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Kass-Steffey adjustment for CCF Alpha Factor model parameters and the moments problem

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1. Introduction

In the probabilistic safety assessment (PSA) for the nuclear power plants, the state-of-knowledge uncertainties for the model parameters are estimated based on the Bayesian framework. The Bayesian framework represents the uncertainties for the model parameters as a probability distribution and its hyperparameters. The current uncertainties in the Bayesian framework are estimated by updating prior uncertainties with observations. When there is plant-to-plant variability, empirical Bayes method has been applied to derive the prior knowledge in the PSA [1]. However, the empirical Bayes method uses point estimation for the hyperparameters and does not take into account the uncertainties in the hyperparameters. Kass and Steffey proposed an adjustment method to consider the uncertainties in the hyperparameters by approximating the moments of the current uncertainty distribution [2].

Common cause failure (CCF) events have been identified as significant risk contributors for the nuclear power plants. The CCF events and their probabilities are typically analyzed by alpha factor model. However, there have been little attentions for the uncertainties in the alpha factors. The alpha factors are typically estimated as point values even though the empirical Bayes and the Kass-Steffey adjustment have been applied to independent component failure data.

This paper derives mathematical formulas of the empirical Bayes method and the Kass-Steffey adjustment for the alpha factors. Furthermore, there is a discussion about the moment problem in the Kass-Steffey adjustment for the multivariate distribution.

2. Alpha factor model parameter estimation

The alpha factor model uses multinomial distribution to analyze CCF event observations. The alpha factors are occurrence probabilities of the multinomial distribution which categorizes the CCF event depending on the number of failure components in the system. Then, the CCF probabilities can be represented as follows:

$$p_i^{(k)} = \frac{1}{\binom{k-1}{i-1}} \alpha_i p_t \tag{1}$$

where k is the number of components in the system, i is the number of failure components, p_t is total component failure probability.

The uncertainty distribution for the alpha factors is typically the Dirichlet distribution because the Dirichlet distribution is conjugate prior for the multinomial distribution. The empirical Bayes estimates the hyperparameters by maximum likelihood method based on the marginal likelihood function which integrates the alpha factors over the parameter space.

The prior distribution with the estimated hyperparameters is updated by the observed data from the interested plant. Kass-Steffey adjustment method approximates the moments of the updated uncertainty distribution with a first order approximation.

$$E_{\boldsymbol{n}}[\alpha_i] \approx E_{\boldsymbol{n},\hat{\boldsymbol{A}}}[\alpha_i] \tag{2}$$

$$Cov_{\boldsymbol{n}}(\alpha_i, \alpha_j) \approx Cov_{\boldsymbol{n},\hat{\boldsymbol{A}}}(\alpha_i, \alpha_j) + DG_i^T \hat{\Sigma} DG_j \tag{3}$$

where $\hat{A}$ is the estimated hyperparameters by the empirical Bayes method, $\boldsymbol{n}$ is the observed data, $\hat{\Sigma}$ is inverse of observed information matrix, and $DG_i = \frac{\partial}{\partial \boldsymbol{A}}\left[E_{\boldsymbol{n},\boldsymbol{A}}[\alpha_i]\right]|_{\boldsymbol{A}=\hat{\boldsymbol{A}}}$.

In the commonly used two-parameter probability distribution, the hyperparameters can be uniquely determined by the expected value and the variance in Eq. (2) and Eq. (3). However, in the Dirichlet distribution, there are $(k-1) + k(k+1)/2$ numbers of equations in Eq. (2) and Eq. (3) even though the number of hyperparameters are k. Omar proposed a moment matching method for the Dirichlet distribution which uses $(k-1)$ equations in Eq. (2) and 1 equation in Eq. (3) [3].

$$A_i = E[\alpha_i]\left[\frac{E[\alpha_j](1-E[\alpha_j])}{var(\alpha_j)} - 1\right] \tag{4}$$

where α_j is a selected alpha factor that the variance in Eq. (3) is used. The estimated hyperparameters depends on the selected alpha factors. Furthermore, the hyperparameters can be estimated by the variances in Eq. (3) only. Therefore, it is required to consider which moments should be used.

3. Application

To demonstrate the effect of the moments, an example observed data set is used. Table I presents the example CCF data set. Table I is a randomly distributed data based on the data collected from 1997 through 2015 that maps to a 4-component system [4].

TABLE I. Observed common cause failure event data

Plant	n_1	n_2	n_3	n_4	n_t
1	1967.4	72.849	9.5476	1.6539	2051.5
2	3010.4	3.1528	3.9418	5.3393	3022.8
3	81.777	0.6515	11.651	7.5341	101.61
4	766.05	4.2852	0.7167	0.3456	771.40

Table II represents the estimated hyperparameters of plant 1 specific parameters that Kass-Steffey adjustment is applied with respect to the moments. Fig. 1 shows the uncertainty distributions for the plant 1 specific alpha factors. It is shown that the uncertainty distributions derived by expected values and single variance are similar each other. However, the uncertainty distribution derived by all the variances has more conservative results for α_2 to α_4.

Table II. Hyperparameters for plant 1 specific parameters

Plant	1			
Hyperparameter	A_1	A_2	A_3	A_4
$E[\boldsymbol{\alpha}], var(\alpha_1)$	1983.6	73.161	9.9649	1.9941
$E[\boldsymbol{\alpha}], var(\alpha_2)$	1983.3	73.148	9.9632	1.9937
$E[\boldsymbol{\alpha}], var(\alpha_3)$	1979.0	72.990	9.9417	1.9894
$E[\boldsymbol{\alpha}], var(\alpha_4)$	1956.1	72.147	9.8268	1.9665
$var(\boldsymbol{\alpha})$	2106.5	83.208	11.333	2.2938

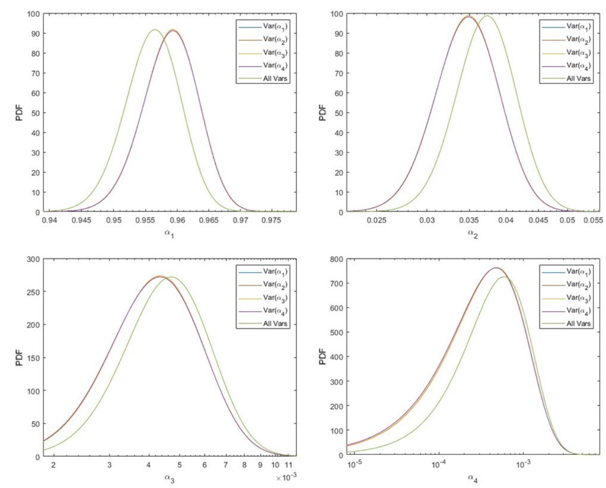

Fig. 1. Uncertainty for the plant 1 alpha factors w.r.t the moments

4. Conclusions and discussion

Common cause failure is a significant risk contributor in the probabilistic safety assessment for the nuclear power plants. Therefore, the uncertainties of the model parameters should be carefully analyzed. In this paper, the uncertainties of the CCF model parameters are analyzed based on the method which is typically used in the independent component failure event. However, contrary to distribution model for the independent component failure event, the equations have multiple solution depending on the selection of the moments. In this paper, the effect of the selection is also presented with an example data set. If expected values and single variance are used, the results are similar regardless of what the variance is. However, if the hyperparameters are estimated based on variances only rather than expected values, the resultant uncertainty distribution provides different insight. Because there is no consensus in the selection of moments, the effect of the moments should be considered when the uncertainty analysis for alpha factor model is performed.

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