges and improve calculation efficiency, this paper aims to develop a new VRT that does not rely on surface sources and supports multi-threading.

2. Methodology and Reactor Overview

2.1 Reference Reactor Description

The Reference Reactor is Lead-Bismuth Eutectic (LBE) cooled fast-spectrum SMR whose name is NCLFR-Oil. Rated power of Reference Reactor is 40 MW_{th} and provided for at least 15 years [2]. The LBE coolant provides effective gamma shielding, while the cladding and reflector are constructed using T-91 stainless steel and Yttria-stabilized zirconia (YSZ) to withstand LBE-induced corrosion. Detailed composition and geometry can be found in Table 1, Table 2[2].

Table 1. Main Design Parameters Description

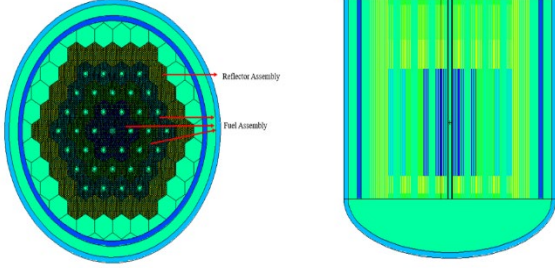
| Parameters | Specification |
|--|--------------------------|
| Thermal power | 40 MW_{th} |
| Fuel | UO ₂ |
| Enrichment (Innermost/Middle/Outer most) | 13.5wt%/16.5wt%/18.5 wt% |
| Cladding | T91 |
| Reflector | YSZ |
| Primary coolant gap | LBE Helium |
| Assembly geometry | Hexagonal |
| Initial reactivity swing | 5247pcm |

Table 2. Design parameters of fuel assembly

| Parameters | Specification |
|--------------------------------|---------------|
| Number of fuel assemblies | 37 |
| Number of pin per one assembly | 198 |
| Equivalent core diameter | 180(cm) |
| Active core height | 90(cm) |
| Pitch to diameter ratio | 1.2 |
| Fuel Pin diameter | 0.56(cm) |
| Assembly geometry | Hexagonal |

As shown in Fig 1, the reactor core consists of 37 fuel assemblies and is surrounded by reflector assemblies, LBE coolant, a barrel, downcomer LBE, and the reactor pressure vessel.

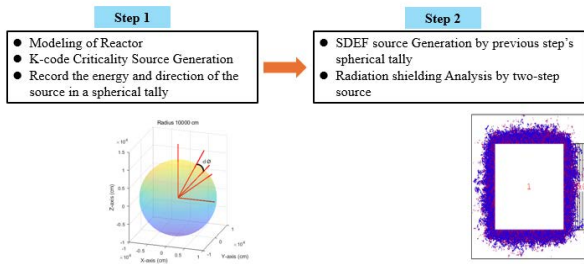
Fig 1. Reactor core configuration, XY view (Left), XZ view (Right)



2.2 Two-Step Variance Reduction Technique

To reduce the relative error in criticality calculations and improve computational efficiency, it is essential to use variance reduction techniques (VRT). Although the SSW/SSR VRT, which involves defining a surface around the reactor and recording source information on that surface, is available, it has the limitation of not supporting multi-threading. Therefore, a new VRT is proposed to address this limitation. The new VRT follows the flow chart presented in Fig 2.

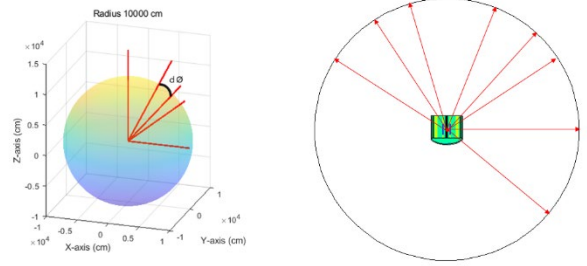
Fig 2. Two-Step Method Flow Chart



In Step 1, For a conservative radiation dose calculation, the reactor is modeled under the assumption of all rods out and steady-state BOC conditions. Then, k-code criticality source (Direct Source) is generated. The radiation transport calculation is performed using this Direct Source. To ensure particles reach the reactor perpendicularly, a large spherical tally surrounding the reactor is defined, as shown in Fig 3. The number of particles and flux can be recorded on this sphere using an f1 or f4 tally. This sphere is divided into equal intervals based on the polar angle ($0 < \text{polar angle} < \pi$) in spherical coordinates and based on Particle energy groups. Assuming azimuthal symmetry due to the reactor's symmetric structure, the flux distribution can be

analyzed. By using the FS card and Tally energy card, the direction and energy distribution of particles reaching the spherical tally can be determined based on the polar angle. To prevent overestimation in the tally due to particles being recorded from scattering outside the reactor vessel, the region outside the reactor vessel is set as void which means no material space.

Fig 3. Spherical Tally for Recording Source's Direction and Energy



In Step 2, a fixed source (Two-step Source) is defined using the direction and energy distribution recorded in Step 1. This source is made by the MCNP6's SDEF card which is used to specify the direction, energy, particle type, position, and shape of the radiation source in MCNP. Radiation shielding materials Outside the Reactor vessel, such as concrete are then modeled while the reactor structures are removed. Radiation shielding analysis is performed using the Two-step Source.

Due to the significant attenuation of gamma rays by the thick LBE coolant and concrete, their contribution to the dose is negligible compared to neutrons. Therefore, the calculation is performed with the "mode n p" option, but only neutrons are recorded in the tally.

However, it is important to note that the tally results must scaled by factors such as C_1 and C_2 according to Eq. (1). $\varnothing_{\text{Direct}}$ is direct source's dose or flux. $\varnothing_{\text{Sdef}}$ is Two-step source's dose or flux. Step1's tally has to be scaled by C_1 because direct source's tally results is normalized per fission neutron [3]. Similarly Step 2's tally has to be scaled by C_2 because number of fixed source particles is normalized to 1.

$$\varnothing_{\text{Direct}} = \varnothing_{\text{Sdef}} C_1 C_2 \quad (1)$$

$$C_1 = \frac{P\nu}{Qk_{\text{eff}}} \quad (2)$$

$$C_2 = \frac{\sum_i \varnothing_{\text{Direct},i}}{\sum_j \varnothing_{\text{Sdef},j}} \quad (3)$$

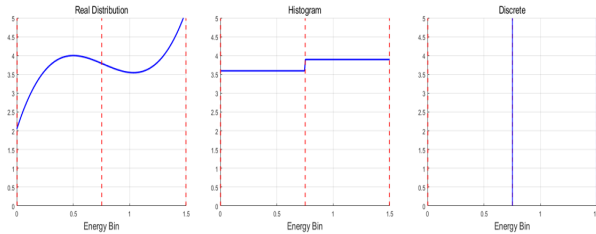
C_1 is number of fission neutron. P is reactor's thermal power. ν is average number of neutrons released per fission. Q is average recoverable energy per fission. k_{eff} is effective multiplication factor. C_2 is the ratio of the

number of neutrons from the direct source to the number of neutrons from the Two-step source passing through the spherical tally. This can be calculated using the number of neutrons that passed through the Step 1 tally.

3. Result and Sensitivity Analysis

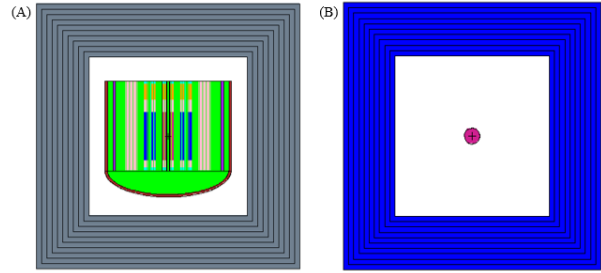
In this study, sensitivity analysis of the source is performed using direction divisions of 64, 128, 256, and 500, and energy divisions of 48, 101, and 202. The direction and energy distributions stored in these divided tallies are used in Step 2 to define the energy and direction of the source. The methods used to define the energy intervals and directions of the source are histogram and discrete mode, respectively. As shown in Fig 4, a histogram represents a continuous spectrum, while a discrete spectrum is represented by line spectrum. For example, Suppose 5 particles are recorded in the energy range $0 < E < 0.75$ Mev. When defining the source using the histogram method, the source is redefined with random energies within this range. For example, particles with energies of 0.1, 0.3, 0.4, 0.6, and 0.7 MeV might be defined. However, when using the discrete method, all 5 particles would be defined with an energy of 0.75 MeV.

Fig 4. Real distribution (Left), Histogram Distribution (Middle), Discrete Distribution (Right)



To verify that the Two-Step source in the Two-Step method accurately simulates the direct source, the dose ratios between the direct source and the Two-Step source are compared based on concrete thickness. As shown in Fig 5, the dose ratios measured on the concrete surface are compared at 10 cm intervals.

Fig 5. Direct source (Left), Two-Step source (Right)



To identify the optimized division, energy is discretely divided into 101 segments as a baseline. Eq. (4) and (5) are used to compare the different division methods. Eq. (4) represents the dose ratio of the Two-Step source to the direct source, serving as an indicator of how accurately the Two-Step source replicates the original direct source. The value L in Eq. (5) is the sum of the squared differences between the dose ratio and 1 across various concrete thicknesses. A smaller L indicates a more accurate division method.

$$\text{Dose Ratio} = \frac{\text{Two-step source dose rate}}{\text{Direct source dose rate}} \quad (4)$$

$$L = \sum_i ((\text{Dose Ratio}_i - 1)^2) \quad (5)$$

The goal of the sensitivity analysis is to identify a Two-Step source that is both conservative (yielding higher doses than the direct source) across all concrete thicknesses and closely replicates the direct source, indicated by a low L value. Fig 6 shows the baseline with energy divided into 101 discrete segments. Among the direction divisions of 64, 128, 256, and 500, the 128 discrete direction division yields the lowest L value of 0.3305, indicating the best replication of the direct source. The average dose error is 18.4%, consistently overestimating compared to the direct source. As seen in Fig. 6 and Fig. 7, when energy is sufficiently divided, there is little difference between dividing direction using the discrete method or the histogram method. Comparing Fig. 6 and Fig. 8, when energy is divided using the histogram method, underestimation occurs, resulting in doses lower than those from the direct source. Consequently, the Two-Step source becomes less reliable. Comparing Fig. 6 and Fig. 9, it is evident that, except for the 256 direction divisions, there is generally low accuracy. This indicates the need for more refined energy group divisions. Fig. 10 represents the best approximation that most accurately replicates the direct source. In this division method, energy is divided into 202 discrete groups, and direction is divided discretely as well. With 128 direction divisions, the LLL value is 0.1816, showing the closest similarity to the direct source. The average dose rate error compared to the direct source is 12.32%, and the required computation time to achieve

the same relative error as the direct source is reduced by 74 times.

In the BOC criticality calculation, the average number of neutrons produced per fission was 2.494, the average recoverable energy per fission was 201 MeV, and the effective multiplication factor was 1.05247. However, since steady-state conditions are assumed, the effective multiplication factor is taken as 1. As a result, the calculated C_1 value from Eq. (2), representing the number of fission neutrons, was $3.098 \text{ E}+18$ neutrons. Additionally, the C_2 value was calculated to be 0.08216.

Fig 6. discrete divisions in direction, 101 discrete divisions in energy

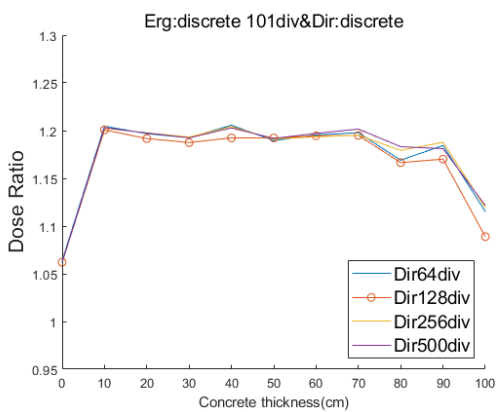


Fig 7. histogram divisions in direction, 101 discrete divisions in energy

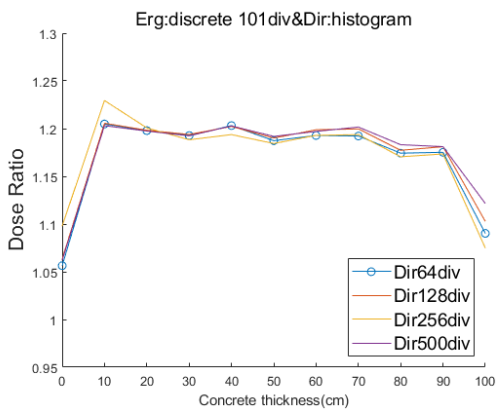


Fig 8. discrete divisions in direction, 101 histogram divisions in energy

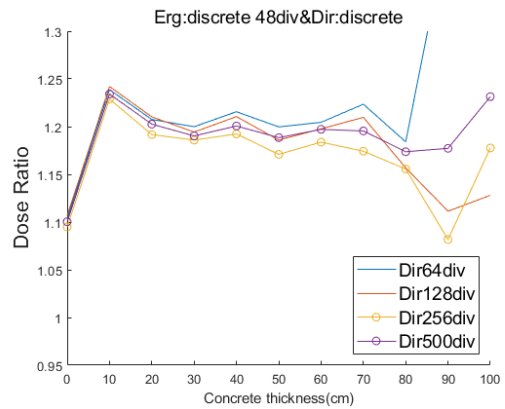


Fig 9. discrete divisions in direction, 48 discrete divisions in energy

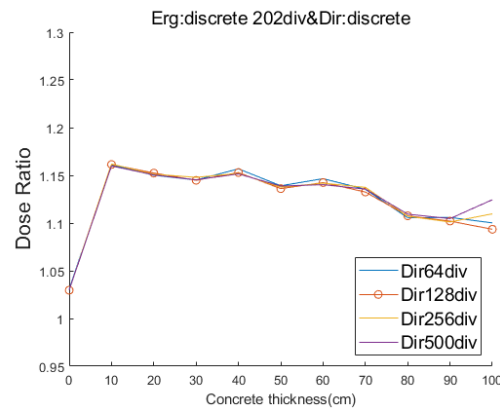
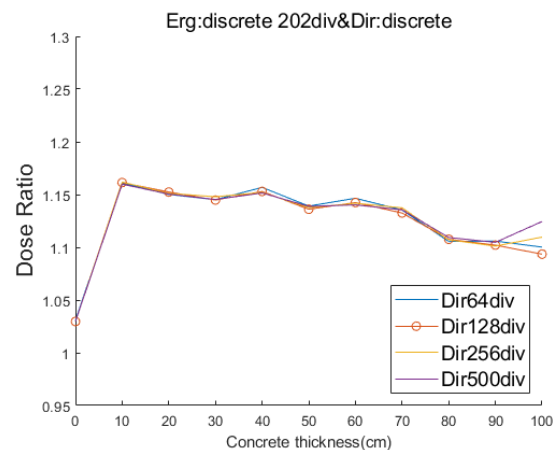


Fig 10. discrete divisions in direction, 202 discrete divisions in energy



4. Conclusions

This study aimed to develop a VRT that can replace the traditional SSW/SSR method for radiation shielding analysis in Small Modular Reactors (SMRs). The

proposed VRT in this paper offers the advantage of multi-threading, improving computational efficiency significantly. The core idea is to record the energy and direction of particles at large Shere surrounding the reactor from the k-code criticality source (Direct source) and redefine them using a Two-Step Source (Fixed Source, SDEF) for subsequent radiation shielding analysis.

Our findings indicate that the best approximation, which closely replicates the direct source while conservatively estimating the dose, is achieved with a 202 discrete energy division and a 128 discrete direction division. This configuration not only provided the lowest L value of 0.1816, indicating a strong similarity to the direct source, but it also reduced the computation time by 74 times while maintaining an average dose rate error of 12.32%.

However, it was observed that using histogram-based energy divisions can lead to underestimation, raising concerns about the reliability of the Two-Step source in certain configurations. Furthermore, the discrepancies in dose rates between the direct and Two-Step sources were primarily attributed to scattering caused by the radiation shielding like concrete.

Overall, this study demonstrates the effectiveness of the Newly proposed VRT in providing a reliable and efficient approach to radiation shielding analysis in SMRs, with significant improvements in computation time and accuracy. However, the selection of appropriate energy and direction divisions remains crucial to ensuring the reliability of the results.

Acknowledgements

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