Development of Revising Accurate Weight Method for Hybrid Monte Carlo Simulation

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***Keywords:** Variance Reduction, Hybrid Monte-Carlo Method, Revising Weight

1. Introduction

With increasing computational power, Monte Carlo (MC) simulation has been widely performed in the radiation transport field. It is well known that the MC method has good accuracy. However, the MC method still has the disadvantage of lower computational efficiency compared to deterministic methods. To increase the calculation efficiency of the MC method, Variance Reduction (VR) techniques have been introduced [1]. Recently, hybrid MC methods were introduced that apply VR techniques using deterministic methods. Among them, the most successfully applied methods are CADIS [2] and FW-CADIS [3] methods. The CADIS and FW-CADIS methods were developed to optimize a single response, such as a source and detector problem, and a global response, such as a dose map, respectively. The hybrid MC methods dramatically improve computational efficiency. However, still takes a huge computational time for large facilities with complex structures such as a Test Blanket Module (TBM) or the other systems in the International Thermonuclear Experimental Reactor (ITER) [4]. In the hybrid MC method, the efficiency can be decreased according to errors caused by the methodology of hybrid MC, and assumptions such as space, angle, and energy division in the deterministic method.

In this study, a Revising Accurate Weight (RAW) method is proposed to adjust the VR parameter by correcting the error of the deterministic method using MC simulation. For verification, results were compared with the FW-CADIS method in two problems

2. Methods and Results

2.1 Hybrid Monte Carlo Method for Single Response

For a single response (detector) problem, the Consistent Adjoint Driven Importance Sampling (CADIS) method is proposed [2]. It determines the weights using the following equation:

$$w(\vec{r},E,\hat{\Omega}) = \frac{R}{\psi^{+}(\vec{r},E,\hat{\Omega})} \tag{1}$$

where ψ^{+} is an adjoint flux calculated by using a single response as the adjoint source and R is a response.

Since calculating Eq.1 by the MC method takes a long time, it is calculated by a deterministic method. Then, the weights are applied in MC simulation. This method is called a hybrid MC method in that it utilizes both methods.

2.2 Hybrid Monte Carlo Method for Global Problem

Cooper and Larsen [5] have suggested that uniform distribution of MC particles can lead the uniformly low statistical uncertainty. The density of MC particle has the following relationship:

$$m(\vec{r}) \approx n(\vec{r}\,)/\overline{w}(\vec{r}) \tag{2}$$

where $n(\vec{r}\,)$ and $\overline{w}(\vec{r})$ are the density of analog particles and average weight in space $\vec{r}$, respectively.

From uniform MC particle distribution, $m(\vec{r})$ is a constant. Then the average weight can be written as follows:

$$\overline{w}(\vec{r}) \propto n(\vec{r}\,)\ or\ \phi(\vec{r}) \tag{3}$$

where $\phi(\vec{r}) = n(r) \times v$ and $\phi(\vec{r})$ is forward scalar flux in space $\vec{r}$.

With the position with the highest flux as a reference value, the weight proposed by Cooper and Larsen can be expressed as the following equation:

$$\overline{\mathrm{w}}(\vec{r}) = \phi(\vec{r})/\max(\phi(\vec{r})). \tag{4}$$

The FW-CADIS method, which is known to be the most efficient in the global problem, set the adjoint source as follows using the concept of Cooper and Larsen:

$$\mathrm{q}^{+}(\vec{r},E) = \sigma_{\mathrm{d}}(\vec{r},E)/R(\vec{r}) \tag{5}$$

where R is a response in space $\vec{r}$, and $\sigma_{\mathrm{d}}(\vec{r},E)$ is the objective function. Weight values of the FW-CADIS method [3] can be obtained by substituting adjoint fluxes from adjoint sources in Eq.(5) into Eq.(1)

2.3 RAW Method for Correcting Particle Weights

For the global problem, Van Wijk et al. [6] proposed a method using relative error (RE) instead of the flux as follows:

$$\overline{w}(\vec{r}) = Min(RE)/RE(\vec{r}) \tag{6}$$

where $Min(RE)$ and $RE(\vec{r})$ are a minimum RE of mesh space and RE in $\vec{r}$ mesh space, respectively.

In hybrid MC simulation, high RE can be generated by their methodology or assumption of the deterministic calculation. Hence, the weight value can be corrected using RE from MC calculation.

In analog MC simulation, RE will be inversely proportional to the square root of the analog particle density or flux [7] as the following relationship:

$$RE(\vec{\mathrm{r}}) \propto 1/\sqrt{\mathrm{n}(\vec{\mathrm{r}})} \;\; or \;\; 1/\sqrt{\phi(\vec{\mathrm{r}})}. \qquad (7)$$

Substituting Eq.(7) into Eq.(3) and rearranging, the following relationship can be derived as follows:

$$\overline{w}(\vec{r}) \propto 1/RE(\vec{r})^2. \qquad (8)$$

With the position with the minimum RE as a reference value, weight values of the hybrid MC method can be revised by using RE of MC simulation as the following equation:

$$\overline{\mathrm{w}}_{\mathrm{raw}}(\vec{r}, E) = \overline{w}_{hy}(\vec{r}, E) \times Min[RE]^2/RE^2(\vec{r}) \qquad (9)$$

where $\overline{\mathrm{w}}_{\mathrm{raw}}$ and $\overline{w}_{hy}$ are the mean weight of the RAW method and the hybrid MC method, respectively.

2.4 Efficiency of MC Simulation for Global Problem

For the global problem, the average Figure of Merit (FOM) [3] and maximum FOM [8] are introduced as the following equations:

$$FOM_{ave} = 1/(\overline{RE}^2 \times T) \qquad (10)$$

and

$$FOM_{max} = 1/(Max[RE_i] \times T) \qquad (11)$$

where $\overline{RE} = 1/N \sum_{i=1}^{N} RE_i$, N is the number of tallies, and RE_i is the RE of i^{th} tally.

2.5 Verification of RAW Method

For the verification of the RAW method, the FW-CADIS method which is built in ADVANTG code [9] was set as a hybrid MC method. The weight values in the weight window file from the FW-CADIS method were revised by the proposed method in this study. MC simulation is performed by MCNP [10] code.

2.5.1 Concrete Cube Problem

The model of the concrete cube problem as shown in Fig. 1 has a regular hexahedron shape with 250 cm thickness. A 1 MeV neutron point source is located in the center of the cube. The concrete with a 2.26 g/cm^3 density was used. Each axis was divided into 25 to create a weight window file for ADVANTG calculation and also, the mesh tally was set equally to revise the weight values. Table I and Fig. 2-3 show the results and the RE distribution of the concrete cube problem.

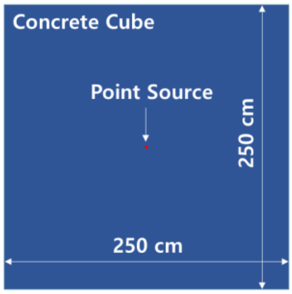
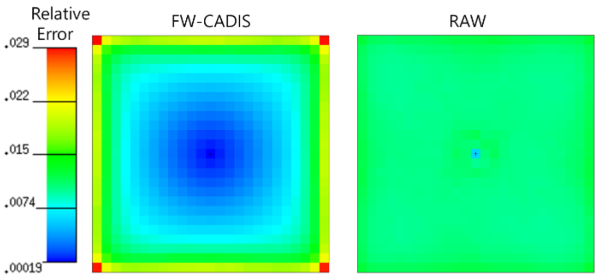


Fig. 1. Calculation model for concrete cube problem

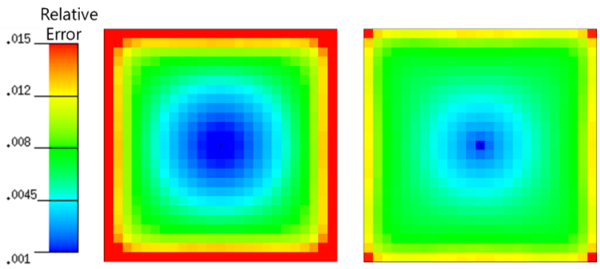
Table I: Results from FW-CADIS and RAW method for concrete cube problem

Method	Total MC Time [Min]	FOM_{ave}	FOM_{max}
FW-CADIS	465	19.9	2.49
RAW	457(23*)	21.1	13.9
Ratio**	-	106.0%	558.2%

*: MC calculation time to get RE for RAW method
**: RAW/FW-CADIS



(a) Set color bar range to maximum and minimum value of FW-CADIS method



(b) Set color bar range to maximum and minimum value of RAW method

Fig. 2. RE distribution obtained by FW-CADIS and RAW method for concrete cube problem

In the case of the FW-CADIS method, Lower REs are distributed near the center of the cube, and higher REs are distributed near the outer boundary of the cube. On the other hand, the RAW method gives uniform RE distribution compared to the FW-CADIS method. The FOM_{ave} values of both methods were similar. However, the FOM_{max} using the RAW method was increased about 5.5 times compared with that of the FW-CADIS method.

2.5.2 Japan Power Demonstration Reactor Problem

For additional verification, the flux distribution of the Japan Power Demonstration Reactor (JPDR) was evaluated. The JPDR problem is an example of ADVANTG code [9] to demonstrate VR efficiency. The geometry and material information of this model are shown in Fig. 3. To generate the weight window file, the x and y axes were divided into 83 and the z axes into 144. Also, the mesh tally was set equally to revise the weight values. Table II and Fig 4 are the results and RE distribution, respectively.

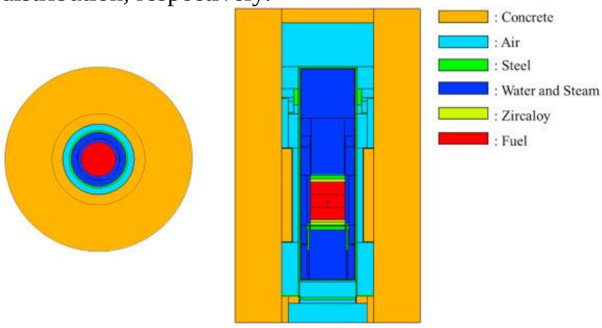


Fig. 3. Geometry and material information for JPDR

Table II: Results from FW-CADIS and RAW method for JPDR problem

Method	Total MC Time [Min]	FOM_{ave}	FOM_{max}
FW-CADIS	39162	2.69×10^{-1}	3.18×10^{-4}
RAW	40660(3919*)	5.99×10^{-1}	3.91×10^{-3}
Ratio**	-	222.9%	1228.8%

*: MC calculation time to get RE for RAW method
**: RAW/FW-CADIS

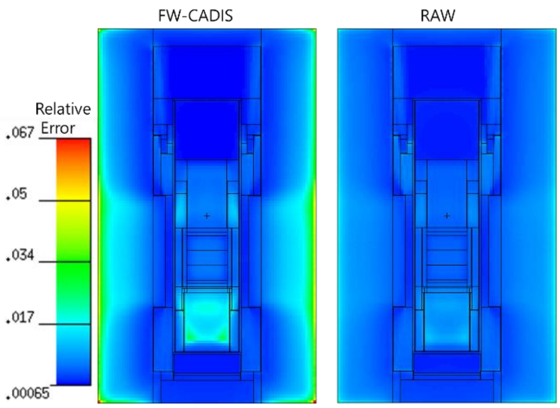


Fig. 2. RE distribution obtained by FW-CADIS and RAW method for concrete cube problem

In this problem, the FOM_{ave} and FOM_{max} values from the RAW method were, respectively, increased by 2.2 and 12.2 times compared to those of the FW-CADIS method.

3. Conclusions

In the hybrid MC method, the methodology and assumption of the deterministic method can generate inefficiencies in MC simulation. In this study, to increase the efficiency of MC simulation, the RAW method was derived to correct the weight values of the hybrid MC methods. To verify the RAW method, two problems were evaluated. The average FOMs did not increase significantly. However, maximum FOMs were greatly increased. Also, the distributions of RE from the RAW method are more uniform than those of the FW-CADIS method. Therefore, it is expected that the RAW method can increase the computational efficiency for responses with high RE. In future work, the proposed method will be applied to TBM nuclear analysis in ITER.

Acknowledgments

This work was supported by the R&D Program through the Korea Institute of Fusion Energy (KFE) and funded by the Ministry of Science and ICT of the Republic of Korea (KFE-IN2403).

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