Investigation of crack characteristics of steel linear plate of containment building using XFEM for the quantification of leakage

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1. Introduction

When a severe accident happens in the containment building of the nuclear power plants, there might be a possibility of the radioactive material leakage through the cracks in the containment building wall. And the results of experimental and analytical researches done in the past shows that leakage rather than burst is most likely failure mode under qusai-static internal pressure [1]. Therefore, the reasonable estimation of the leakage can be one of the crucial factors for evaluating the effects of radiation to the environment in case of severe accidents in the nuclear power plants. From this perspective, we first need to consider the characteristics of cracks in the reinforced concrete wall and liner plate attached inside of the concrete wall of the containment building wall in order to quantify the leakage in a reasonable manner. The dominant failure mechanism has been found to be liner crack caused by the interaction of the liner and its anchorage system with the concrete which the anchors of liner plate are embedded because major stiffness discontinuity is expected in this region [2]. Therefore, in this paper, we aim to see the crack characteristics of the liner plate and the feasibility of the quantification of leakage through liner plate based on a numerical method utilizing XFEM.

2. Methods and Results

In this section the numerical techniques used to investigate the crack characteristics of the liner plate are described.

2.1 Features of XFEM

The Extended Finite Element Method(XFEM) is an advanced numerical technique that enhances the classical Finite Element Method(FEM) by incorporating enrichment functions to model discontinuities and singularities within elements. Despite the widespread use and success, conventional FEM faces challenges when dealing with certain types of problems:

- Discontinuities: FEM struggles to accurately represent strong discontinuities (e.g. cracks) and weak discontinuities (e.g. interface of different materials) without extremely fine mesh refinement. (refer to Fig. 1)

- Singularities: Problems involving stress singularities, such as crack tips in fracture mechanics, require special treatment in conventional FEM.
- Moving boundaries: Simulating evolving geometries, like crack propagation, often necessitates remeshing, which can be computationally expensive and prone to errors.
- Mesh dependency: The accuracy of FEM solutions can be highly dependent on mesh quality and refinement, especially near discontinuities and singularities.

So, the XFEM was developed to address these limitations of conventional FEM. XFEM can model discontinuities independent of the mesh structure and represent singularities accurately without excessive mesh refinement. It also enable the simulation of evolving discontinuities without remeshing. These unique features can lead to the improvement of solution accuracy while maintaining computational efficiency.



Fig. 1 Modeling of strong and weak discontinuities in XFEM technique [3]

2.2 Key Concepts

The core behind the XFEM is to enhance the standard finite element approximation space with problem-specific enrichment functions as shown in Eq. (1) [4]

$$u(x) = \sum_{i=1}^{N} N_i(x) \bar{u}_i + \sum_{j=1}^{M} N_j(x) \Psi(x) \bar{a}_j$$
(1)

N: the set of all nodal points of domain $N_i(x)$: the standard FE shape functions \bar{u}_i : standard nodal displacement M: the set of nodes of elements located on the discontinuity Γ_d

 N_j : shape functions of enriched part

 $\Psi(x)$: enrichment function

 \bar{a}_i : the nodal DOF corresponding to the enrichment function

The first term of right side of Eq. (1) is conventional FEM displacement field and the second term shows the enrichment functions

These enrichment functions are designed to capture the known behavior of the solution near discontinuities or singularities. Enrichment means adding special functions to the finite element approximation to represent discontinuities or singularities. There are two types of enrichment functions in XFEM. One is the Heaviside functions which is used to represent strong discontinuities such as cracks. The other is Asymptotic crack-tip functions to model the singular behavior near crack tips in fracture mechanics problems as represented in Eq.(2)

$$u(x) = \sum_{i \in S} N_i(x)\overline{u}_i + \sum_{j \in S_H} N_j(x) \left\{ H(\varphi(x)) - H(\varphi(x_j)) \right\} \overline{a}_j$$

+ $\sum_{k \in S_C} N_k(x) \sum_{\alpha=1}^4 (\Psi^{\alpha}(x) - \Psi^{\alpha}(x_k)) \overline{b}_k^{\alpha}$ (2)

 \bar{u}_i : Nodal DOF for conventional shape functions $N_i(x)$ Heaviside enrichment term:

 $H(\varphi(x))$: Heaviside function

 \bar{a}_j : Nodal enriched DOF (jump discontinuity)

 S_H : Nodes belonging to elements cut by crack

x_j: Position of node *j*

Crack tip enrichment term:

 $\Psi(x)$: Crack tip asymptotic functions to model singularity \bar{b}_k^{α} : Nodal DOF (crack tip enrichment)

S_C: Nodes belonging to elements containing crack tip

: Crack
• : Nodes in S_H B
\blacksquare : Nodes in S_C

Fig. 2 Crack representation with associated XFEM nodes

2.2 Numerical Examples

A simple three point beam bending simulations was performed to confirm the unique characteristics of XFEM. Beam length(L) and height(h) are assumed as 60 mm and 10 mm respectively. The finite element model was generated using 2D 4-node bilinear plane strain element (CPE4) in Abaqus software. The material is Tin Alloy and its material properties are summarized in Table 1.



Fig. 3 Simulation model

	Table 1.	Material	properties ((Tin Alloy))
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Elastic modulus	Poisson's ratio	Fracture Energy
(MPa)		(N/mm)
1.0E4	0.36	35

The punch pushes down the center of the beam by 10 mm. and when the maximum principal strain reaches 0.3, then the damage is initiated.



Fig. 4 Crack initiation and propagation



Fig. 5 Crack surface

As shown in the figures above, the crack was initiated when the maximum principal stress reaches its damage criteria and continues to propagate as the load increases. Even if the crack occurrence point is not specified in advance, the crack point is determined automatically at the point that meet the damage criteria. There is no need to mesh refinement around crack tip and crack propagation area as depicted in Fig. 5. This enables to reduce the complexity of model preparation process dramatically as well as the computational time compared to other fracture simulation approaches.

2.3 Fracture simulation for liner plate embedded in concrete

In order to investigate the feasibility of estimating the leakage through liner plate crack in a quantitative manner, the analysis model was constructed as shown in Fig. 6.

The concrete inner radius and thickness was assumed as 2.0 m and 1.2 m., and the liner plate's outer radius is 2.0 m and its thickness is 0.006 m (6 mm). The L-angle attached to the backside of linear plate. The major material properties are summarized in Table 2. The internal pressure of 3 MPa was applied inside surface of the liner plate and the bottom face of the model was fully constrained. The type of damage evolution is based on power law energy damage criterion, mixed mode

Conventional FEM terms:

behavior for the power law(power=1), and liner softening was applied. The normal mode, first direction shear mode and second direction shear mode fracture energy was given as 42200 N/m.



Fig. 6 Analysis Model: (a) assembly of concrete wall and liner plate, (b) liner plate with L-angle

Concrete				
Elastic modulus (GPa)	Poisson's ratio	Density (Kg/m ³)	Compressive strength (MPa)	Tensile strength (MPa)
29.14	0.17	2351	34.3	18.96
	Concrete Damage Plasticity Parameters			
Dilation angle	Eccentricity	fb0/fc0	k	Viscosity parameter
36	0.1	1.16	0.6667	0.001
Liner plat	e			
Elastic modulus (GPa)	Poisson's ratio	Density (Kg/m ³)	Fracture Energy (N/mm)	Damage Criteria (MPa)
200	0.3	7800	42200	100 (max. principal stress)

Table 2 Material properties

The analysis results showed that the maximum principal stress occurred in the region of intersection between embedded L-angle and the liner plate as expected. So the crack is initiated at the point where the stress meets the damage criteria shown in Table 2.



Fig. 7 Maximum principal stress



Fig. 8 Crack initiation and propagation pattern

And it can be seen that cracks propagate along the path where the maximum principal stresses occurred.

Ultimately, in order to calculate the amount of leakage in the liner plate, it is necessary to calculate the crack area based on the width and length of the crack as depicted in Fig. 9.



Fig. 9 Idealization of flow of pressurized gas through a liner plate tear [5]

XFEM provides two signed distance functions \emptyset and ψ to describe the geometry of the crack as shown in Fig. 10. We can estimate the crack width and length by utilizing these values and the relevant nodal position information



Fig. 10 Representation of nonplanar crack in 3-dimensions by two singed functions

In this example, the values of singed function are calculated at specific nodal points as below, here, ϕ^+ and ϕ^- mean the distance from the crack surface to the adjacent nodes which are located in the positive and negative direction with respect to the crack surface.

Table 3 crack width

signed distance function values			original
ϕ^+	ϕ^-	$\phi^+ - \phi^-$	nodal distance (d)
0.0106577	-0.0043383	0.014996	0.015

When we compare the value $(\phi^+ - \phi^-)$ and d in the Table 3, the difference is almost negligible. This means that the crack width is extremely small even there is a visible crack. Actually the crack in this example was not a full penetrating crack. The crack depth is almost 40% of the thickness of the linear plate. Therefore, this result is understandable.

3. Conclusions

In this study, we sought to verify the feasibility of simulating the crack characteristics of a liner plate embedded in concrete using XFEM. Although a closer review considering more diverse analysis conditions and boundary conditions is necessary, it was basically confirmed that crack analysis of a liner plate including an anchor area of a complex three-dimensional shape using XFEM can provide reasonable results. Through this, we plan to conduct further research to derive the correlation between crack area, internal pressure level, and geometric characteristics of the liner plate anchor to quantify the amount of leakage through the cracks of liner plate embedded in the concrete of the nuclear power plant containment building under postulated severe accident conditions.

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