Theoretical Approach to the Impact of Space Charge on Field Emission Models

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1. Introduction

Electron emission mechanism under strong electric fields has been explained by Fowler and Nordheim as a result of quantum tunneling of electrons[1]. Dyke and Dolan extended and re-generated formula which can be used as an approximation in numerous studies for materials with metallic properties, such as carbon nanotubes(CNTs)[2-4].

The field emission effect is typically described by a relationship between current density and electric field, which includes an unknown parameter known as the Fowler-Nordheim(FN) coefficient. Many studies conduct simulations using FN coefficients obtained from experimental results. However, when validating the field emission equation through experiments, discrepancies arise in certain regions due to the influence of space-charge effects. it is significant that the FN coefficients used in these simulations must exclude space charge effects. This paper proposes a method for deriving the pure FN plot coefficient by excluding the space-charge effects that impact field emission.

2. Methods

2.1 Field emission without space charge

The formula of field emission model is commonly expressed as follows

$$J = aE_0^2 \exp\left(\frac{-b}{E_0}\right) \qquad (1)$$

where E_0 represents the electric field at the cathode, J denotes the current density, and a and b are the FN coefficients, which is assumed to be $a = 1.54E-6\ [A \cdot V^{-2}], b = 6.83E+9\ [V^{-1} \cdot m]$

To compare with the measured values in experiments, the equation can be modified to express the relationship between voltage and current by considering the distance between the electrodes as d. If Φ represents the voltage across the electrodes, the traditional model, which does not account for space-charge effects, assumes that E is constant within the electrodes, leading to the linear relationship as $\Phi = E_0 d$ and formula can be modified as

$$\ln\left(\frac{J}{(\Phi/d)^2}\right) = -b\left(\frac{\Phi}{d}\right)^{-1} + \ln(a) \qquad (2)$$

From the equation (2), the FN coefficients can be determined by performing linear regression. However, if the space charge effect shields the electric field, Φ can no longer be calculated as $\Phi = E_0 d$, leading to discrepancies between the actual emission characteristics and the expected behavior.

2.2 Space charge effect calculation

By applying the charge continuity, Poisson equation and the principle of energy conservation to the two infinite parallel plate electrodes with an applied voltage $\Phi = \phi(d)$, the electric field in respect to position can be derived as

$$|E(r)| = \phi'(r) = \sqrt{4AJ\sqrt{\phi(r)} + E_0^2} \qquad (3)$$

Equation (3) can be solved analytically, leading to the following result [5],

$$6A^2J^2d - E_0^3 = \left(2AJ\sqrt{\Phi} - E_0^2\right)\sqrt{4AJ\sqrt{\Phi} + E_0^2} \qquad (4)$$

where A is defined as $\frac{1}{\epsilon_0}\sqrt{\frac{m_e}{2e}}$.

Figure 1 indicating solution of the implicit equation (4) without a trivial solution $J = 0$. This figure demonstrates the presence of a Space-Charge-Limit (SCL) current, indicating that it cannot exceed the first-order electric field, $E_{linear} = \Phi/d$.

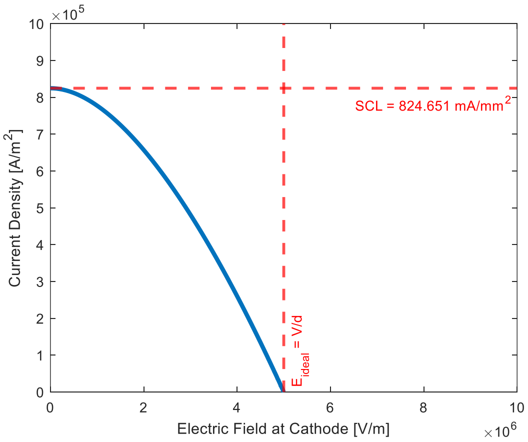

Fig. 1. Solution of the steady-state equation with space-charge where d = 1 mm, Φ = 5 kV.

Since the current under SCL value always has a steady-state solution, it can be concluded that when the current density is given for a specified voltage across both ends, a modified electric field exists. Therefore, by measuring the steady current flowing through the CNT diode, the effective electric field on the cathode surface within the tube can be determined from the graph in Figure 1.

2.3 Field emission model affected by space charge

Let's assume that the field emission model for the electric field at the cathode surface follows Equation (1). The current density can be calculated using the electric field at the cathode surface, and by combining this with Equation (4), the voltage Φ across the electrodes required to satisfy the given electric field can be determined.

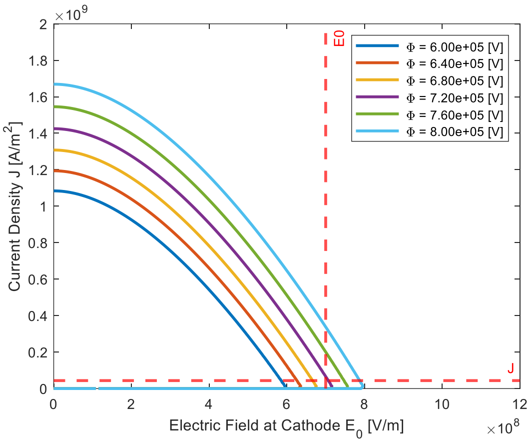

Fig. 3. Self-consistent solution with space charge of electric field and current density equation (4) in respect to voltage Φ

Figure 3 provides the example process for obtaining the steady-state voltage solution corresponding to a given electric field at the cathode. In the case of $E_0 = 7E + 8\,[V \cdot m^{-1}]$, the corresponding voltage is approximately $\Phi = 7.2E + 5\,[V]$, which is slightly higher than the ideal calculated value of $E_0 d = 7E + 5\,[V]$. Therefore, we can plot each graph corresponding to (E_0, J) and $(\Phi/\mathrm{d}, \mathrm{J})$ for examining space charge effect.

3. Results and Discussions

3.1 Field emission comparison result

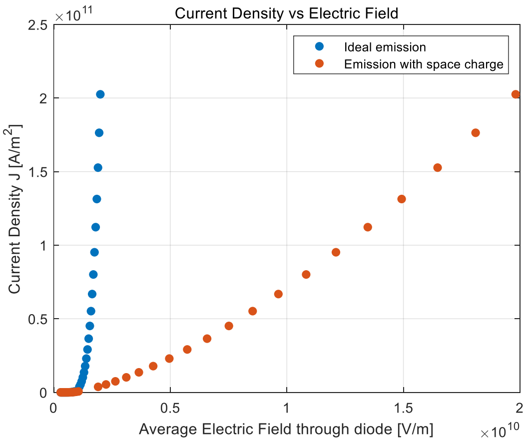

Fig. 4. Two field emission models plotted in respect to average electric field E_0 and current density J.

Figure 4 compares the ideal emission model with the model equilibrated by space charge effects. The coefficients required for the calculations were set using the FN coefficients a and b, with a gap d of 1 mm. The figure shows that in the presence of space charge, a higher voltage than what is predicted by the linear model is required to produce the same electric field strength at the cathode surface as in the ideal model.

3.2 Explicit and asymptotic solution

The equation (4) also can be expressed by explicit solution with respect to current density as the equation (5) shows.

$$J = \begin{cases} \dfrac{1}{2} J_{SCL} \left\{ 1 + \sqrt{1 - \dfrac{27}{4} \left(\dfrac{E_0 d}{\Phi} \right)^2 \left(1 - \dfrac{E_0 d}{\Phi} \right)} \right\} & \left(J \geq \dfrac{1}{2} J_{SCL} \text{ or } E_0 \leq \dfrac{2}{3} \dfrac{\Phi}{d} \right) \\ \dfrac{1}{2} J_{SCL} \left\{ 1 - \sqrt{1 - \dfrac{27}{4} \left(\dfrac{E_0 d}{\Phi} \right)^2 \left(1 - \dfrac{E_0 d}{\Phi} \right)} \right\} & \left(J \leq \dfrac{1}{2} J_{SCL} \text{ or } E_0 \geq \dfrac{2}{3} \dfrac{\Phi}{d} \right) \end{cases} \tag{5}$$

It is important to note that this equation changes its solution form when a characteristic electric field solution exists that satisfies $E_0 = (2/3)(\Phi/d)$. This form of equation (5) indicates that the asymptotic behavior of the solution differs under conditions of extremely strong or weak electric fields.

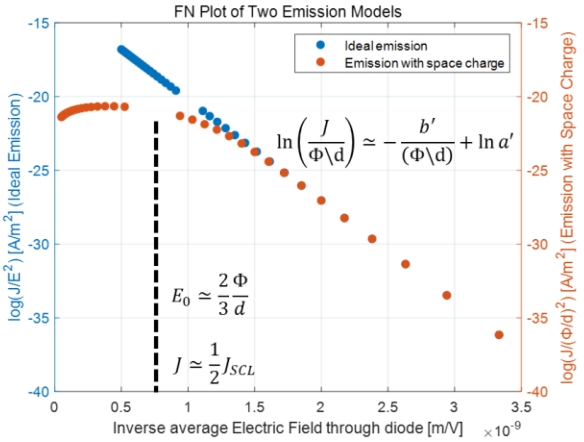

Fig. 5. Logarithm FN plot in two field emission model.

Figure 5 shows the relationship between the electric field E_0 at the cathode surface and the estimated current density J as predicted by the Field Emission model. When weak electric fields exists, modified FN formula can be deduced from asymptotic formula of the equation (5) shown as

$$\ln\left(\frac{J}{\Phi\backslash \mathrm{d}}\right) \simeq -\frac{b'}{(\Phi\backslash \mathrm{d})} + \ln a' \tag{6}$$

, where modified FN coeffcients a' and b' are

$$a' = a\left(1 - \frac{16}{27}\frac{J}{J_{SCL}}\right)^2,\ b' = b\left(1 - \frac{16}{27}\frac{J}{J_{SCL}}\right)^{-1} \tag{7}$$

Note that equation (7) implies that the space charge can be considered as a “negative” field enhancement factor, reducing the electric field at the cathode surface..

5. Conclusions

When experimentally verifying the field emission formula, expressed as the relationship between current density and electric field, it is essential to consider that the electric field and voltage may not be directly proportional due to the effects of space charge. In field emission simulations, where FN coefficient needed from the cathode surface is used, it is advisable to apply the FN coefficient that excludes space charge effects to achieve more reasonable results.

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