Comparison of the steam turbine off-design performance between the MARS-KS code and Stodola cone law

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1. Introduction

As the demand for decarbonized energy sources continues to rise, the role of nuclear power is becoming increasingly significant. Small modular reactors (SMRs), in particular, offer promising potential for deployment in large industrial facilities or in regions requiring decentralized power generation. These reactors are perceived as safer, particularly with the incorporation of passive safety systems. Additionally, SMRs are anticipated to have the capability to operate under varying load conditions.

Power generation systems included in nuclear reactors, including SMRs, typically utilize a steam Rankine cycle. Throttling control, which adjusts the steam volumetric flow rate, serves as the primary control strategy for managing load variations. For both quasi-steady and transient analyses of the steam cycle, it is crucial to understand the turbine's pressure ratio and efficiency when the mass flow rate changes at a constant rotational speed. This is because the partial power output and the off-design performance of the feedwater heater train are directly influenced by the turbine operating performance.

In recent years, there has been significant research aimed at integrating artificial intelligence (AI) into control methodologies to enhance the analysis of power cycle transitions, moving beyond conventional control theories [1, 2]. The MARS-KS code, developed for analyzing transients in nuclear systems, is also being prepared to support the adoption of the advanced control system. However, it is recognized that MARS-KS exhibits a room for improvement in modeling secondary system components, such as steam turbine to be able to model the whole SMR system and evaluate the advanced control system.

Therefore, it is essential to verify the accuracy of steam turbine in MARS-KS code, which are critical components of the power generation system, to ensure reliable transient analysis for simulating load-following operations in SMRs controlled by AI. The target component of this study is the 50 MWe class steam turbine developed by NuScale, the first reactor to receive U.S. NRC standard design approval for an SMR.

2. Methods and Results

2.1 MARS-KS code

MARS-KS (Multi-dimensional Analysis of Reactor Safety - Korea Standard) is a thermal-hydraulic safety analysis code developed by the Korea Atomic Energy Research Institute (KAERI). This code is primarily employed for the safety evaluation of nuclear reactor systems under both operational and accidental conditions. MARS-KS integrates the capabilities of RELAP5 and COBRA-TF, enabling it to model one-dimensional and three-dimensional fluid flow, heat transfer, and reactor kinetics with good accuracy [2]. It is extensively utilized in the South Korean nuclear industry for the design and safety assessment of pressurized water reactors.

There was a reported issue of steam turbine modeling in MARS-KS. Until version 1.4 of the MARS-KS code, the law of energy conservation did not hold true across the turbine stage. Turbine power output should be the product of the enthalpy difference across the turbine and the mass flow rate. However, upon reviewing the results from MARS-KS, discrepancies were identified. This issue arose because the previous versions assumed that the density before and after the turbine was the same, thereby applying an isentropic process assumption [4]. The problem was solved by implementing the Modified PV term option along with energy conservation equation [5].

2.2 Stodola cone law

Stodola cone law, a fundamental principle in the analysis of steam turbines, describes the relationship between the mass flow rate and the pressure ratio in a turbine stage. According to this law, for a given turbine stage, the mass flow rate through the turbine is approximately proportional to the square root of the difference between the squares of the upstream and downstream pressures as shown in Equations 1, 2 and Fig. 1 [6, 7]. In the context of steam turbines, Stodola cone law provides a reliable method for estimating the performance of turbine stages, especially under varying load conditions. It enables engineers to predict how changes in pressure ratios across the turbine stages will affect the mass flow rate, which in turn influences the turbine's efficiency and output power. By applying

Stodola cone law, it is possible to optimize the design of turbine blades and nozzles to ensure efficient energy conversion under a wide range of operating conditions.

$$\frac{\dot{m}_{off}}{\dot{m}_{d}} = \frac{\frac{P_{in,off}}{\sqrt{T_{in,off}}}}{\frac{P_{in,d}}{\sqrt{T_{in,d}}}} \sqrt{\frac{1 - \left(\frac{P_{out,off}}{P_{in,off}}\right)^{2 - \eta\left(\frac{\gamma-1}{\gamma}\right)}}{1 - \left(\frac{P_{out,d}}{P_{in,d}}\right)^{2 - \eta\left(\frac{\gamma-1}{\gamma}\right)}}} \cong \sqrt{\frac{1 - PR_{off}^{2 - \eta\left(\frac{\gamma-1}{\gamma}\right)}}{1 - PR_{d}^{2 - \eta\left(\frac{\gamma-1}{\gamma}\right)}}} - Equation 1$$

$$PR_{off} = \left(1 + \left(\frac{\dot{m}_{off}}{\dot{m}_{d}}\right)^{2} \left(PR_{d}^{2 - \eta\left(\frac{\gamma-1}{\gamma}\right)} - 1\right)\right)^{\frac{1}{2 - \eta\left(\frac{\gamma-1}{\gamma}\right)}} - Equation 2$$

$$\dot{m}$$

$$cheked operation \qquad p_{in}$$

Fig. 1. Elipsoidal curve from Stodola cone law not choked (left) and choked (right) for small number of stages.

2.3 Design condition of target system

In 2020, NuScale obtained standard design approval for its Small Modular Reactor from the U.S. Nuclear Regulatory Commission (NRC). This study focuses on the 50 MWe steam Rankine cycle turbine within the system as shown in Fig. 2. The steam cycle comprises a total of eight turbine sets, where the inlet conditions for the first and second sets are in the superheated state, while the turbine sets from the third to the eighth operate in a liquid-vapor two-phase state. The design conditions for these turbine sets are detailed in Table I. The first and sixth turbine sets were selected for comparison between MARS-KS and the Stodola cone law. The Stodola cone law is known to have relatively higher accuracy for turbines with lower pressure ratios. Additionally, the similar pressure ratios of these two turbines allow for a comparison between the operation in the superheated steam state and the saturated state.



Fig. 2. Power conversion system block flow diagram. [8]

Fable I:	Conditions	of steam	turbine	stages	in N	JuScale
	Powe	r Module	e 50MW	e		

	Inlat tomp	Inlet	Mass	Inlet	Pressure
	rr	pressure	flow rate	quality	ratio
	[K]	[kPa]	[kg/s]	[-]	[-]
1^{st}	580.0	3397	67.07	1.000	1.237
2^{nd}	558.3	2747	67.07	1.000	4.997
3 rd	428.6	549.8	63.74	0.962	2.065
4 th	402.7	266.3	58.71	0.928	1.837
5 th	383.5	144.9	57.73	0.915	2.112
6 th	361.8	66.56	53.74	0.887	1.200
7 th	357.1	55.47	52.55	0.898	1.938
8 th	341.2	28.62	51.19	0.897	3.308

2.3 Results

The differences in pressure ratio when evaluating the off-design performance of the NuScale turbine sets using MARS-KS and the Stodola cone law are presented in Table II and Table III. Table II corresponds to the first set operating under superheated conditions, while Table III corresponds to the sixth set operating under two-phase conditions.

Table II: Comparison between MARS-KS and Stodola cone law (1st turbine set of Nuscale 50MWe cycle)

Mass flow	MARS-KS	Stodola	Difference
rate factor	PR	PR	(%)
2	1.68	1.79	-6.50
1.7	1.54	1.61	-4.23
1.5	1.45	1.49	-2.84
1.4	1.40	1.44	-2.19
1.3	1.36	1.38	-1.58
1.2	1.32	1.33	-1.04
1.1	1.28	1.28	-0.57
1	1.24	1.24	0.00
0.9	1.20	1.20	0.14
0.8	1.16	1.16	0.36
0.7	1.13	1.12	0.48
0.6	1.10	1.09	0.51
0.5	1.07	1.06	0.46

Table III: Comparison between MARS-KS and Stodola cone law (6th turbine set of Nuscale 50MWe cycle)

Mass flow rate factor	MARS-KS PR	Stodola PR	Difference (%)
2	1.43	1.63	-12.20
1.7	1.37	1.47	-7.03
1.5	1.32	1.38	-4.18
1.4	1.30	1.34	-2.97
1.3	1.27	1.29	-1.92
1.2	1.24	1.25	-1.01
1.1	1.21	1.22	-0.27
1	1.18	1.18	0.00
0.9	1.16	1.15	0.75
0.8	1.13	1.12	0.96
0.7	1.10	1.09	1.00
0.6	1.08	1.07	0.93
0.5	1.06	1.05	0.76

When the mass flow rate factor is around 50%, the difference between the two is less than 4%, but when the mass flow rate is doubled, the difference increases to 6% and 12%.

As shown in Fig. 3, for the same mass flow rate factor, there is less discrepancy between MARS-KS and Stodola cone law in the superheated state compared to the saturated state. This phenomenon can be due to the specific heat characteristics of the two-phase flow, which the Stodola equation may not adequately reflect in the equation.



Fig. 3. Difference of pressure ratio between MARS-KS and Stodola cone law.

3. Conclusions

The load variation in small modular reactors leads to changes in the mass flow rate of steam. Additionally, when the compression work is considered sufficiently small, it is sometimes assumed that the mass flow rate and the power output of the generation system are linearly related. Therefore, using the steam turbine unit in MARS-KS, it is evaluated how accurately that it could simulate performance changes with respect to mass flow rate by comparing with Stodola cone law. It was observed that when the mass flow rate increased by up to two times, the discrepancy between the two models reached up to 12%. However, in the low flow rate range, where most load variation transient analyses will be conducted, the difference between the two was less than 1%. This suggests that MARS-KS is capable of performing load variation transient analysis with reasonable accuracy for now.

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