Monte Carlo sampling for seismic probabilistic safety assessment involving correlated and uncorrelated seismic failures

U Sung Moon[a], Woo Sik Jung[a]*
[a]*Nuclear Engineering, Sejong University, 209 Neungdong-ro, Gwanggin-gu, Seoul, Korea*
**Corresponding author: woosjung@sejong.ac.kr*

Keywords: *probabilistic safety assessment, correlated seismic failure, Monte Carlo sampling*

1. Introduction

In the probabilistic safety assessment (PSA) of nuclear power plants, most component failures are considered independent, excluding the following three: human failure events, common-cause failures, and correlated seismic failures.

Correlated seismic failures become dependent when multiple components fail simultaneously because of an earthquake event. Component failures with high correlations are combined into a single component failure for application in the fault tree. The combination probabilities of correlated seismic failures can be calculated using multivariate normal (MVN) integration [1], which was proposed in the NUREG-7237 [2] report. However, the limitations of MVN integration are as follows: (1) The calculation becomes impossible when the number of failed components increases beyond a certain threshold. (2) The probability of a failure combination cannot be calculated when correlated and uncorrelated failures coexist. (3) Calculations cannot be performed across multiple types of distributions, such as the standard normal distribution, log-normal distribution, exponential distribution, and others. To address these limitations, this paper proposes Monte Carlo sampling to calculate the failure combination probabilities for both correlated and uncorrelated failures.

In a prior study [1], the CORrelation Explicit (COREX) software program, developed based on MVN integration by Sejong University in 2019, was used to calculate the combined probabilities of correlated seismic failures. However, considering the aforementioned limitations of MVN integration, the method was updated to calculate the combined probabilities of correlated seismic failures via Monte Carlo sampling [2]. In the present study, the failure combination probabilities for both correlated seismic failures and uncorrelated failures (random failures) were calculated, as described in the following sections.

2. Monte Carlo sampling method

2.1 Failure probability for a single component

The basic principle of Monte Carlo sampling involves sampling the Capacity A of a component T times. If A is less than a specified ground acceleration a, the component is considered to have failed, and vice versa. If Component A fails a certain number of times F out of the total number of samples T, the failure probability $P(a)$ is expressed as

$$P(a) = P(A < a) \approx F/T. \qquad (1)$$

Eq. (1) represents the procedure for calculating the failure probability of a single component. As described earlier, this calculation can be performed regardless of whether correlations are considered. An example of calculating the individual failure probabilities for multiple components is also shown in Eq. (2):

$$\begin{aligned}
P(a) &= P(A_1 < a) \approx F/T \\
P(a) &= P(A_2 < a) \approx F/T \\
&\vdots \\
P(a) &= P(A_k < a) \approx F/T \\
P(a) &= P(A_{k+1} < a) \approx F/T \\
&\vdots \\
P(a) &= P(A_{n-1} < a) \approx F/T \\
P(a) &= P(A_n < a) \approx F/T \\
&A_1 \ldots A_k{:}\ Correlated\ seismic\ failures \\
&A_{k+1} \ldots A_n{:}\ Randomfailures.
\end{aligned} \qquad (2)$$

In Eq. (2), the capacities of correlated failed components $A_1 - A_k$ are sampled from an MVN distribution, while the capacities of uncorrelated failed components $A_{k+1}-A_n$ are sampled from a log-normal distribution.

2.2 Failure probability of multiple components

Eqs. (1) and (2) represent the procedure for determining the individual failure probabilities for uncorrelated components. Conversely, the failure combination probability for correlated components is determined using AND/OR logic. When the failures of components are combined using OR logic, the set of capacities $[A_1, A_2, \ldots]$ is sampled T times. If at least one component's capacity is less than the ground acceleration a, it is considered an OR logic failure. If such failures occur F times, the OR logic combination probability is calculated using Eq. (3):

$$P(a) = P(A_1 < a \ \cup \ A_2 < a \ \cup \cdots) \approx F/T. \qquad (3)$$

As explained earlier, if the capacity A of each component is less than the ground acceleration a, the component is considered to have failed and is represented by “1”; otherwise, it is represented by “0.” A simple example of applying OR logic is shown in Eq. (4):

$$[A_1, A_2, A_3, \dots] = [\dots, 0, 1, 0 \dots]. \tag{4}$$

As shown in Eq. (4), based on OR logic, a failure is considered to occur when all the correlated components fail. Therefore, if OR logic failure occurs F times out of T times, the failure combination probability is F/T.

When the failures of components are combined using AND logic, the set of capacities $[A_1, A_2, \dots]$ is sampled T times. If at least one component's capacity is less than the ground acceleration a, it is considered an AND logic failure. If such failures occur F times, the AND logic combination probability is determined using Eq. (4) and (5):

$$P(A_1 < a \ \cap \ A_2 < a \ \cap \ \cdots) \approx F/T, \tag{5}$$

$$[A_1, A_2, A_3, A_4, \dots] = [1,1,1, \dots] \; for \; all \; failures. \tag{6}$$

As shown in Eq. (6), based on AND logic, a failure is considered to occur when all the correlated components fail. If AND logic failure occurs F times out of T times, the failure combination probability is F/T.

The procedures described in Sections 2.1 and 2.2 were used to determine the failure combination probability for correlated components. Section 2.3 explains the procedure of Monte Carlo sampling in detail.

Unlike MVN integration, in which the correlations among components are considered, Monte Carlo sampling incorporates correlations by applying the Cholesky decomposition of the covariance matrix (Eq. (8)) to log-normally distributed component capacities [4]. Thus, the correlations are considered during the sampling process. The seismic capacity of components can be determined using Eqs. (7)–(11). Additionally, after sampling the failure probabilities of the components, the failure combination probability can be calculated by dividing the number of failures of correlated components by the total number of samples.

2.3 Procedure of Monte Carlo sampling

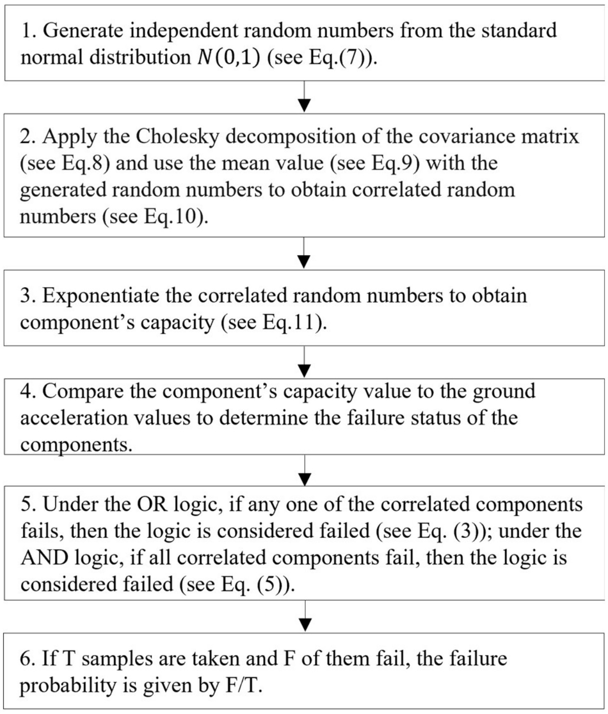

Fig. 1. Flowchart illustrating Monte Carlo sampling procedure

The process depicted in Fig. 1 can be explained as follows:

$$Z^t = [Z_1, Z_2, \dots, Z_n]^t \tag{7}$$

(Step 1) Generate random numbers from a standard normal distribution that do not reflect correlation. These random numbers will be used in the next step to create correlated values.

$$\Sigma = \begin{bmatrix} \beta_1^2 & \beta_1^2 & \cdots & \beta_{1n}^2 \\ \beta_{21}^2 & \beta_2^2 & \cdots & \beta_{2n}^2 \\ \cdots & \cdots & \cdots & \cdots \\ \beta_{n1}^2 & \beta_{n2}^2 & \cdots & \beta_n^2 \end{bmatrix}$$
$$= \begin{bmatrix} \beta_1 & 0 & \cdots & 0 \\ \beta_{21} & \beta_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ \beta_{n1} & \beta_{n2} & \cdots & \beta_n \end{bmatrix} \begin{bmatrix} \beta_1 & \beta_{12} & \cdots & \beta_{1n} \\ 0 & \beta_2 & \cdots & \beta_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \beta_n \end{bmatrix} \tag{8}$$
$$= CC^t,$$

$$\mu^t = [\mu_1, \mu_2, \dots, \mu_n]^t$$
$$= [ln(A_{1m}), ln(A_{2m}), \dots, ln(A_{nm})]^t, \tag{9}$$

$$X = CZ + \mu. \tag{10}$$

(Step 2) Calculate the correlated values in Eq. (9) using the random numbers generated in Step 1 and Eqs. (6)–(8). Here, C is the lower triangular matrix obtained via Cholesky decomposition of the covariance matrix in

Eq. (6). Cholesky decomposition is used to transform independent standard normal random numbers into correlated random numbers through the process expressed in Eq. (10).

$$A^t = [A_1, A_2, \dots, A_n]^t \\ = [exp(X_1), exp(X_2), \dots, exp(X_n)]^t. \quad (11)$$

(Step 3) Apply the exponential function to the correlated random values to obtain the capacities, since the seismic capacity values of the components follow a log-normal distribution [3], as shown in Eq. (11).

(Step 4) Determine if the calculated capacity value for each component is less than the given ground acceleration a. If so, the component is considered to have failed, referencing Eq. (11).

(Step 5) Apply OR logic as shown in Eq. (3). OR logic failure is considered to have occurred if any one of the correlated components fails. Apply AND logic as shown in Eq. (5). AND logic failure is considered to have occurred if all the correlated components fail.

(Step 6) Calculate the failure probability by taking T samples and determining the number of failures F. The failure probability is F/T.

Monte Carlo sampling offers the following advantages: (1) Calculation remains possible even as the number of failure components increases. (2) The probability of a failure combination can be calculated even when correlated and uncorrelated failures coexist. (3) Calculations can be performed across various types of distributions, such as the standard normal distribution, log-normal distribution, exponential distribution, and others.

Monte Carlo sampling has been integrated into COREX [1]. Therefore, COREX can be used to perform logical calculations based on the MVN integration method as well as the versatile Monte Carlo sampling method, which imposes no limit on the number of failure components.

3. Application

The combined probabilities of correlated seismic failures and random failures were calculated using the Monte Carlo sampling method described in Section 2, and the calculations were performed with one million samples.

Table 1. Correlated random failures (X_1–X_3) [1]

Ground acceleration	a	1.0
Median capacity	A_{1m}	0.8
	A_{2m}	1.0
	A_{3m}	1.2
Covariance	$\beta_{R1} = \beta_{U1}$	0.4
	$\beta_{R2} = \beta_{U2}$	0.5
$\beta^2 = \begin{bmatrix} 0.32 & 0.08 & 0.18 \\ 0.08 & 0.5 & 0.32 \\ 0.18 & 0.32 & 0.72 \end{bmatrix}$	$\beta_{R2} = \beta_{U2}$	0.6
	$\beta_{R12} = \beta_{U12}$	0.2
	$\beta_{R13} = \beta_{U13}$	0.3
	$\beta_{R23} = \beta_{U23}$	0.4

$\beta_{R1} = \sqrt{\beta_{Ri}^2 + \beta_{Ui}^2}$ and $\beta_{ij} = \beta_{ji} = \sqrt{\beta_{Rij}^2 + \beta_{Uij}^2}$

Table 1 presents the input conditions used in the initial study [1] to calculate the combined probabilities of correlated seismic failures through MVN integration. Based on these conditions, we aimed to calculate the failure combination probabilities of correlated seismic failures and random failures via Monte Carlo sampling.

Table 2. Noncorrelated random failures (X_4–X_6)

Ground acceleration	a	1.0
Median capacity	A_{4m}	1.0
	A_{5m}	1.2
	A_{6m}	1.4
Standard deviation $\beta^2 = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.72 & 0 \\ 0 & 0 & 0.98 \end{bmatrix}$		

Tables 1 and 2 summarize the input data for COREX. Table 1 presents the conditions for the three correlated components, while Table 2 compiles the input data for the uncorrelated components. In this study, sampling was conducted on six components, three correlated and three uncorrelated.

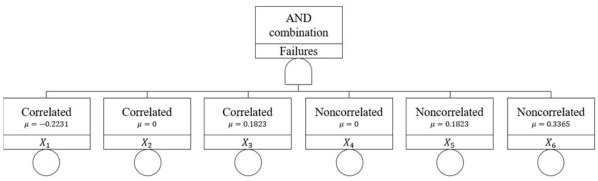

Fig. 2. AND combination of failure components

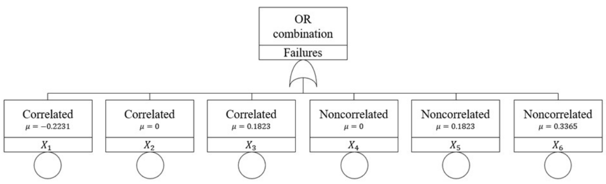

Fig. 3. OR combination of failure components

The failure combination probability for the six components was calculated using AND/OR logic, as shown in Figs. 2 and 3. The failure probability of the correlated components was calculated as explained in Section 2, while that of the uncorrelated components (random failures) was determined through sampling from a log-normal distribution (assuming that the capacity of the components followed such a distribution).

Table 3. Sampling validation

	Mean value (input)	Mean value (sampled random numbers)
X_1	−0.2231	−0.2224
X_2	0	0.0019
X_3	0.1823	0.1786
X_4	0	0.0052
X_5	0.1823	0.1895

X_6	0.3365	0.3321

Table 3 shows the validation results for the Monte Carlo sampling for X_1–X_6. The small margin of error indicates that the sampling was conducted appropriately.

Table 4. Failure combination probabilities calculated via Monte Carlo sampling

AND-based method		OR-based method	
P_1	0.653113	P_1	0.653113
P_2	0.499853	P_2	0.499853
P_3	0.414818	P_3	0.414818
P_4	0.499394	P_4	0.499394
P_5	0.400770	P_5	0.400770
P_6	0.365010	P_6	0.365010
P_{12}	0.356083	P_{1+2}	0.796883
P_{13}	0.325077	P_{1+3}	0.742854
P_{14}	0.325855	P_{1+4}	0.826652
P_{15}	0.262151	P_{1+5}	0.791732
P_{16}	0.238425	P_{1+6}	0.779697
P_{23}	0.294584	P_{2+3}	0.620087
P_{24}	0.249782	P_{2+4}	0.749465
P_{25}	0.200238	P_{2+5}	0.700385
P_{26}	0.182716	P_{2+6}	0.682146
⋮	⋮	⋮	⋮
P_{123456}	0.017089	$P_{1+2+3+4+5+6}$	0.966714

Table 4 presents the failure combination probabilities calculated via Monte Carlo sampling based on AND and OR logic. As seen in Table 4, Monte Carlo sampling allows for the calculation of failure combination probabilities for both correlated and uncorrelated failures, enabling the computation of all combinations. As this method has been implemented in COREX, it can calculate failure probabilities through MVN integration and Monte Carlo sampling, as well as the failure combination probabilities for both correlated and uncorrelated failures.

4. Conclusions

In this study, the Monte Carlo sampling method was integrated into COREX, a software tool developed to calculate the failure probabilities of correlated seismic failures. This method offers the following advantages: (1) It enables calculations regardless of the number of correlated components. (2) It also enables calculations regardless of the distribution of component capacities. (3) It enables calculations even when the fault tree contains both correlated and uncorrelated failures.

By leveraging advantages (2) and (3), we calculated the failure probabilities and failure combination probabilities of correlated seismic failures, the failure probabilities of random uncorrelated failures, and the combined failure probabilities of both types of failures. Moreover, we incorporated these functions into the existing COREX program, which could previously calculate failure probabilities through only MVN integration. Thus, COREX can now perform calculations via both Monte Carlo sampling and MVN integration.

These methods are significantly more accurate and versatile than the previous approach used in seismic PSA, which involved calculating failure probabilities by converting highly correlated components into a single component.

Therefore, using the updated version of COREX will ensure more accurate core damage frequency calculations compared with traditional seismic PSA.

Acknowledgement

This work was supported by a Korea Foundation Of Nuclear Safety (KOFONS) grant funded by the Nuclear Safety and Security Commission (NSSC), Republic of Korea (No. RS-2022-KN067010 and RS-2021-KN050610).

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